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VACHTSEVANOS G J ECE (404)894-6252

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Title: METHODOLOGY FOR FAULT DETECTION & IDENTIFCATION OF TURBO-JET ENGINE FAILURES

PROJECT ADMINISTRATION DATA

OCA contact: E. Faith Gleason 894-4820

Sponsor technical contact	Sponsor issuing office
ROBERT J. HANSEN, CODE 121	BONNIE DAMANSKI, CODE 1512B:BCD
(202)696-4315	(202)696-2598
OFFICE OF THE CHIEF NAVAL RESEARCH	OFFICE OF NAVAL RESEARCH
800 NORTH QUINCY STREET	800 NORTH QUINCY STREET
ARLINGTON, VA 22217-5000	ARLINGTON, VA 22217-5000
	FAX: (703)696-0993

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GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

TELEPHONE: (404) 894- 6252

November 27, 1989

Dr. James G. Smith (Code 1211)
Program Manager, Applied Research &
Technology Directorate
Office of Naval Research
Arlington, Virginia 22217-5000

Dear Dr. Smith:

Enclosed please find a summary report on research performed under ONR contract number N00014-89-3113 titled "A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design."

Since research activity on this project was initiated in September 1989, this report covers the reporting period from September 1, 1989 to December 1, 1989 and details some preliminary results while pointing out future research directions.

I will be pleased to provide you with any additional information required.

Very Sincerely,

George Vachtsevanos
Principal Investigator
Professor
School of Electrical Engineering

Enclosure

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Annual Letter Report for FY 89

A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design

(Contract No: N00014-89-J-3113)

Sponsor: **Office of Naval Research**
Applied Research and Technology Directorate

Principal Investigator: **Dr. George Vachtsevanos**
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Tel: (404)894-6252

December 2, 1989

A HYBRID ANALYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN

CONTRACT NO: N00014-89-J-3113

PI: Dr. George Vachtsevanos
Georgia Institute of Technology

In September 1989, the Office of Naval Research awarded contract No. N00014- 89-J-3113 to the Georgia Institute of Technology to develop fault-tolerant control strategies for large scale dynamical systems. Specifically, the technical issues under investigation are: Development of a structure-based modeling methodology of large scale systems which possesses features of structure flexibility, i.e. a component or subsystem may be deleted in a simple way that does not substantially disturb the global control strategy; development of robust failure detection and fault identification algorithms for single and multiple faults that are maximally sensitive to true failure conditions but insensitive to noise, thus reducing the possibility of false alarms; a technique to restructure the system dynamics by isolating the faulty components; a method that will lead to a structural control law which reconfigures the original system controller in order to meet the primary performance objective of guaranteed stability while the system is operating in a degraded mode; finally, these algorithmic developments are to be demonstrated on an actual physical system to be designated jointly by ONR and Georgia Tech.

The technical approach pursued to meet these objectives is outlined as follows: The underlying factor of the fault-tolerant methodology capitalizes upon structural features of the large scale system and employes a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. In the modeling area, the topological description of many complex dynamical processes may be cast in the form of structurally interconnected subsystems. The large scale system is composed of a number of linearly interconnected subsystems. In every subsystem, a control law is applied which consists of a local and a global feedback term, thus resulting in a two-level hierarchical control strategy. We examine first large scale systems which are described by linear state equations and exhibit this two-level hierarchical structure. The analysis will be extended eventually to nonlinear systems of a similar structure. A methodology for fault diagnosis is introduced which is based upon a combination of signal redundancy and detection/estimation procedures. An expert system, consisting of a multivalued rule base and an appropriate inferencing mechanism, assesses the aggregate of fault symptoms and determines the sensitivity of a failure condition. An innovative approach to multiple failure detection uses qualitative simulation techniques to decide on the impact of a component failure on other (healthy) system components. The proposed fault detection and identification scheme incorporates such additional functionalities as fault trending and the estimation of the "best" value of critical variables and parameters under failure states. Assume that the presence of a failure has been detected by the FDI algorithm and means for isolating the faulty or potentially faulty components are available, then the system restructuring function is undertaken by a supervisory controller which modifies the original (healthy state) description of the system by changing the appropriate entries of the local subsystem and interconnection matrices.

Finally, we propose the development of a two-level structural dynamic hierarchical approach to address the control reconfiguration problem of a faulted large scale system. The approach uses a structural state model, the Block Arrow Structure, which consists of N independent linear subsystems interconnected with a control common linear dynamic system. The objective is to find a time-invariant decentralized feedback controller which minimizes a quadratic cost functional and has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

A HYBRID ANALYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN
CONTRACT NO: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

Research Accomplishments

During this reporting period 9/1/89 - 12/1/89, the research team focused attention on the issues of modeling and fault detection and identification of dynamical Large Scale Systems(LSS).

1 LSS modeling

In the modeling area, a review of available techniques was completed and such schemes as aggregation and perturbation analysis, hierarchical and decentralized approaches and the descriptive variable method were assessed as to their suitability in fault tolerant control. The basic objective here is to develop mathematical modeling tools for large scale systems that possess attributes of modularity and structural flexibility so that a failed component or subsystem may be easily isolated without seriously affecting the global behavior of the system. More specifically, the failed system is to be restructured and the control actions reconfigured so that stability(survivability) is assured for the duration of the emergency.

We are developing a two-level structural dynamic hierarchical approach to address the control reconfiguration problem. We seek a structural control law and, by effectively combining information with control action, to move closer to create a structural intelligent control concept. We outline below the innovative features of this approach.

Suppose that the large scale system consists of l interconnected nonlinear subsystems of the form

$$\dot{z}_i = f_i(z_i, t) + g_i(z_1, \ldots, z_l, t) \qquad i = 1, 2, \ldots, l \tag{1}$$

where

$$z_i \in R^{n_i} \text{ is the state vector of the } i\text{th subsystem}$$

$$f_i : R^{n_1} \times J \to R^{n_1},$$

$$g_i : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_l} \times J \to R^{n_i}$$

Each subsystem, when isolated, is given by

$$z_i = f_i(z_i, t) \tag{2}$$

The function $g_i(z_1, \ldots, z_l, t)$ represents the interconnections of the ith subsystem with the remaining subsystems.

In every subsystem of the form (1), a control law is applied which consists of a local and a global feedback term. Specifically, for the ith subsystem (1), the control input is

$$u_i(t) = u_i^l(t) + u_i^g(t) \tag{3}$$

If the controllers are linear, then they have the form:

$$u_i^l(t) = k_{ii} z_i(t) \tag{4}$$

$$u_i^g(t) = \sum_{j=1, j\neq i}^{l} k_{ij} z_j(t) \tag{5}$$

where $k_{ij} \in R^{m_i \times m_j}$ and $u_i(t) \in R^{m_i}$.

A two-level hierarchical control is obtained by assigning proper interconnections and feedback loops to the large scale system. At the lower level we have l subsystems, S_i, $i = 1, 2, \ldots, l$, which are described in state space by the equations:

$$S_i : \dot{z}_i(t) = f_i(z_i, t) + h_i(u_i, t) + f_{i0}(z_0), \quad i = 1, 2, \ldots, l \tag{6}$$

where

$$h_i : R^{m_i} \times J \rightarrow R^{n_i}$$

$$f_{i0} : R^{n_0} \times J \rightarrow R^{n_i}$$

and $z_0 \in R_{n0}$ is the state vector of the coordinator system in the upper level. The upper level is occupied by the coordinator system S_0 described by the state space model:

$$S_0 : \dot{z}_0(t) = f_0(z_0, t) + \sum_{i=1}^{l} f_{0i}(z_i, t) + h_0(u_0, t) \tag{7}$$

where $u_0(t)$ is the control law of the coordinator and

$$h_0 : R^{m_0} \times J \rightarrow R^{n_0}$$

The structural implications of the two-level model on fault-tolerant control become evident when we consider a linearized version of the system dynamics. Here, we use a structural state model which consists of N independent linear subsystems $S_1, S_2, \ldots, S_N$ interconnected with a control common linear dynamic subsystem S_0. The mathematical model of the time- invariant large scale system, in a decomposed form, is

$$S_i : \dot{z}(t) = A_i z_i(t) + B_i u_i(t) + A_{i0} z_0(t) \tag{8}$$

$$\text{with} \qquad z_i(t_0) = z_{i0}, \quad z_0(t_0) = z_{0_0}$$

$$S_0 : \dot{z}_0(t) = A_0 z_0(t) + B_0 u_0(t) + \sum_{i=1}^{N} A_{0i} z_i(t) \tag{9}$$

where $z_i(t) \in R^{n_i}, z_0(t) \in R^{n_0}$ and $u_i(t) \in R^{r_i}, u_0(t) \in R^{r_0}$ are the state and control vectors for subsystems S_i and S_0, respectively.

The standard overall description of (8),(9) is

$$\dot{z}(t) = Az(t) + Bu(t), \qquad z(t_0) = z_0 \tag{10}$$

$$A = \begin{bmatrix} A_1 & 0 & \dots & A_{10} \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & A_l & A_{l0} \\ A_{01} & \cdots & A_{0l} & A_0 \end{bmatrix} \in R^{n\times n}, \qquad B = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & B_l & 0 \\ 0 & & 0 & B_0 \end{bmatrix} \in R^{n\times r}$$

Since the dynamics matrix (10) consists of Block elements arranged in an Arrow Structure, the system (10) is called a Block Arrow Structure(BAS) decentralized large scale system.

By appropriately defining a quadratic cost functional, a time-invariant BAS decentralized feedback controller may be designed which has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

We are currently investigating critical properties of LSS, from the fault- tolerant control point of view, that include restructurability and reconfigurability. Specifically, we have identified the following key properties for restructurable/reconfigurable systems:

- Flexibility: based upon the strength of subsystem interconnections (interlevel or intralevel).
- Stabilizability: in terms of structural controllability and structural observability concepts.

Our investigations thus far indicate that the modeling approach pursued is representative of a large class of engineered systems, that, because of critical performance requirements, require some degree of fault tolerance in the control design.

2 Fault Detection and Identification

A major effort has been undertaken, during this initial phase of the project, to develop an integrated methodology to fault detection and identification that capitalizes upon a combination of conventional techniques and artificial intelligence. New and innovative concepts are introduced in the FDI approach that maximize symptomatic evidence, account for sensor and system uncertainties and combine failure data even when the latter are of a conflicting nature.

FDI techniques are considered for sensor, actuator and component failures. Both single and multiple faults are treated. The research objective is to develop a systematic and thorough FDI procedure that is maximally sensitive to failures while avoiding false alarms. The approach is systematic because it relies on a modular architecture to (a) trigger efficiently the FDI routines, (b) validate sensor data, (c) combine failure evidence from such diverce sources as analytic redundancy, dectection/estimation theory and limit checking, (d) utilize expert system tools and Dempster-Shafer evidential theory to manage uncertainty and assess the symptomatic evidence, i.e. detect and identify faulty components, and (e) finally, provide trending information as well as the best value of critical variables and parameters for monitoring and control purposes.

This research team has previously developed FDI procedures based upon parity space by utilizing analytic redundancy. We will highlight below only some recent developments in detection/estimation and evidential theory. Symptoms derived from this technique will augment the failure evidence available from the parity space procedure to assure a robust FDI approach.

2.1 New Directions in FDI

Decision strategies for FDI based on a Bayesian approach were reviewed. Bayes' estimation theory using minimum mean squared estimation and joint detection/estimation(JDE) procedures as well as test methods such as the M-hypothesis, generalized likelihood and probability ratio tests were critically assessed as to their effectiveness in FDI and their computational cost and complexity. Trade-offs were considered and robustness issues were addressed. Such Bayesian methods as the multiple model approach were examined in detail using a general linear discrete-time multiple dynamic model. Here, failure modes are modeled as switching parameters and it is assumed that the number of failure models is finite and compact. The ith model of the linear process is represented by

$$x_i(t+1) = A_i(q,\theta_i)x_i(t) + B_i(q,\theta_i)u(t) + w_i(t) \tag{11}$$

$$y(t) = H_i(q,\theta_i)x_i(t) + v_i(t) \tag{12}$$

where $u(t), y(t)$ are the actuator input and the measured output, respectively; $x_i(t)$ is the state of the ith model; A_i, B_i, and H_i are the associated system matrices; θ_i is the parameter vector of the process which may be time-varying; q is the shift operator defined by $q^{-1}x(t+1) = x(t)$; $w_i(t) \propto N[0, Q_i]$ is the white Gaussian process noise; $v_i(t) \propto N[0, R_i]$ is the white Gaussian measurement noise.

The minimum mean squared estimate(MMSE) can be represented by

$$\hat{x}^{MS}(t|t) = E(x(t)|Y(t)] = \arg\min_{x(t)} E(x(t) - \hat{x}(t|t))(x(t) - \hat{x}(t|t))^T$$

$$\text{where } Y(t) = [y(t), y(t-1), \ldots, y(t-n)]^T$$

The ith Kalman filter equations are easily derived from these assumptions. The computational shortcomings of both optimal and suboptimal procedures following these Bayesian methods are well documented in the techniical literature. The statistical ratio test paradigms may be circumvented by using a Markov chain with an associated transition probability matrix. Consider a joint detection/estimation algorithm based on this approach. Each state of the model may be described by an integer index $s(t)$. We define a windowed Markov chain state sequence $I_i(t)$ by

$$I_i(t) = \{s(t) = i, s(t-1), \cdots, s(t-n+1)\} \tag{13}$$

where the window size is n, $s(t) \in \{1, 2, \cdots, M\}$, and M is the total number of stochastic models. $s(t)$ determines or identifies the structure of the failure model at time t.

Suppose that, given a stochastic process $s(t)$ with a finite number of possible integers, the associated Markov chain exists such that

$$p(s(t)|s(t-1), \cdots, s(0)) = p(s(t)|s(t-1)) = \pi(s(t), s(t-1)) \tag{14}$$

$$p(t) = \Pi p(t-1) \tag{15}$$

where $p(\cdot) \in R^M$ stands for probability of the state sequence, $\Pi = \{\pi(i,j)\} \in R^{M \times M}$ for all states $\{s(0), \cdots, s(t)\}$ and for all $t \geq 0$. $\pi(i,j)$ is referred to as the transition probality from the model state j to the next state of the model i. Now, we can consider state estimation and structural detection for this partitioned systems.

Given the probability density function $f(\cdot)$ of the state $x_i(t)$, the estimate of the ith model is

$$\hat{x}(t|t) = E[x_i(t)|Y(t), I_i(t)] \tag{16}$$

and the detection/estimation approach can be summarized as follows:

- **state estimation:**

$$\hat{x}(t|t) = \sum_{i=1}^{M} \hat{x}_i(t|t)p(I_i(t)|Y(t)) \tag{17}$$

$$\text{where } p(I_i(t)|Y(t)) = \frac{f(Y(t)|I_i(t))p(I_i(t))}{\sum_{j=1}^{M} f(Y(t)|I_j(t))p(I_j(t))} \tag{18}$$

- **structural detection**

$$p(i|Y(t)) = \sum_{\forall I_i(t)} p(I_i(t)|Y(t)) \tag{19}$$

where $f(\cdot|\cdot)$ is a conditional pdf of $Y(t)$ given the sequence $I_i(t)$.

It is noted that this state estimation approach requires a bank of Kalman filters for the fixed window and the number of states must be the same in each failure model. These problems may be alleviated by employing an alternate approach which is based on multiple parametric models. The proposed approach relies on the error equation of auto-regressive moving average (ARMA) models. The ith model of the system is described by

$$A_i(q,\theta) = q^{-d}B_i(q,\theta)u(t) \tag{20}$$

where $A_i(q,\theta)$ is a monic polynomial, $B_i(q,\theta)$ is a stable polynomial, and d is the system delay. If the process is considered to be linear, then from the Kalman filter interpretation for the time-varying system, the parameter estimate $\hat{\theta}_i(t)$ can be calculated as follows:

$$\hat{\theta}_i(t) = E[\theta_i(t)|Y(t)] = \hat{\theta}_i(t-1) + K_i(t)\{y(t) - \phi_i(t)^T\hat{\theta}_i(t-1)\} \tag{21}$$

where $\hat{\theta}_i(t) = [a_1, \cdots, a_n; b_1, \cdots, b_m]^T$, $\phi_i(t) = [-y(t-1), \cdots, -y(t-n); u(t-1), \cdots, u(t-m)]^T$, and $K_i(t), P_i(t)$ are the familiar Kalman gain and covariance matrices, respectively.

Now, parameter estimation and structural detection may be accomplished by using the following computations:

- **parameter estimation:**

$$\hat{\theta}(t) = \sum_{i=1}^{M} \hat{\theta}_i(t)p(I_i(t)|Y(t)) \tag{22}$$

$$\text{where } p(I_i(t)|Y(t)) = \frac{f(Y(t)|I_i(t))p(I_i(t))}{\sum_{j=1}^{M} f(Y(t)|I_j(t))p(I_j(t))} \tag{23}$$

- **structural detection**

$$p(i|Y(t)) = \sum_{\forall I_i(t)} p(I_i(t)|Y(t)) \tag{24}$$

The generalized likelihood ratio (GLR) test is shown to be appropriate here to execute the detection and estimation strategy.

A new strategy to the detection/estimation problem is based on possibilistic methods and is implemented using expert system tools. A new failure detection scheme is obtained by utilizing Zadeh's fuzzy set theory in constructing a decision-making rule base. Two possible approaches are considered: The first one involves the design of membership functions for the a posteriori probability or likelihood ratio of each failure model followed by a determination of the fuzzy relational mapping for the rules. The second refers to the collection of all measures of the failure modes in evidence, which may be functions of standardized normal variables, to evaluate statistical ratio tests for them, and finally, to fire those rules that satisfy a particular failure mode.

For, the fuzzy failure detection scheme, some appropriate measure such as the a posteriori probability or likelihood of each failure model is used. To illustrate the point, the fuzzy rulebase consists of rules of the form:

- If 'p1 is large' and 'p2 is small', then 'prob model is 1' or
- If 'r1 is large' and 'r2 is small', then 'likely model is 1'

where p1 and p2, r1 and r2 are the a posteriori probabilities and likelihood ratios for models 1 and 2, respectively. The goal here is to find the fuzzy relational mapping or fuzzy relational matrix for failure detection. The advantages of this methodology are a significant reduction in decision errors by using linguistic variables and a considerable saving in computational time through parallel (disjunctive) rule implementation.

The coupling of the new FDI scheme to the multiple parametric model approach is achieved as follows: For each kth component of the parameter vector θ_i, the parametric space is divided into several crisp cells that represent some specific symptoms or failure evidence in the associated parameter. These cells can be chosen according to probabilistic specifications such as mean, variance, etc. Next, under real time parameter estimation conditions, these values for each model in a particular cell can be used to determine the failure mode at hand and infer structural detection through the output of the fuzzy rulebase.

Failure detection may also be accomplished using production rules. In this case, the decision depends on a description of crisp failure situations under the assumption that all possible failure modes are completely specified.

A typical rule in the production rulebase for failure detection of impulse lines is

- If 'reading is full-scale', then 'line is blocked'.

This last methodology holds a great deal of promise when all possible failure modes are considered.

Presently, simple dynamical systems are modeled and both Bayesian and possibilistic approaches are simulated with the intent of arriving at the optimum detection/estimation algorithm that satisfies the performance criteria and yet is computationally efficient.

The final thrust of our current research activity is concerned with the study of the Dampster-Schafer mathematical theory of evidence and its possible implications to the FDI problem. Dempster-Schafer theory provides an excellent tool for combining several pieces of evidence from a knowledge source. We intend to use this tool in order to combine failure symptomatic evidence resulting from the application of parity space, possibilistic detection/estimation and limit checking techniques. The power of D-S theory is in the representation of subjective(as opposed to probabilistic) uncertainty, mainly by virtue of its expression of ignorance. The groundwork for applying D-S theory to the FDI problem has been laid and we expect to report on this task in the near future.

In summary, substantial progress has been made in conceptualizing and developing new and innovative techniques to address the FDI and modeling problems of dynamical large scale systems that undergo component or sensor failures. The distinguishing charasteristic of our approach is a blend of analytical and symbolic representations. Computational tools are used to develop, simulate and demonstrate the power and effectiveness of these techniques. For the remainder of this fiscal year, we will continue to pursue this research direction so that we may produce a viable and generic package of FDI routines that can be applied to any complex physical system.

A HYBRID ANALYTICAL/INTELLIGENT
APPROACH TO FAULT-TOLERANT
CONTROL SYSTEM DESIGN
CONTRACT NO: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

Presentations and Technical Contributions

1. In September 1989, a presentation was given to the staff of NASA's MSFC at Huntsville, Alabama by Drs. Vachtsevanos and Arkin on possible applications of fault-tolerant control technology to the space station program.

2. On 11/9/89, Dr. G. Vachtsevanos was invited to present a seminar to the technical staff of Rockwell International Corp., the Science Center at Thousand Oaks, California on fault-tolerant control techniques. He held technical discussions with Rockwell personnel on issues of fuzzy logic and the application of fault-tolerant control techniques to aerospace systems.

3. In November 1989, discussions were held and a statement on fault- tolerant control was mailed to Nothrop Corp.

4. Interest on our fault-tolerant control activities has been expressed by the Federal Aviation Administration. A position paper on this subject was mailed to FAA research staff.

5. In June 1989, Dr. Vachtsevanos chaired an invited session on Intelligent Control at the 1989 Automatic Control Conference and presented a paper entitled "Fault Tolerant Control of Dynamical Large Scale Systems". A copy of the paper is attatched to this report.

6. In December 1989, Dr. Vachtsevanos will present a paper, co-authored by Dr. H. Kang, on "Model Reference Fuzzy Control" at the 28th IEEE Conference on Decision and Control, Tampa, Florida.

7. A paper entitled "Stability and Robustness of a Nonlinear Fuzzy Controller Based on the Phase Portrait Assignment Algorithm", co-authored with H. Kang, has been submitted for publication to the IEEE Trans. on Systems, Man and Cybernetics.

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Project Participants

The parincipal Investigator of this project is

Dr. George Vachtsevanos, Professor of Electrical Engineering at the Georgia Institute of Technology, Atlanta, Georgia.

Dr. Ronald Arkin, Assistant Professor at the School of Information and Computer Science, the Georgia Institute of Technology, serves as a co- investigator to the project.

Dr. Hoon Kang, a recent Ph.D. graduate at the Georgia Institute of Technology, is participating in project activities as a post-doctoral fellow.

Three graduate students in the School of Electrical Engineering at Georgia Tech are currently receiving partial support from this project.

FAULT-TOLERANT CONTROL OF DYNAMICAL LARGE SCALE SYSTEMS

George Vachtsevanos & Young-Tae Kim
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250

Manolis Christodoulou
Technical University of Crete
Chenia, Greece

ABSTRACT

A fault-tolerant design procedure for large scale engineered systems is introduced. The proposed methodology is based on a hybrid approach that capitalizes upon beneficial attributes of conventional systems and control-theoretic techniques, as well as methods of artificial intelligence. The paper focuses on a fault propagation algorithm that assesses quickly the impact of a failure on other healthy components and subsystems. Qualitative reasoning techniques are employed to determine the state transitions. An example from a major subsystem of the space station is used to illustrate the algorithmic developments.

1. INTRODUCTION

Modern dynamical systems, such as aircraft and space vehicles, nuclear power plants, and many industrial and manufacturing processes, are complex systems that place a heavy burden on performance monitoring, status evaluation, and control strategies. As the technological sophistication of these systems increases, they are required to be more fault tolerant so that the system availability is kept high at the maximum possible rate.

The problem of designing fault-tolerant control systems has been addressed in the recent past in terms of two general and fundamental approaches [1-5]. The first one relies upon modern control-theoretic techniques to recast the fault tolerant system design into a classical control problem, while the second is based on decision-theoretic concepts related to the field of artificial intelligence in order to provide partial answers to the problem. A critical review of both AI-based and control-theoretic techniques for fault-tolerant control reveals that there is no unified methodology to integrate those diverse issues of system modeling, fault detection and isolation, fault propagation, system restructuring, and controller reconfiguration.

This need is addressed by introducing a problem-focused system integration philosophy that capitalizes upon a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. AI techniques allow a marked reduction of the set of hypotheses, thus permitting traditional control-theoretic approaches to be applied efficiently.

When one or more failures or abrupt parameter changes are detected, the system enters an emergency mode. The proposed fault-tolerant control system design procedure takes over in that event and entails the following tasks (Figure 1):

- Fault Detection and Identification (FDI)
- Fault Propagation (FP)
- System Restructuring
- Controller Reconfiguration.

A methodology for fault diagnosis of large scale dynamical systems is introduced which is based upon a combination of signal redundancy techniques and fuzzy logic.

Suppose that a fault or parameter deviation occurs in some subsystem of a large scale process. Then the deviation propagates to adjacent subsystems before the faulty component is isolated. The fault might propagate through an adjacent subsystem without causing any further fault, it might be absorbed in this subsystem (depending on its transition pattern), or it might cause this subsystem to fail. When a faulty behavior or a marked deviation in the system's behavior is observed, then it will be necessary to track the current behavior pattern of the system and its major subsystems and take fault-tolerant action before a permanent failure occurs and cripples the system. This paper addresses this issue by recognizing the unconventional nature of the problem. For a fault propagation model to be effective, it must be capable of predicting the impact of a failure on other system components well in advance before the actual effect is realized. Only then may the system have opportunity to restructure itself by isolating failed or potential faulty subsystems so that overall system survivability is assured.

2. FAULT PROPAGATION

A fault propagation model is defined here to be a representation of the propagation of input system variable deviations that originate at the faulty unit to other system components. A fault is initiated in a unit which by definition is unhealthy, propagates through a set of units which are generally healthy (although in some units, an additional enabling fault is a condition for further propagation), and terminates in a unit or units which are thereby rendered unhealthy.

Fault Tree Analysis (FTA) [6] has been classically used to discover the paths to failure in complex systems.

In a different approach [7], it is possible to model the subsystems of the large scale system as nonvariant stochastic automata with a transition matrix between the input and output states.

The proposed approach to modeling of fault propagation begins with a system structural diagram and decomposes it into its constituent units or components; it then represents the system as a causal network via a qualitative simulation.

Qualitative Reasoning (QR) has been recently an area of intense research activity within the artificial intelligence and cognitive sciences [8-11]. Investigators in these two fields have studied qualitative causal reasoning models of physical systems or mechanisms.

Considering the nature and complexity of large scale dynamical systems, Kuipers' approach to qualitative simulation seems to be the most appropriate. We propose to augment his original approach by including explicitly constraints that represent conservation of energy via scalar Lyapunov functions. This addition eliminates most spurious behaviors and it is hoped that will lead to predicting the single correct behavior of the faulted system.

2.1 QUALITATIVE SIMULATION

The goal of qualitative physics is to provide a theoretical framework for understanding the behavior of physical systems.

From the qualitative structure of the physical system, Qualitative Simulation (QSIM) uses a local propagation strategy to predict the qualitative behavior of a mechanism. The qualitative structure of a mechanism is described by a set of qualitative constraints. Each constraint specifies a relationship that holds between two or more functions of the mechanism. Each function has a qualitative state consisting of a qualitative value and direction of change. A qualitative value can be a landmark value such as 0 or it can be an open interval between two landmark values. At first, QSIM determines a complete initial state by propagating known qualitative values and directions of change (given the initial condition) through the given constraints before proceeding to predict possible behaviors. With a complete set at a distinguished time point, QSIM uses a strategy of proposing and filtering sets of qualitative transitions to predict all possible subsequent states in the following open time interval. A number of real valued parameters, which vary continuously over time, characterize a physical system. Each parameter can be considered as a function $f:[a,b] \to R^*$, where $R^* = [-\infty,\infty]$, the extended real number line. The qualitative behavior of f on $[a,b]$ is the sequence of qualitative states of f:

$$QS(f,t_0), QS(f,t_0,t_1), QS(f,t_1), QS(f,t_1,t_2)\ldots, QS(f,t_{n-1},t_n), QS(f,t_n)$$

Constraints holding between parameters in the structural description of the system serve to limit the possible combinations of qualitative behavior. Constraints are expressed as predicates rather than as functions for two reasons: First, they will be used as predicates in the QSIM algorithm to test the consistency of sets of qualitative values. Second, if a constraint were to be treated as a function, it is unclear how to define precisely the function's range. At large there are two types of constraints which are used in QSIM:

- Arithmetic constraints
- Qualitative function constraints.

An example of an arithmetic constraint is:

- $ADD(f,g,h)$: iff $f(t) + g(t) = h(t)$ for every $t \in [a,b]$.

Qualitative function constraints are defined as follows:

- M^+ is a two-place predicate on reasonable functions $f,g:[a,b] \to R^*$.

$M^+(f,g)$ is true, iff $f(t) = H(g(t))$ for all $t \in [a,b]$, where H is a function with domain $g([a,b])$ and range $f([a,b])$, differentiable with $dH(x)/dt > 0$ for all x in the interior of the domain. Similarly, M^- can be defined except that $dH(x)/dt < 0$.

We propose additional constraints related to energy functions. A scalar Lyapunov function, associated with a subsystem of the engineered large scale system, is viewed as an energy function.

Let $V(x)$ be a candidate Lyapunov function with $\dot{V}$ its time derivative. Qualitative constraints may be imposed by asserting the following conditions:

(1) If $\dot{V} < 0$, the system tends to be stable and thus its energy decreases.

(2) If $\dot{V} = 0$, the system is marginally stable and its energy remains constant.

(3) If $\dot{V} > 0$, the system may be unstable, thus its energy may be increasing.

2.2 FROM STATE SPACE TO QUALITATIVE SPACE

A large scale engineered system is considered to be compared of a number of linearly interconnected linear or nonlinear subsystems. A control law is applied to each subsystem which consists of a local and a global feedback term. Let us focus attention first to a typical subsystem without regard to interconnections. We will define some useful concepts for linear systems only although the extension to the nonlinear case is straightforward except for the number and the nature of the qualitative constraints.

Consider the state space model of a controllable system described as:

$$\dot{x} = Ax + Bu$$
$$y = Cx \qquad (1)$$

where x is an n-dimensional state vector and A,B,C are matrices with appropriate dimensions.

With the controllability assumption, the system equations may be written in the controller canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u \quad (2)$$

Or, more explicitly as:

$$\begin{aligned} x_1 &= x \\ x_2 &= \dot{x}_1 \\ &\vdots \\ x_n &= \dot{x}_{n-1} \\ \dot{x}_n &= -a_1 x_n - a_2 x_{n-1} - \cdots - a_n x_1 + u \end{aligned} \quad (3)$$

Equations (3) define n+1 qualitative variables and their derivatives. We seek the qualitative behavior of these variables starting from some given initial values. The qualitative value of a variable is either a symbolic or numeric value and its direction of change, i.e. increasing, descreasing, or steady.

A set of constraint equations defined on the basis of constituent component models and their interconnections (the state equations), as well as functional and global (energy) relations, specify the transition from state space to qualitative space. The qualitative operations do not define a mathematical field. Only a few qualitative solutions or combinations are consistent with the equations. The set of qualitative constraints is intended to limit the behavioral modes or interpretations of the system to the correct ones.

From the above relations, an appropriate set of qualitative constraints may be derived as:

DERIV(x_1,x_2),DERIV(x_2,x_3),...,DERIV(x_{n-1},x_n).

The last equation of the set is expressed qualitatively as:

DERIV(x_n,K)

where K is composed of addition constaints as depicted in Figure 2a. A total of n-derivative, n-multiplicative, and n-additive constraints are required to convert from the state space model to the qualitative model, as shown in Figure 2b.

The tree of behavioral modes can be pruned by the consideration of functional and energy-type constraints. A transformation of the state model to a Schwartz matrix form provides the most appropriate setting for the scalar Lyapunov function as a qualitative constraint criterion.

Similar qualitative state transition diagrams can be derived for multivariable systems.

2.3 FAULT PROPAGATION TO OTHER MAJOR SUBSYSTEMS OF THE LSS

When a system undergoes structural perturbations (faults), the interactions between constituent subsystems are expected to have major effects on stability. In assessing the impact of propagating faults on other (healthy) subsystems, we will use the following assertion: If the subsystems are stable to a degree larger than the strength of the interconnection, then the composite system is stable.

Consider a large scale unforced system described by:

$$\dot{z} = f(z,t) \quad (4)$$

which is assumed to consist of N-subsystems:

$$\dot{z}_i = f_i(z_i,t) + g_i(z,t) \quad , \quad i = 1,\ldots,N \quad (5)$$

and assume that the origin e = 0 is an equilibrium state.

When the i-th subsystem is completely decoupled, then:

$$\dot{z}_i = f_i(z_i,t) \quad (6)$$

i.e. the i-th subsystem is totally isolated from the rest of the system.

Assume a Lyapunov function $v_i(z_i,t)$ for the isolated subsystems (6). Use a weighted sum:

$$r(z,t) = \sum_{i=1}^{N} c_i v_i(z_i,t) \quad (7)$$

as a potential Lyapunov function for the large scale system. The c_i's are positive constants.

In stability studies of large scale systems, we define a positive definite function $w_i(z_i)$ and we check for:

$$\dot{v}_i(z_i,t) \lessgtr -a_i\{w_i(z_i)\}^2 \quad (8)$$

The constant a_i is considered as the degree of stability in view of the fact that it gives a lower bound for the decrease in $\dot{v}$ with respect to $w_i^2(z_i)$. Next we check the interconnection terms:

$$|g_i(z,t)| \lessgtr \sum_{j=1}^{N} b_{ij} w_j(z_j) \quad (9)$$

where the nonnegative constants b_{ij} indicate the interconnection strength in the sense that they provide an upper bound with respect to $w_j^2(z_j)$. Therefore, a qualitative measure of the impact of a faculty j-th subsystem on the behavior of the i-th subsystem may be obtained by comparing the interconnection strength:

$$|g_i(z,t)| \text{ to } \dot{v}_i(z_i,t) \ .$$

In terms of the constants a_i and b_{ij}, we may form the N x N matrix $E = (e_{ij})$, where $e_{ii} = a_i - b_{ii}$, $e_{ij} = -b_{ij}$, $i \neq j$, and compute the leading principle minors of E; a check on their sign and relative magnitude will determine again the fault propagation impact. The evolution of the time derivatives of the Lyapunov functions and their relative values compared to the

interconnections strength over the first distinguished time points after the occurrence of a fault will provide the information required to eliminate spurious behaviors.

3. AN EXAMPLE

The proposed qualitative simulation approach is demonstrated, in outline form, with an example. The test system is the Thermal Control System of the space station's common module. Jointly with the Boeing Aerospace Company, a hierarchical control algorithm and fault diagnostic routines for this major large scale system of the space station have been developed [12]. The algorithms have been implemented and successfully tested on a subscale laboratory prototype constructed by Boeing. In our example, a fault condition is hypothesized and propagated through the system's major components. If certain threshold or tolerance limits are exceeded, then the affected subsystems are isolated, before the hierarchical controller is reconfigured, in order to preserve the system integrity. A simplified schematic of the Thermal Control System is shown in Figure 3.

The qualitative simulation begins with a structural description which consists of a set of constraints holding among time varying, real valued parameters. The behavioral description consists of a finite set of time points representing the qualitative distinct states of the system and values for each parameter at each time point. This description consists of the ordinal relations (i.e. >, <, and =) holding among the different values in the behavioral description. Certain values are thermal distinguished or landmark values and play a special role in the qualitative simulation. Beginning with a set of assertions about the initial state of the system, the process takes place through the propagation/prediction cycle.

Algebraic relations of the system may, therefore, be represented as qualitative constraints. Let us demonstrate the procedure by considering the example of component HX-2 (Figure 4a). The algebraic equations for HX-2 are:

$$\begin{aligned} Q_2 &= \dot{m}_2 C_v \Delta T_2 \\ \Delta T_2 &= T_{o2} - T_{12} \\ \dot{m}_2 &= (1-\beta)\,\dot{m}_0 \\ T_{13} &= T_{o2} - (1-\beta)\,\Delta T_2 \end{aligned} \tag{10}$$

where β denotes the bypass control valve variable (i.e. $\beta = 0$ means closed valve and $\beta = 1$ means fully opened valve). And constraints for HX-2 may be written as:

$$(C1)\Delta T_2 < 5^\circ F \quad , \quad (C2) T_{12} = (70 \pm 2.5)^\circ F \tag{11}$$

We assume that C_v is constant. Qualitative constraints relations for HX-2 are as follows:

$$\begin{aligned} &(QC1) M^-(\beta,\dot{m}_2) \quad , \quad (QC2) MULT(\dot{m}_2,\Delta T_2,Q_2) \\ &(QC3) MULT(1-\beta,\Delta T_2,T) \quad , \quad (QC4) ADD(T_{12},T,T_{13}) \end{aligned} \tag{12}$$

A qualitative constraints model for HX-2 is shown in Figure 4b. We can begin the process with active states at the distinguished time $t = (t_0,t_1)$. We assume that the qualitative states of each variable at time t are as follows:

$$QS(\beta,t_0,t_1) = [(0,1),dec] \tag{q1}$$

$$QS(\dot{m}_2,t_0,t_1) = [(0,\dot{m}_0),inc] \tag{q2}$$

$$QS(Q_2,t_0,t_1) = [(0,Q_2^*),inc] \tag{q3}$$

$$QS(\Delta T_2,t_0,t_1) = [(0,5),inc] \tag{q4}$$

$$QS(T,t_0,t_1) = [(a,b),inc] \tag{q5}$$

$$QS(T_{12},t_0,t_1) = [67.5,72.5),std] \tag{q6}$$

At the distinguished time $t = (t_1,t_2)$, these states undergo the following transitions:

$$QS(\beta,t_1,t_2) = \begin{bmatrix} (0,\ 1),dec \\ (0,\ 1),inc \\ (0,\ 1),std \end{bmatrix} \begin{matrix} (q1\text{-}1) \\ (q1\text{-}2) \\ (q1\text{-}3) \end{matrix}$$

$$QS(\dot{m}_2,t_1,t_2) = \begin{matrix} [(0,\dot{m}_0),dec] \\ [(0,\dot{m}_0),inc] \\ [(0,\dot{m}_0),std] \end{matrix} \quad \begin{matrix} (q2\text{-}1) \\ (q2\text{-}2) \\ (q2\text{-}3) \end{matrix}$$

$$QS(Q_2,t_1,t_2) = \begin{matrix} [(0,Q_2^*),dec] \\ [(0,Q_2^*),ind] \\ [(0,Q_2^*),std] \end{matrix} \quad \begin{matrix} (q3\text{-}1) \\ (q3\text{-}2) \\ (q3\text{-}3) \end{matrix}$$

$$QS(\Delta T_2,t_1,t_2) = \begin{bmatrix} (0,\ 5),dec \\ (0,\ 5),inc \\ (0,\ 5),std \end{bmatrix} \begin{matrix} (q4\text{-}1) \\ (q4\text{-}2) \\ (q4\text{-}3) \end{matrix}$$

The assumed initial qualitative state at time (t_0,t_1) has three possible transitions at time (t_1,t_2). Let us further assume that a failure has occurred at t_1 and the faculty component was identified to be a valve stuck in one position. A stuck valve means that β = const(std) (i.e. (q1) can undergo a transaition to only (q1-3) at time (t_1,t_2)). Let us investigate next how this fault propagates through the system by checking the qualitative constraints under possible state transitions. Here we will check only for (QC1) and (QC2) to show how the search is conducted and the effect of the failure event on other components is determined.

$$(QC1) M^-(\beta,\dot{m}_2) \ ,$$

from the assumption that β = const

(q1-3)(q2-1) : not consistent
(q1-3)(q2-2) : not consistent
(q1-3)(q2-3) : consistent

$$(QC2) MULT(\dot{m}_2,\Delta T_2,Q_2) \ ,$$

by checking (QC1) we get (q2-3) for $QS(\dot{m}_2,t_1,t_2)$.

Among nine possible transitions, we determine readily that there are three consistent ones as: (q2-3)(q4-1)(q3-1),

(q2-3)(q4-2)(q3-2), or (q2-3)(q4-3)(q3-3). Therefore, failure of the valve causes a variation of the temperature difference dependent on the absorbed heat. Similarly, we can check the other qualitative constraints (QC3,QC4) related to this particular system. Energy function related constraints will also be used to narrow the behavioral interpretations and optimize the search algorithm.

The fault propagation model thus provides the information sought as to the impact of this fault condition on other system components.

4. CONCLUSIONS

The proposed fault propagation procedure proceeds as follows:

(1) Construct the qualitative model of the physical system.

(2) Estimate the initial qualitative state of the faulty system.

(3) Using qualitative simulation, find all possible behaviors.

(4) Construct the constraints related to the total energy function of the system.

(5) Using the constraints in (4), eliminate spurious behaviors.

(6) Determine the genuine behavior caused by the failure.

The key difficulty in employing such a qualitative technique to assess the impact of fault propagation is due to the generation of behavioral modes that are not consistent with the physical behavior of the system. Functional and energy related constraints that augment arithmetic qualitative constraints seem to effectively reduce spurious behaviors, thus leading to the correct interpretation.

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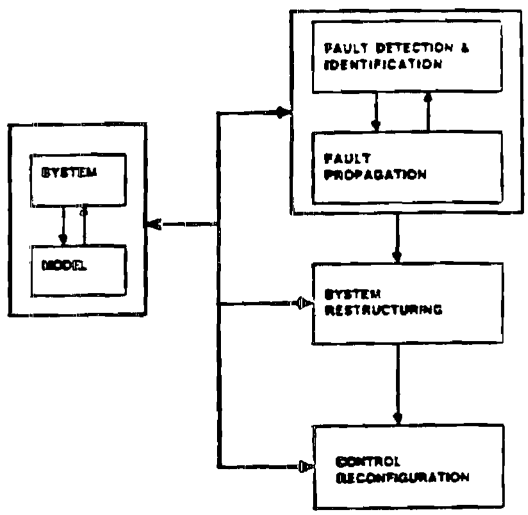

Figure 1. Block Diagram of Proposed Fault Tolerant Control Philosophy

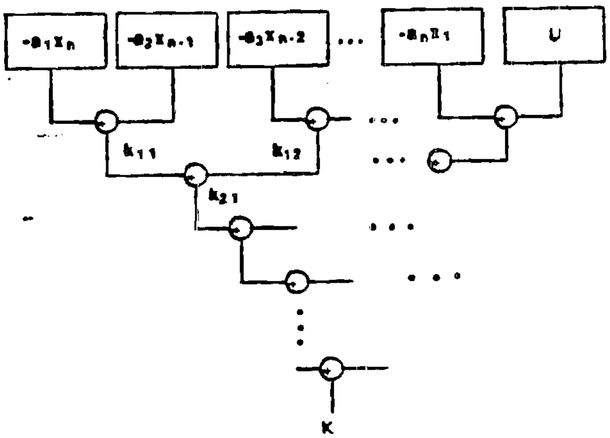

Figure 2a. Qualitative Constraints from $\dot{x}_n = -a_1x_n - a_2x_{n-1} - \dots$

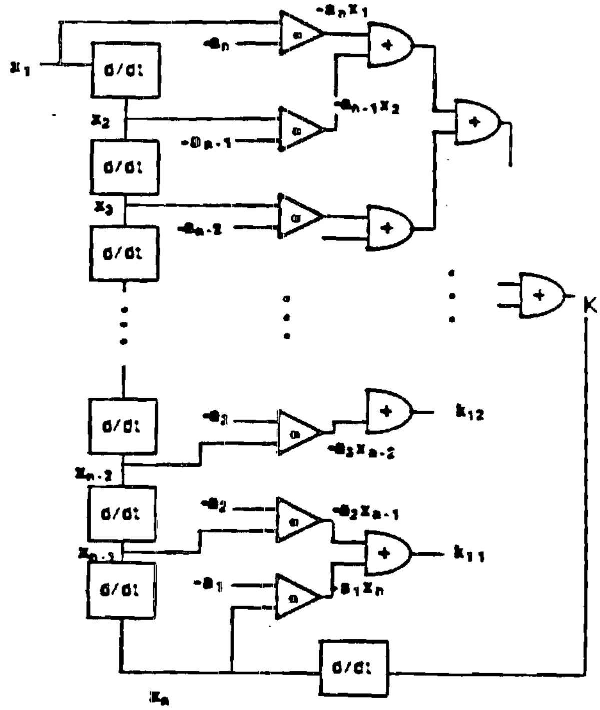

Figure 2b. Block Diagram of Qualitative Constraints Model

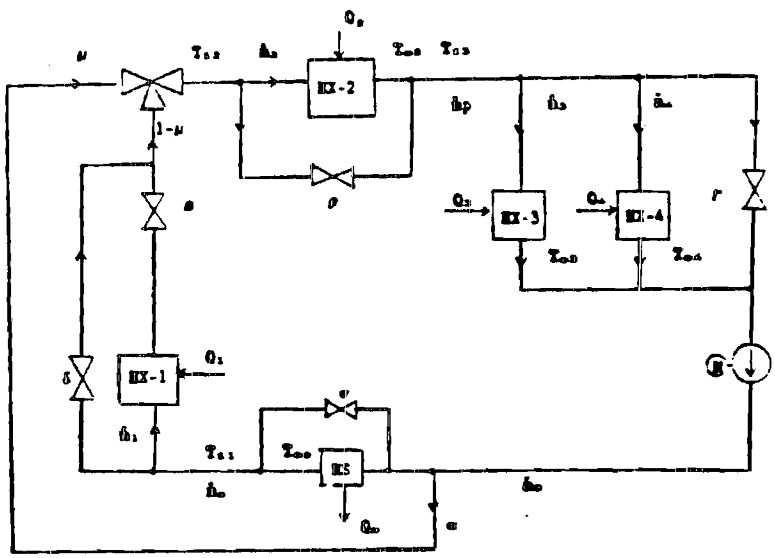

Figure 3. A Schematic of the Thermal Control System

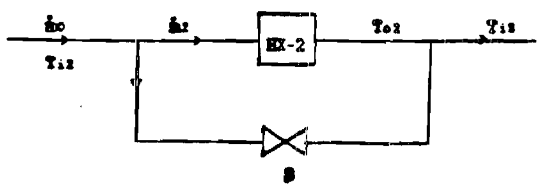

Figure 4a. Block Diagram of Heat Exchanger 2

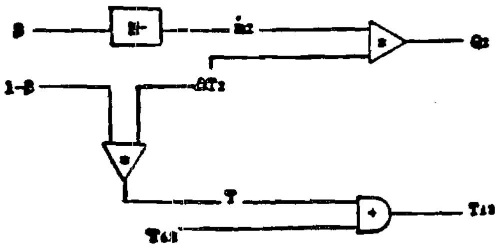

Figure 4b. Qualitative Constraints Model of HX-2

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Stability and Robustness of A Nonlinear Fuzzy Controller Based on The Phase Portrait Assignment Algorithm

by

Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering

Georgia Institute of Technology

Atlanta, Georgia 30332

(404)-894-6252 (404)-894-3834

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Abstract

This paper presents a new design technique and the associated stability analysis of fuzzy logic controllers for a class of nonlinear systems. The design approach is called 'the phase portrait assignment algorithm' (P^2A^2), and it establishes a stabilizing rulebase by utilizing fuzzy measures as introduced by Zadeh. This method can substitute for classical control paradigms when the latter suffer from dynamic, parametric, and stochastic uncertainties. The P^2A^2 technique not only provides a bridge between qualitative reasoning and asymptotic stability of nonlinear feedback control systems, but also gives an insight into the dynamical system behavior. A single-link robot arm is used to illustrated the performance and advantages of the proposed fuzzy control mechanism.

Keywords: fuzzy-logic controller, nonlinear feedback system, stability analysis

1 Introduction

The introduction of fuzzy sets and possibility measures by Zadeh [1,2] made it possible to cope with the approximate estimation and control problem under large-grain uncertainty. A fuzzy set is defined as a class of objects with a continuum of grades of membership using certainty or confidence factors in order to handle uncertainties or complexity in a particular domain of knowledge [3] and a fuzzy number can be described by linguistic terms such as 'large', 'medium', 'small', 'very small', and so on, whose fuzziness provides many degrees of freedom in dealing with uncertainty as a non-uniform possibility distribution [3]. Furthermore, a fuzzy confidence interval represents information that reduces (expands) the uncertainty by using lower and upper bounds as the presumption level or the α-cut increases (decreases). The characteristic functions of linguistic variables represent the associated crisp subsets by using uniform distributions of the feasible subset (possibility = 1) and the non-feasible subset (possibility = 0). Referring to the characteristic functions, membership functions are determined by using non-uniform possibility distributions.

Since Zadeh's introduction of fuzzy mapping and control [4], fuzzy concepts can be applied to the control of dynamic systems whereas stability of a fuzzy feedback control system has been an important issue. Mamdani [5] and Tong[6] suggested means for the application of fuzzy systems to control engineering. Kickert et al.[7] approximated a nonlinear fuzzy-logic controller with a multi-level relay in a simple case which was analyzed by the describing function method since an oscillation was anticipated. Tong [8] showed some results of asymptotic properties of fuzzy feedback systems. Mamdani et al.[9] introduced a self-organizing control system of a fuzzy rule modifier by using a performance table. The concept of L_2-stability was examined in [10] by using Nyquist criteria of a compensated system and a fuzzy-logic controller. Kania et al.[11] introduced the concept of energistic stability for fuzzy dynamic systems. Chen et al.[12] applied Hsu's cell-to-cell mapping theory [13] to the global behavior of the cell state space implicitly by grouping the cells. Finally, Kang et al.[14] emphasized evidential aspects of the nonlinear system dynamics and the transformation from crisp relations to fuzzy relations by using the vector field of the phase plane approach.

The objective of this paper is to provide a general method for constructing rule sets for fuzzy feedback controllers so that global asymptotic stability may be guaranteed with respect to a class of nonlinear systems. This is based upon 'the principle of increasing precision with decreasing intelligence' [15] where the basic idea is to relate evidence of the asymptotic system behavior to crisp subsets by using vector fields, explicitly, and then, to fuzzify each crisp subset (non-fuzzy cell). Periodic behavior of the closed-loop system may be avoided by applying the proposed algorithm. To preview the organization of the paper, Section 2 deals with the design procedure and stability analysis of the proposed scheme whereas convergence and robustness are also considered. A single-link robot arm is used to show the performance of the fuzzy regulator in Section 3 of this paper. Finally, Section 4 contains conclusions and a discussion

of the contributions.

2 Nonlinear Fuzzy Control

A design procedure for a fuzzy expert control mechanism, 'the phase portrait assignment algorithm' (P^2A^2), is proposed for a closed-loop feedback system consisting of a multi-input nonlinear process and a fuzzy rulebase controller as shown in Figure 1. The compositional rule of inference and the modified mean-of-maxima defuzzifier are used for deciding the control feedback input.

2.1 Nonlinear Fuzzy-Logic Feedback Control Systems

Let a nonlinear system be described by a vector differential equation of the form [16]

$$\dot{x}(t) = f(x(t), u(t), t) \qquad t \geq t_0 \tag{1}$$

where $x(t) \in \mathcal{R}^n$ represents the state vector with $x_0 = x(t_0)$; $u(t) \in \mathcal{R}^m$ is the control input vector; and $f(\cdot,\cdot,\cdot)$ denotes a nonlinear function. In brief,

$$\begin{aligned} & x(\cdot) : \mathcal{R}_+ \longrightarrow \mathcal{R}^n \\ & u(\cdot) : \mathcal{R}_+ \longrightarrow \mathcal{R}^m \\ & f(\cdot,\cdot,\cdot) : \mathcal{R}^n \times \mathcal{R}^m \times \mathcal{R}_+ \longrightarrow \mathcal{R}^n \end{aligned} \tag{2}$$

where $\mathcal{R}_+$ is a subset of $\mathcal{R}^1$ such that $\mathcal{R}_+ = [t_0, \infty)$. A general method of constructing a fuzzy rule set is necessary so that the rulebase updates the control input $u(t)$ to ensure global asymptotic stability with respect to a given nonlinear system. The rulebase can be represented as follows:

- rule 1: if (x_1 is $L^1_{x_1}$) and $\cdots$ and (x_n is $L^1_{x_n}$) then (u_1 is $L^1_{u_1}$) and $\cdots$ and (u_m is $L^1_{u_m}$).

$$\vdots \qquad\qquad \vdots \tag{3}$$

- rule s: if (x_1 is $L^s_{x_1}$) and $\cdots$ and (x_n is $L^s_{x_n}$) then (u_1 is $L^s_{u_1}$) and $\cdots$ and (u_m is $L^s_{u_m}$).

where x_j ($j = 1..n$) and u_k ($k = 1..m$) are fuzzy representations for $x_j(t)$ and $u_k(t)$, the elements of $x(t)$ and $u(t)$, respectively; $L^i_{x_j}$ and $L^i_{u_k}$ are linguistic variables such as 'positive large', 'negative small', and so on. The number of rules 's' depends on the number of possible linguistic variables 'ℓ', and $s = \ell^n$ when all possible combinations are considered but this number is finally determined by the P^2A^2. Reduction in the number of rules is possible only when the control $u(t)$ has no influence on the system behavior for some rules [17].

Consider the constraints of the control inputs and the states.

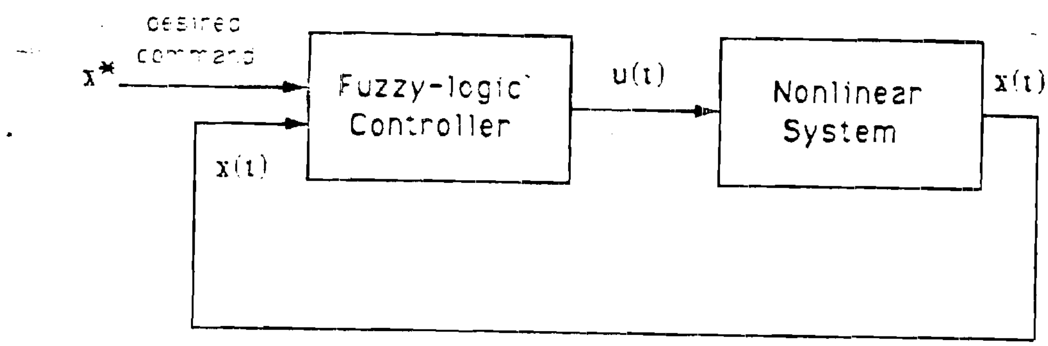

Figure 1: Block diagram of a fuzzy-logic feedback system

Assumption 2.1 (admissible controls) : *The control input $u(t)$ is bounded within some range*

$$u_{k\,\min} < u_k(t) < u_{k\,\max} \qquad \text{for } k = 1..m \tag{4}$$

implying that the maximum and minimum values, $u_{k\,\max}$ and $u_{k\,\min}$, of the actuator are bounded. If the set of admissible controls is defined by

$$\mathcal{U}(t) = \{u_k(t) : t \in [t_0, t_1], k = 1..m\} \in \mathcal{R}^m \tag{5}$$

then the above constraints may be described by

$$\mathcal{U}_c(t) = \{u(t) : \|u(t) - \bar{u}\|_p < M_u, t \in [t_0, t_1]\} \tag{6}$$

for some fixed vector $\bar{u} = \{\bar{u}_k\}$ where M_u is a finite positive real number; $\bar{u}_k$ may be $\frac{1}{2}(u_{k\,\min} + u_{k\,\max})$; and $\|\cdot\|_p$ represents p-norm.

Assumption 2.2 (the domain-of-interest in the state space) : *The domain-of-interest (DOI) in the phase plane is considered to provide global asymptotic stability for a local positive definite function. The state $x(t)$ is bounded as follows:*

$$x_{j\,\min} < x_j(t) < x_{j\,\max} \qquad \text{for } j = 1..n \tag{7}$$

where $x_{j\,\max}, x_{j\,\min}$ are real numbers. The set of allowable state trajectories is denoted by

$$\mathcal{X}_s(t) = \{x_j(t) : t \in [t_0, t_1], j = 1..n\} \in \mathcal{R}^n \tag{8}$$

and the DOI may be described by

$$\mathcal{X}_c(t) = \{\|x(t) - \bar{x}\|_p < M_x, t \in [t_0, t_1]\} \tag{9}$$

for some fixed vector $\bar{x} = \{\bar{x}_j\}$, where M_x is a finite positive real number and $\bar{x}_j$ may be $\frac{1}{2}(x_{j\,\min}+x_{j\,\max})$.

Note that $\|x(t)\|_\infty$ is related to the Cartesian coordinates while $\|x(t)\|_2$ is related to polar or spherical coordinates.

Definition 2.1 (extended reachability) : *Let S_u and S_x be subsets of $\mathcal{U}_c(t)$ and $\mathcal{X}_c(t)$, respectively, such that*

$$S_u = \{u(t) : \|u(t) - u^*\|_p < \delta_u\} \tag{10}$$

$$S_x = \{x(t) : \|x(t) - x^*\|_p < \delta_x\} \tag{11}$$

for fixed vectors $u^* \in \mathcal{R}^m$, $x^* \in \mathcal{R}^n$, and for small positive real numbers δ_u, δ_x. Extended reachability is stated as follows:

For the nonlinear system (1) with the initial state $x_0 = x(t_0)$, there exists S_u such that S_x is reachable (accessible) from x_0 at $t = t_0$ with respect to S_u for some finite time $t = T$.

Remark : Roughly speaking, extended reachability implies that $u(t)$, within a δ_u-neighborhood of u^*, leads $\mathcal{X}_s(t)$ to the δ_x-neighborhood of x^* in a finite time. In short, if $u(t) \in S_u$ for $t \in [t_0, T]$,

$$\|x_0 + \int_{t_0}^{T} f(x(\tau), u(\tau), \tau)d\tau - x^*\|_p < \delta_x. \tag{12}$$

Extended reachability relaxes the existing reachability condition by accommodating the fuzzified subsets of $x(t)$ and $u(t)$.

Some definitions relating to fuzzy sets and fuzzy numbers are appropriate for a better understanding of the theoretical developments in this paper (see [18]):

Definition 2.2 *An ordinary subset, A, of a referential set, X, is defined by its 'characteristic function' $\mu_A(x) \in \{0, 1\}, \quad \forall x \in X$.*

A feasible set $F(X)$ is a crisp set described by a characteristic function. A linguistic description in X such as 'large' resides in a feasible subset $F(X) \subset X$ such that $\mu_{F(X)}(x) = 1$ if $x \in F(X)$ and $\mu_{F(X)}(x) = 0$ otherwise.

Definition 2.3 *A fuzzy subset, A, from the universe of discourse, X, is defined by its characteristic function $\mu_A(x) \in [0, 1]$, $\forall x \in X$, which is called the 'membership function'.*

Now consider the following mappings of the fuzzy controller in (3):

$$\mathcal{X} = \{(x, \mu(x))\}; \quad \mu : X \longrightarrow [0,1]; \quad x \in X$$

$$\mathcal{U} = \{(u, \mu(u))\}; \quad \mu : U \longrightarrow [0,1]; \quad u \in U$$

$$R_F : \mathcal{X} \longrightarrow \mathcal{U} \tag{13}$$

where $\mathcal{X}, \mathcal{U}$ are families of fuzzy sets defined on X, U, respectively, and R_F is a fuzzy relation which can be interpreted as a mapping with its domain in $\mathcal{X}$ and a space of values constrained in $\mathcal{U}$. Let $A \in \mathcal{X}$ and $B \in \mathcal{U}$, then a formal fuzziness system is defined as

$$A \circ R_F = B \tag{14}$$

where '$\circ$' denotes the operator for the compositional rule of inference. A membership function may represent a non-uniform possibility distribution for the associated linguistic variable described by the characteristic function, and moreover, a fuzzy set is an invariant and consonant set as the presumption level increases.

Consider a controller structure of the fuzzy relational system represented by

$$u(t) = r(x(t)) \qquad t \geq t_0 \tag{15}$$

where $r(\cdot) : \mathcal{R}^n \longrightarrow \mathcal{R}^m$ is a memoryless nonlinear mapping described by the compositional rule of inference (3). Now the fuzzy feedback system can be represented by

$$\dot{x}(t) = f(x(t), r(x(t)), t) \triangleq g(x(t), t). \tag{16}$$

Our goal is to find a fuzzy controller $r(\cdot)$ that stabilizes the nonlinear feedback system $\dot{x}(t) = g(x(t), t)$, and to provide a generic procedure of constructing the stabilizing rule set by using fuzzy logic. If we denote the fuzzy assignments (fuzzy sets) by

$$A_{ij} = x_j \text{ is } L^i_{x_j} = L^i_{x_j}(x_j) \qquad (i = 1..s; j = 1..n) \tag{17}$$

$$B_{ik} = u_k \text{ is } L^i_{u_k} = L^i_{u_k}(u_k) \qquad (i = 1..s; k = 1..m) \tag{18}$$

then the fuzzy relation R_F of the parallel implications in (3) can be obtained as [3]

$$R_F = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s) = \bigvee_{i=1}^{s} (A_i \Rightarrow B_i) \tag{19}$$

where $A_i = \bigwedge_{j=1}^{n} A_{ij}$ and $B_i = \bigwedge_{k=1}^{m} B_{ik}$ with $\bigwedge$ and $\bigvee$ denoting the minimum and maximum operators, respectively.

Definition 2.4 *A 'fuzzy implication' is a mapping $S_i : A_i \Longrightarrow B_i$ and the membership function for S_i is $\mu_{S_i}(u,x) = \mu_{A_i}(x) \bigwedge \mu_{B_i}(u)$ where $x \in X$ and $u \in U$.*

If we consider a fuzzy relational matrix, the quantization of the support in X determines the size of the matrix, and the element of R_F is

$$R_F(p,q) = \mu_{A_i}(x_p) \bigwedge \mu_{B_i}(u_q) \tag{20}$$

where p, q are integers: x_p, u_q are the quantized representatives in X, U, respectively.

Definition 2.5 *The compositional rule of inference can be stated as follows [7]: let the i-th fuzzy implication be $\Sigma_i : A_i \Longrightarrow B_i$ whose membership function is*

$$\mu_{\Sigma_i}(y,x) = \mu_{A_i}(x) \wedge \mu_{B_i}(y) \tag{21}$$

then, for any implication Σ_i^* such that $\Sigma_i^* : A_i^* \Longrightarrow B_i^*$, the membership function for B_i^* is defined by

$$\mu_{B_i^*}(y) = \bigvee_x \{\mu_{A_i^*}(x) \wedge \mu_{\Sigma_i}(y,x)\} \tag{22}$$

and R_F may be represented by

$$R_F = \bigcup_{i=1}^{s} \Sigma_i. \tag{23}$$

The design of membership functions is an important issue closely linked to the possibility distribution of linguistic variables. Fuzzy membership functions should satisfy the properties of convexity and normality [18]. One approach to the design of membership functions is to make use of the S, Π, and Z curves as proposed by Zadeh [2]. The parameters of these functions should be carefully determined according to the scope of each linguistic variable [17]. We define the S, Π, and Z membership functions as follows:

$$S(x;a,b,c) = \begin{cases} 0 & x < a \\ 2(\frac{x-a}{c-a})^2 & a < x < b \\ 1 - 2(\frac{x-c}{a-c})^2 & b < x < c \\ 1 & x > c \end{cases} \tag{24}$$

$$\Pi(x;b,c) = \begin{cases} S(x; c-b, c-\frac{b}{2}, c) & x < c \\ S(x; c, c+\frac{b}{2}, c+b) & x > c \end{cases} \tag{25}$$

$$Z(x;a,b,c) = 1 - S(x;a,b,c) \tag{26}$$

For example, A_{11} = 'x_1 is positive large' can be described by

$$\mu_{A_{11}}(x_1(t)) = S(x_1(t); a, b, c). \tag{27}$$

The parameters of the membership functions are a, b, and c. The transition points, defined as the value of the universe of discourse for which the grade of membership is equal to $\frac{1}{2}$, are $x = b$ for S, Z; $x = c \pm \frac{b}{2}$ for Π. The maximum grade of 1 occurs at $x = c$ for all these functions and the minimum value of 0 at $x = a$ for S, Z; $x = c \pm b$ for Π. These design parameters are chosen to include the state trajectories of $x(t)$ for all x_0 in the DOI [19]. Typical S, Π, Z curves are shown in Figure 2. Other kinds of membership functions are triangle, trapezoid, arctangent, normal distribution functions, and so forth.

A number of mathematical properties describe the relationship between membership functions and the

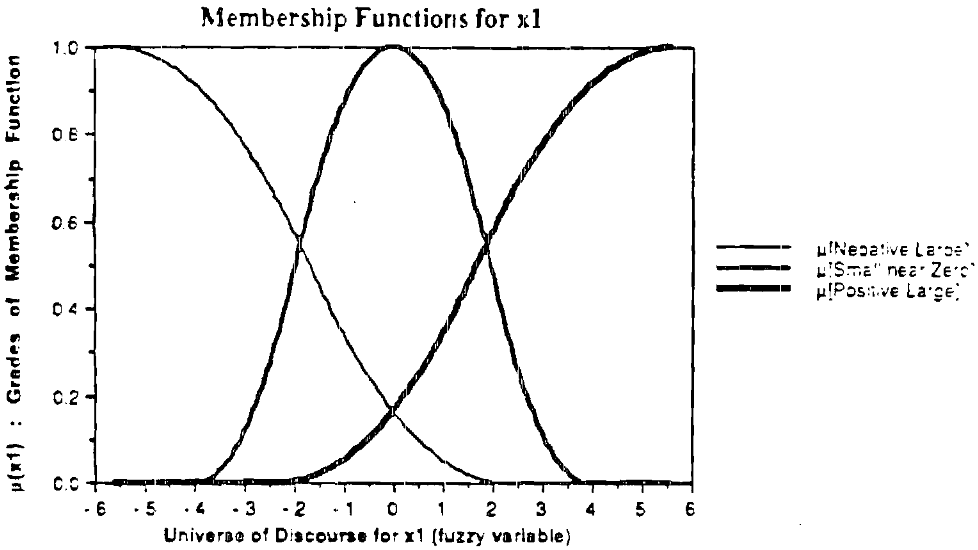

Figure 2: S, Π, Z membership functions

inferred control action. Kania and his co-workers [20] proposed an α-stability property, an α-decision stability property, an α-β strong decision stability property, a good mapping property, a γ-good mapping property, and the associated conditions. Some definitions and mathematical properties of the formal fuzziness system are shown as follows [20,21]:

Definition 2.6 *The formal fuzziness system (14) has the α-stability (α-S) property with respect to some family $\mathcal{A}$ and some feasible subset $F(U) \in U$ if $\mu_B(u) \leq \alpha$ $\forall u \in U - F(U)$ for every fuzzy set $A \in \mathcal{A}$ of $A \circ R = B$ where the operator $\circ$ is defined in (14).*

Theorem 2.1 *Consider the system where $R = A \Rightarrow B$ is given. Then $\bigvee_{i=1}^{n} \mu_A(x_i) \leq \alpha$ or $\mu_B(u_j) \leq \alpha$ $\forall u_j \in U - F(U)$ if and only if the system (14) has the α-S property with respect to a family $\mathcal{A}$.*

Definition 2.7 *A system* (14) *has the α-decision stability (α-DS) property with respect to some family $\mathcal{A}$ of fuzzy sets defined on X if the system has the α-S property with respect to $\mathcal{A}$ and* $\max_{U-F(U)} \mu_{A\circ R}(u) < \max_{F(U)} \mu_{A\circ R}(u)$.

The α-DS property assures that the grade of membership in the feasible subset $F(U)$ is greater than that in the nonfeasible subset $U - F(U)$. The next definition is a stronger version of the α-DS property [20]:

Definition 2.8 *A system* (14) *has the α-β strong decision stability (α-β SDS) property with respect to some family $\mathcal{A}$ if and only if the system* (14) *has the α-DS property with respect to $\mathcal{A}$ and* $\bigvee_{u\in U} \mu_{A\circ R}(u) \geq \beta > \alpha$.

A membership function should follow the α-DS ($\alpha - \beta$ SDS) property for the feasible subset of a linguistic variable, and for every rule, the γ-good mapping property should hold.

Definition 2.9 *Let* $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ *be given. Then R has the good mapping (GM) property if and only if* $A_i \circ R = B_i$ *for every* $i = 1, ..., s$.

Definition 2.10 *Let* $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ *be given where* $A_i \in \mathcal{X}$ *and* $B_i \in \mathcal{U}$ *for* $i = 1, \cdots, s$. *Then a fuzzy relation R has the γ-good mapping (γ-GM) property in the weaker sense if and only if* $A_i \circ R = \tilde{B}_i$ *for every* $i = 1, ..., s$ *where* $B_i \subseteq \tilde{B}_i$, *and* $\mu_{\tilde{B}_i} = \mu_{B_i}$ *for* $\mu_{B_i} \geq \gamma, 0 \leq \gamma \leq 1$.

<u>Remark</u>: Note that, when $\gamma = 0$, the γ-GM property reduces to the GM property as defined above [21]. It is recommended that from the γ-GM property, at the boundary of each crisp subset, overlapping of the associated adjacent membership functions takes place where the grade of membership is $\gamma = 0.5$ [22]. This overlapping concept is important in the sense that we can minimize the decision errors by overlapping adjacent membership functions where $\mu_A(x_i) = 0.5$.

2.2 Design: The Phase Portrait Assignment Algorithm (P^2A^2)

Let us consider the P^2A^2 for a nonlinear fuzzy controller described by (3) or (15). The following steps describe how to establish the fuzzy rulebase that guarantees aymptotic stability of the closed-loop system consisting of (1) and (15). The key *point* here is that the procedure entails three basic steps: first, the classification of *integral* manifolds according to the evidence in the phase plane, second, the selection of nonfuzzy cells from the crisp relations, and finally, the transformation of each crisp relation into a fuzzy relation.

- (i) Find $u^* \in S_u$ with which $x(t)$ converges to or passes through S_x in a finite time T, such that $x(T) \in S_x$. There may be several u^*'s for some x_0's that satisfy the extended reachability condition.

These trajectories are called 'the primary-attracting manifolds' (the PAM's). This concept is closely related to the switching curves [23,24].

- (ii) Divide an admissible control element $u_k(t)$ into m_k regions resulting in $\bar{u}_{kr}$'s ($r = 1..m_k$) in such a way that some u_k^*'s $\in \{\bar{u}_{kr}\}$ where $\bar{u}_{kr}$ represents the mean of $u_k(t)$ in the r-th region. A linguistic variable $L_{u_k}^i$ is defined and is assigned to each $\bar{u}_{kr}$. The maximally assignable controls are mm_k constant $\bar{u}_{kr}$'s.

- (iii) Plot the state trajectories $C_t(x_0, \bar{u}_{kr}) \in \mathcal{X}_s(t)$ for every x_0 in the DOI and for each $\bar{u}_{kr}$. The PAM's are denoted by $C_t^*(x_0, u_k^*)$. Define 'the primary-attracting cells' (the PAC's) R^*'s $\in \mathcal{R}^n$ that cover the PAM's $C_t^*(x_0, u_k^*)$ in the DOI and assign u_k^* to R^*. The size of R^* partially determines the number of fuzzy rules as the size of the DOI is fixed.

- (iv) Plot the state trajectories $C_t(x_0, \bar{u}_{kr})$ for every x_0 and for some $\bar{u}_{kr}$'s ($\neq u_k^*$). If such $\bar{u}_{kr}$'s lead to or intersect with $C_t^*(x_0, u_k^*)$ either directly or indirectly (via another $C_t(x_0, \bar{u}_{kr})$) in a finite time, then these $C_t(x_0, \bar{u}_{kr})$'s are defined as 'the secondary-attracting manifolds' (the SAM's) $C_t^\dagger(x_0, u_k^\dagger)$ while these $\bar{u}_{kr}$'s are denoted by $u_k^\dagger$'s.

- (v) Define 'the secondary-attracting cells' (the SAC's) $R^\dagger \in \mathcal{R}^n$ that include $C_t^\dagger(x_0, u_k^\dagger)$'s by assigning $u_k^\dagger$ to each $R^\dagger$. $R^\dagger$'s are chosen in a such way that $C_t^\dagger(x_0, u_k^\dagger)$'s may have the shortest path, if possible, to $C_t^*(x_0, u_k^*)$ or to nearby $C_t^\dagger(x_0, u_k^\dagger)$ that eventually leads to $C_t^*(x_0, u_k^*)$ and so that $u_k^\dagger$'s may not change abruptly between $R^\dagger$'s. Also, $C_t^\dagger(x_0, u_k^\dagger)$'s may not form a closed-cycle.
  <u>Remark</u> : Steps (iv) and (v) may be difficult since there are many choices for assigning the SAC's. But there should be only one u_k^* or $u_k^\dagger$ for each R^* or $R^\dagger$, respectively.

- (vi) Repeat steps (iv) and (v) until the whole DOI is filled with R^*'s and $R^\dagger$'s. The rest of the manifolds $C_t(x_0, \bar{u}_{kr})$ not satisfying $C_t^*(x_0, u_k^*)$ or $C_t^\dagger(x_0, u_k^\dagger)$ are defined as 'the diverging manifolds' (the DVM's) $C_t^\triangle(x_0, \bar{u}_{kr})$ and these x_0's and $\bar{u}_{kr}$'s should be avoided. Similarly, 'the diverging cells' (the DVC's) that include $C_t^\triangle(x_0, \bar{u}_{kr})$'s are defined as $R^\triangle$'s.

- (vii) The final attracting cells, R^*'s (PAC's) and $R^\dagger$'s (SAC's), constitute the <u>cells</u> of the P^2A^2. The total number of cells is equal to the total number of rules. On the boundary of the DOI, all the manifolds $C_t^\dagger(x_0, u_k^\dagger)$ and $C_t^*(x_0, u_k^*)$ should be directed toward the inner regions.
  <u>Remark</u> : $C_t(x_0, \bar{u}_{kr})$ can be obtained analytically by finding the n-dimensional vector fields for the nonlinear system as described above; heuristically by exploiting the expertise of experts; or experimentally for unknown complex systems. Figure 3 shows an example of the attracting cells, R^*'s and $R^\dagger$'s, and the DVC's $R^\triangle$'s for a simple second-order system.

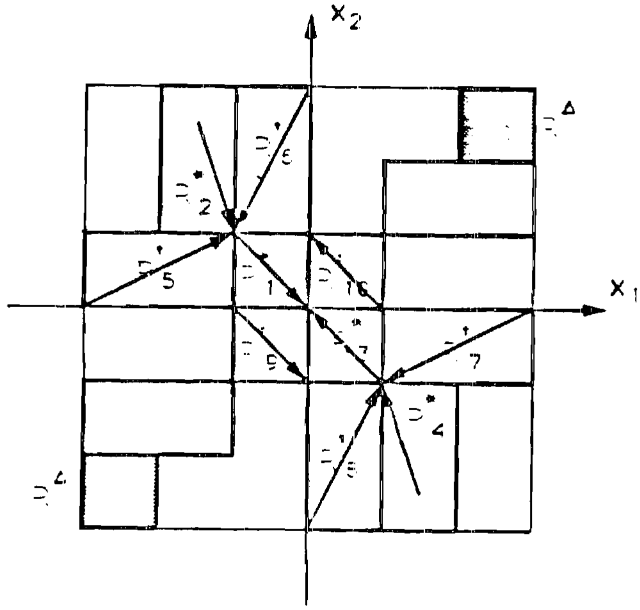

Figure 3: Typical cells and the vector fields of the P^2A^2

- (viii) If there are s cells in the DOI, the i-th cell is defined by the pair $c\{R[i], \bar{u}[i]\}$ where $\bar{u}[i] \in \mathcal{R}^m$ is a control vector of the chosen m constant elements among $\{u_k^*, u_k^\dagger; k = 1..m\}$ for $i = 1..s$, and for each cell, a linguistic characterization is performed by assigning n $L^i_{x_j}$'s (for $j = 1..n$) to each $R[i] \in \{R^*, R^\dagger\}$ in the DOI. Every fuzzy assignment $\{A_{ij}, B_{ik}\}$ in the i-th rule is determined according to the above steps, since each admissible control $\bar{u}_{kr}$ has its own linguistic description $L^i_{u_k}$ for $k = 1..m$ for each $R[i]$. Each cell $R[i]$ in the DOI is assigned to its particular linguistic characterizations $\{L^i_{x_j} : j = 1..n\}$ by using the connectivity of the i-th cell $c\{R[i], \bar{u}[i]\}$. The membership functions are described by $\mu_{A_{ij}}(x_j(t))$ for A_{ij} = 'x_j is $L^i_{x_j}$' and $\mu_{B_{ik}}(u_k(t))$ for B_{ik} = 'u_k is $L^i_{u_k}$'.

- (ix) The control input $u_k(t)$ in (15) is determined *via* defuzzification by

$$u_k(t) = \frac{\sum_{i=1}^{s} d_i(x(t))\bar{u}_k[i]}{\sum_{i=1}^{s} d_i(x(t))} \qquad \text{for } k = 1..m \tag{28}$$

where $\bar{u}_k[i]$ is the k-th element of $\bar{u}[i]$ and $d_i(x(t))$ is the degree of fulfilment for the i-th rule defined by

$$d_i(x(t)) = \wedge_{j=1}^{n} \mu_{A_{ij}}(x_j(t)) \qquad \text{for } i = 1..s \tag{29}$$

$$d_i(\cdot) : \mathcal{R}^n \longrightarrow [0,1] \tag{30}$$

and $\Sigma_{i=1}^{s} d_i(x(t)) \neq 0$ is required for the existence of a solution of the defuzzification scheme. It is noted that

$$r_k(\cdot) = \frac{\Sigma_{i=1}^{s} \bar{u}_k[i] d_i(\cdot)}{\Sigma_{i=1}^{s} d_i(\cdot)} \tag{31}$$

where $r_k(\cdot)$ represents the k-the element of $r(\cdot)$ in (15).

<u>Remark</u> : The number of cells in the DOI is the same as the number of rules in the fuzzy rule set. When the fuzzy characterization functions are mapped into the fuzzy membership functions, α-DS property [20] should hold for the feasible subsets. Moreover, for each linguistic variable, overlapping of the adjacent membership functions is necessary to ensure that $\Sigma_{i=1}^{s} d_i(x(t)) \neq 0$ for any $x(t)$ in the DOI (the γ-GM property) [17]. Note that the DOI is decomposed into two kinds of cells, PAC's and SAC's. The crisp relational mapping R_C may be represented using the crisp cells as

$$R_C = \bigcup_{i=1}^{s} c\{R[i], \bar{u}[i]\} \tag{32}$$

where $R_C : \mathcal{R}^n \longrightarrow \mathcal{R}^m$. If we introduce the fuzziness transformation $\mathcal{F}$, the fuzzy relational mapping (19) may be defined by

$$R_F = \mathcal{F}[R_C] = \mathcal{F}[\bigcup_{i=1}^{s} c\{R[i], \bar{u}[i]\}]. \tag{33}$$

Lemma 2.1 *If the γ-GM property and the α-DS property (or the α-β SDS property) hold, then there exists a $\delta_u[i] > 0$ for all $x(t) \in R[i]$ such that*

$$|u_k(t) - \bar{u}_k[i]| < \delta_{u_k}[i] \qquad \text{for } k = 1..m \tag{34}$$

or

$$\|u(t) - \bar{u}[i]\| < \delta_u[i] \tag{35}$$

where $\bar{u}[i] = \{\bar{u}_k[i] : k = 1..m\}$ and $\delta_u[i] = \max_k\{\delta_{u_k}[i]\}$.

<u>Proof</u> : Let overlapping (the γ-GM property) occur at $\gamma = \alpha$ for the design, then there exists only one $d_i(x(t))$ such that $d_p(x(t)) > \alpha$ if $i = p$, that is, the p-th rule is the most contributing fuzzy rule at time t,

$$d_p(x(t)) = \bigwedge_{j=1}^{n} \mu_{A_{pj}}(x_j(t)) = \alpha_p > \alpha \qquad \text{if } i = p \tag{36}$$

$$d_i(x(t)) = \bigwedge_{j=1}^{n} \mu_{A_{ij}}(x_j(t)) = \alpha_i \ll \alpha \qquad \text{if } i \neq p \tag{37}$$

and from the α-DS property

$$\Sigma_{\substack{i \neq p \\ i=1}}^{s} \alpha_i \ll \alpha_p \tag{38}$$

is satisfied by approximation subject to the minimum operators. From (28),

$$u_k(t) = \frac{\alpha_p \bar{u}_k[p] + \Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}$$

$$= \frac{\alpha_p}{\Sigma_{i=1}^{s} \alpha_i} \bar{u}_k[p] + \frac{\Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}. \tag{39}$$

Since $\alpha_p \cong \Sigma_{i=1}^{s} \alpha_i$ from (38),

$$u_k(t) = \bar{u}_k[p] + \triangle_k[p] \tag{40}$$

where $\triangle_k[p] \triangleq \frac{\Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}$. Therefore,

$$|u_k(t) - \bar{u}_k[p]| < \delta_{u_k}[p] \tag{41}$$

where $\delta_{u_k}[p] = |\triangle_k[p]|$ for $k = 1..m$.

Remark : If the granularity of the fuzzy rulebase is dense enough, (38) is strongly confirmed, and therefore, $\delta_{u_k}[p]$ in (41) is reduced. Moreover, reduction in the fuzzy rules is possible as long as the system behavior is not affected by deleting some rules.

Theorem 2.2 (global convergence) *Let the extended reachability condition hold for a nonlinear system described by (1) and let the P^2A^2 be applied to the control input $u(t)$. Then, for any x_0 in the DOI of the state space,*

$$||x(t) - x^*||_p < \epsilon \tag{42}$$

is satisfied for $t \geq T$ where $\epsilon > 0$ and $x(t)$ is globally convergent to an ϵ-neighborhood of x^*.

Proof : Let the index 'i' of the most contributing rule be omitted for simplicity. The proof is divided into two parts.

case (i) $x_0 \in R^*$: $x(t)$ is found by

$$x(t) = x_0 + \int_{t_0}^{t} f(x(\tau), u(\tau), \tau) d\tau. \tag{43}$$

In the worst case, $u(\tau) = u^* + \triangle^* \in S_u$, and from Lemma 2.1 of the P^2A^2, $x(t)$ continues to remain in R^*'s reaching S_x in a finite time $t_x = t_1 - t_0$. From (12),

$$||x(t_1) - x^*||_p = ||x_0 + \int_{t_0}^{t_1} f(x(\tau), u^* + \triangle^*, \tau) d\tau - x^*||_p < \delta_x \tag{44}$$

at $t = t_1 > t_0$.
case (ii) $x_0 \in R^\dagger$: There exists $t = t_2 > t_0$ such that $x(t)$ is transferred from $R^\dagger$ to R^* by applying the worst-case control $u(t) = u^\dagger + \Delta^\dagger$ from Lemma 2.1 of the P^2A^2 such that

$$x(t_2) = x_0 + \int_{t_0}^{t_2} f(x(\tau), u^\dagger + \Delta^\dagger, \tau)d\tau \in R^* \tag{45}$$

and then, similarly from (i), we can use $u(t) = u^* + \Delta^* \in S_u$, resulting in $x(t) \in S_x$ at time $T = t_2 + t_x$. Therefore,

$$\|x(t) - x^*\|_p < \epsilon \tag{46}$$

in a finite time $t \geq T$ where $\epsilon = \delta_x > 0$.

2.3 Analysis: Stability of The Nonlinear Fuzzy-logic Controller

The fuzzy-logic control system is analyzed by using Lyapunov's direct method [25] in this section. Since the P^2A^2 forces the vector field of the dynamic equation (1) to be directed toward S_x by using the fuzzy-logic rulebase control $u(t)$, the following proposition is considered for stability analysis of nonlinear fuzzy control systems considering the vector fields.

Proposition 2.1 *Let the extended reachability condition hold and let the P^2A^2 be applied to (1). If $x(t) \in R^*$, then for some x^*,*

$$\dot{x}(t) = \Psi(t)(x(t) - x^*) + \xi_x \tag{47}$$

is satisfied $\forall t \in \mathcal{R}_+$ where the eigenvalues of $\Psi(t)$ are negative, i.e., $\lambda_i[\Psi(t)] < 0$ for i=1..n, $\Psi(t) = \Psi(x(t), u(t), t) \in \mathcal{R}^{n \times n}$; and $\xi_x \in \mathcal{R}^n$ is a point in the σ-neighborhood of $\dot{x}(t)$ when $x(t) = x^*$ such that

$$\|\dot{x}(t) - \xi_x\|_p|_{x(t)=x^*} < \sigma. \tag{48}$$

Proof : Since $x(t) \in R^*$, the vector field of $\dot{x}(t) = f(x(t), u^*, t)$ is directed toward $x^* \in S_x$ by applying the fuzzy control input $u(t) = u^* + \Delta^* \in S_u$ from the P^2A^2. If $x_i(t), \xi_{xi}, x_i^*$ are the elements of $x(t), \xi_x, x^*$, respectively, then the following is satisfied.

$$\frac{\dot{x}_i(t) - \xi_{xi}}{x_i(t) - x_i^*} < 0 \qquad \forall t \in [t_0, t_1] \tag{49}$$

for $i = 1..n$. Thus, for some function of time $\Psi_i(t) < 0, \forall t \in \mathcal{R}_+$.

$$\dot{x}_i(t) = \Psi_i(t)(x_i(t) - x_i^*) + \xi_{xi} \tag{50}$$

where $\Psi_i(t)$ may be a diagonal element of $\Psi(t)$. Therefore, in general,

$$\dot{x}(t) = \Psi(t)(x(t) - x^*) + \xi_x \tag{51}$$

where $\lambda_i[\Psi(t)] < 0$ for $i = 1..n$ and $\forall t \in [t_0, t_1]$. t_1 can be extended to infinity by considering piecewise time-scale analysis in the different R^*'s.

Remark : It is noted that (49) is related to a Lipschitz condition of (1) when $u(t) = u^*$ in such a way that the uniqueness of the solution to (1) is guaranteed if $\Psi_i(t)$ is finite $\forall t \in \mathcal{R}_+$ [16].

Theorem 2.3 *Let a local positive definite function $v(t)$ in R^* be defined by*

$$v(t) = (x(t) - x^*)^T \mathcal{P}(x(t) - x^*) \tag{52}$$

where $\mathcal{P}$ is a symmetric positive definite matrix. If the P^2A^2 is applied to (1) with the extended reachability condition, x^* is asymptotically stable with a ball attractor S_z for all $x_0 \in R^*$, $\forall t \in [t_0, \infty)$.

Proof : Considering $\dot{v}(t)$.

$$\dot{v}(t) = \dot{x}(t)^T \mathcal{P}(x(t) - x^*) + (x(t) - x^*)^T \mathcal{P}\dot{x}(t) \tag{53}$$

and if we define $z(t) = x(t) - x^*$ then from (47) in Proposition 2.1.

$$\dot{v}(t) = (\Psi(t)z(t) + \xi_x)^T \mathcal{P}z(t) + z(t)^T \mathcal{P}(\Psi(t)z(t) + \xi_x)$$

$$= -z(t)^T \mathcal{Q}(t)z(t) + 2\xi_x^T \mathcal{P}z(t) < 0 \tag{54}$$

is required for asymptotic stability where $\mathcal{Q}(t) \in \mathcal{R}^{n\times n}$ is a positive definite matrix defined by

$$\mathcal{Q}(t) = -(\Psi(t)^T \mathcal{P} + \mathcal{P}\Psi(t)). \tag{55}$$

Therefore, for asymptotic stability,

$$z(t)^T \mathcal{Q}(t)z(t) > 2\xi_x^T \mathcal{P}z(t) \tag{56}$$

should be satisfied and this is implied by

$$\|z(t)\|_p > \frac{2\lambda_{\max}[\mathcal{P}]\|\xi_x\|_p}{\inf_t \lambda_{\min}[\mathcal{Q}(t)]} \tag{57}$$

where $\lambda_{\max}[A], \lambda_{\min}[A]$ represent the maximum and minimum eigenvalues of a matrix A. If $S_z \in \mathcal{R}^n$ is defined by

$$S_z = \{z(t) : \|z(t)\|_p < \delta_x\} \tag{58}$$

where $\delta_z = \frac{2\lambda_{\max}[\mathcal{P}]\|\xi_z\|_p}{\inf_t \lambda_{\min}[\mathcal{Q}(t)]} > 0$, then S_z is a ball attractor outside of which $\dot{v}(t) < 0$ holds with asymptotic stability, and it is easy to realize $S_z = S_x$.

Remark : Note that $\lambda_{\min}[\mathcal{Q}(t)]$ may change but $\inf_t \lambda_{\min}[\mathcal{Q}(t)] < 0$ is satisfied by Proposition 2.1. If $\|\xi_x\|_p$ is small enough compared with $\inf_t \lambda_{\min}[\mathcal{Q}(t)]$, the attractor (58) reduces to a point.

Corollary 2.1 *Let the extended reachability condition and the P^2A^2 be satisfied for a multi-input nonlinear system in (1). Then x^* is globally asymptotically stable with a ball attractor S_x for all x_0 in the DOI and* $\forall t \in [t_0, \infty)$.

Proof : (i) If $x(t) \in R^\dagger$, there exists a control in the δ_u-neighborhood of $u^\dagger$ that transfers $x(t)$ from $R^\dagger$ to R^* in a finite time by the P^2A^2.
(ii) If $x(t) \in R^*$, there exists a control in the δ_u-neighborhood of $u^* \in S_u$ that forces $x(t)$ to converge to S_x by applying Theorem 2.3 and x^* is asymptotically stable.
From (i) and (ii), for all x_0 in the DOI, global asymptotic stability is guaranteed.
Remark : (i) is referred to as 'transferring capability' since $x(t) \in R^\dagger$ is moved to $x(t) \in R^*$ in a finite time by using the P^2A^2. The corollary 2.1 can be applied to the fuzzy regulators for nonlinear systems.

Theorem 2.4 (fuzzy robustness) *Given the nonlinear system in (1), let the extended reachability condition hold and let the P^2A^2 be applied to (1). The size of the attractor in (58) can be reduced to the minimum-size attractor S_x if we can select* $\xi_x^* = \min_x[\xi_x]$.

Proof : The proof is obvious from Theorem 2.3 since $S_x(S_x)$ in (58) can be minimized if there exist a sequence of u^* so that the minimum Euclidean distance can be obtained by applying the P^2A^2 such that $\xi_x = \xi_x^*$ in (47).
Remark (fuzzy robustness) : As the proposed fuzzy controller deals with fuzzified cells in the DOI, the controller is robust against disturbances as long as dynamic and parametric uncertainties of nonlinear systems, or some external noise, are bounded within some allowable regions. It is asserted that any state $x(t)$ outside the ϵ-neighborhood of x^* is attracted to S_x in a finite time, and therefore, the bounded region can allow the corresponding magnitude of any disturbance with the same performance.

3 Example: Simulation of A Single-Link Robot Arm

As an illustrative example, consider a single-link robot arm shown in Figure 4 with a mass of 1.0 [Kg] at the end of the link of 1.0 [m] length. The dynamics can be derived by using the Lagrange-Euler equation of motion [26].

$$u(t) = m\ell^2\ddot{\theta}(t) - mg\ell\sin\theta(t) \qquad (59)$$

and since $m = 1[Kg]$, $\ell = 1[m]$, and $g = 9.8[m/sec^2]$ if we define $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$, the following vector equations can be obtained

$$\dot{x}_1(t) = f_1(x(t), u(t), t) = x_2(t)$$

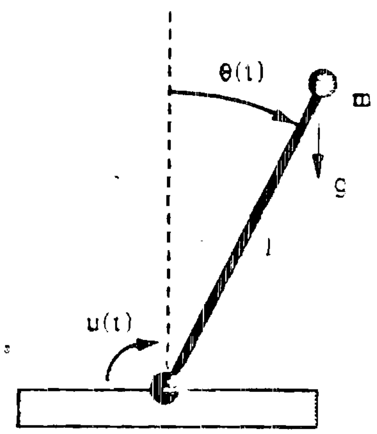

Figure 4: Single-link robot arm

$$\dot{x}_2(t) = f_2(x(t), u(t), t) = 9.8 \sin x_1(t) + u(t). \tag{60}$$

The direction of the vector field of (60) is found by

$$\varphi(t) = \tan_2^{-1}(f_1, f_2) \tag{61}$$

where $\tan_2^{-1}(\cdot,\cdot)$ can access any direction in the (x_1, x_2) coordinates. Making use of the vector field $\varphi(t)$ in the P^2A^2, the fuzzy rulebase for the control input $u(t)$ in (3) and (15) can be constructed to regulate $x_1(t)$ and $x_2(t)$ to zero ($x^* = 0$) as follows:

- R_1: if (x_1 is PL) and (x_2 is PL) then (u is NL)
- R_2: if (x_1 is PL) and (x_2 is SZ) then (u is NS)
- R_3: if (x_1 is PL) and (x_2 is NL) then (u is SZ)
- R_4: if (x_1 is SZ) and (x_2 is PL) then (u is NS)
- R_5: if (x_1 is SZ) and (x_2 is SZ) then (u is SZ)
- R_6: if (x_1 is SZ) and (x_2 is NL) then (u is PS)
- R_7: if (x_1 is NL) and (x_2 is PL) then (u is SZ)

- R_8: if (x_1 is NL) and (x_2 is SZ) then (u is PS)
- R_9: if (x_1 is NL) and (x_2 is NL) then (u is PL)

where we use abbreviations for various linguistic variables: 'PL' for 'positive large', 'PS' for 'positive small', 'SZ' for 'small near zero', 'NS' for 'negative small', and 'NL' for 'negative large'. Membership functions for the fuzzy variables x_1, x_2, and u, are the S, Π, and Z curves. The above 9 rules guarantee global asymptotic stability in the sense of Lyapunov.

In the simulations, the initial conditions are chosen to be $x_1(0) = 30$ [deg] and $x_2(0) = 0$. The maximum value for the control input $u(t)$ is $\pm 300[Nm](M_u)$ while the boundary values for $x(t)$ are $\pm\frac{\pi}{32}$ for $x_1(t)$; $\pm 2\pi$ for $x_2(t)$. In the case of 9 rules, Figure 5 (a) and (b) show the non-fuzzy cells and the phase portrait of the nonlinear system (60) for different $\bar{u}[i]$'s of the fuzzy rulebase established by the P^2A^2 utilizing the vector field where the PAC's and the SAC's are represented by R^*'s and $R^\dagger$'s, respectively. The results of 9 rules and those of 25 rules of the fuzzy-logic controller are compared in Figures 6, 7, and 8. Figure 9 shows actual state trajectories of 9 rules and 25 rules. As mentioned earlier, more rules usually imply higher convergence rates since the granularity of the DOI reduces the fuzziness in the phase plane. Moreover, an impact force of $100[N]$ is applied to the end of the link during $0.01[sec]$ starting at $t = 4.0[sec]$ to consider some uncertainty at the joint.

Although the simulations exclude the actuator dynamics, if the actuator modes are fast enough compared to the robot dynamics, the 9 rules can be applied without further modifications. If not, the number of states should be increased by adding the actuator dynamics to the fuzzy rulebase. For more information on joint flexibility in connection to actuators, see [27].

4 Conclusions and Open Problems

The research work reported in this paper supports a connection between artificial intelligence and analytic methods by linking the 'if-then' rules of the control input to a systematic design procedure called the P^2A^2 for nonlinear systems. Advantages of fuzzy-logic controllers include robustness and flexibility in modifying the particular domain by providing qualitative reasoning for a specific control decision. Also, we can reduce the decision errors by transforming crisp sets into fuzzy sets.

Global asymptotic stability of nonlinear fuzzy-logic control is considered by using Lyapunov's direct method. Some difficult problems associated with fuzzy control, such as ambiguity in completeness of the fuzzy rulebase,updating and calibrating the rulebase controllers, and exhaustive use of the linguistic rules, can be solved by applying the P^2A^2. We can easily change, add, or delete some rules according to the organization level.

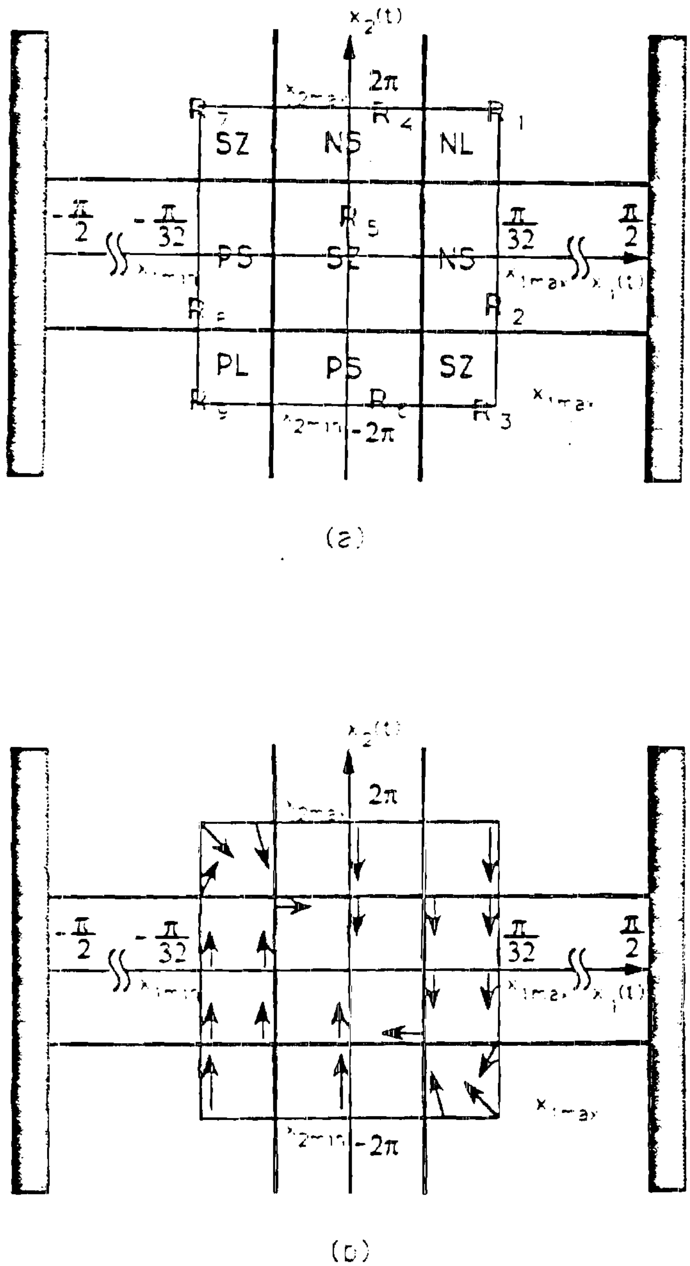

Figure 5: The cells (a) and the phase portrait (b) of $x_1(t)$ and $x_2(t)$ from the P^2A^2

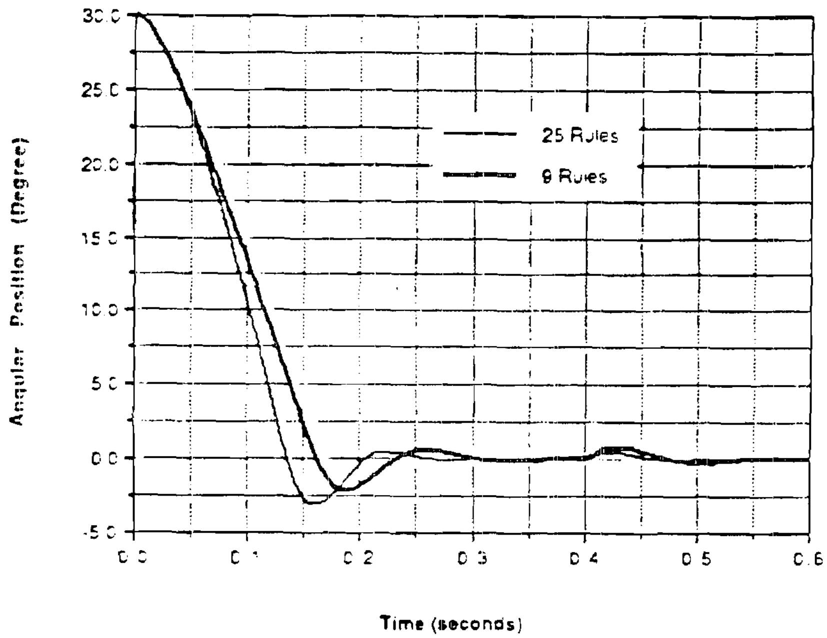

Figure 6: Angular position of the single-link robot arm

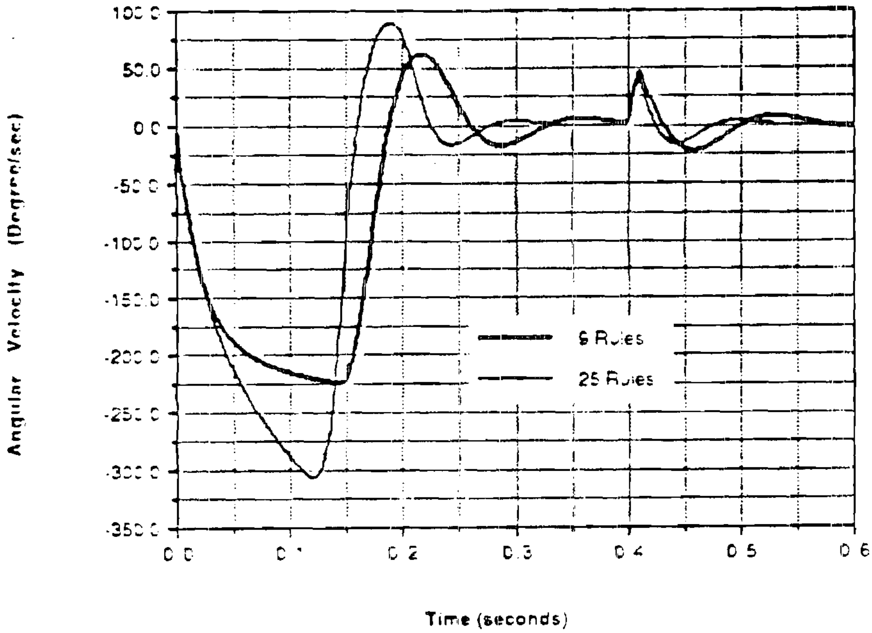

Figure 7: Angular velocity of the single-link robot arm

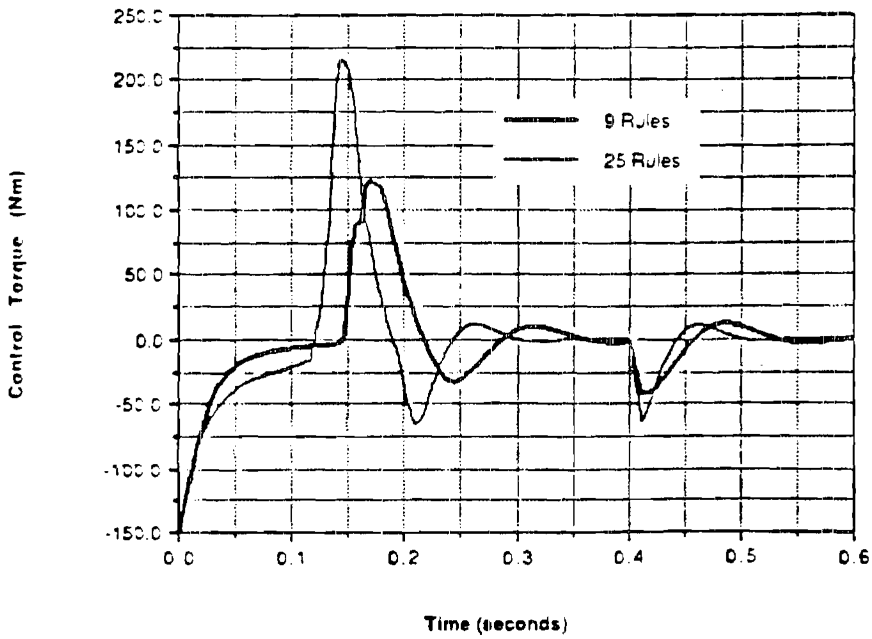

Figure 8: Fuzzy control torque input applied to the single-link robot arm

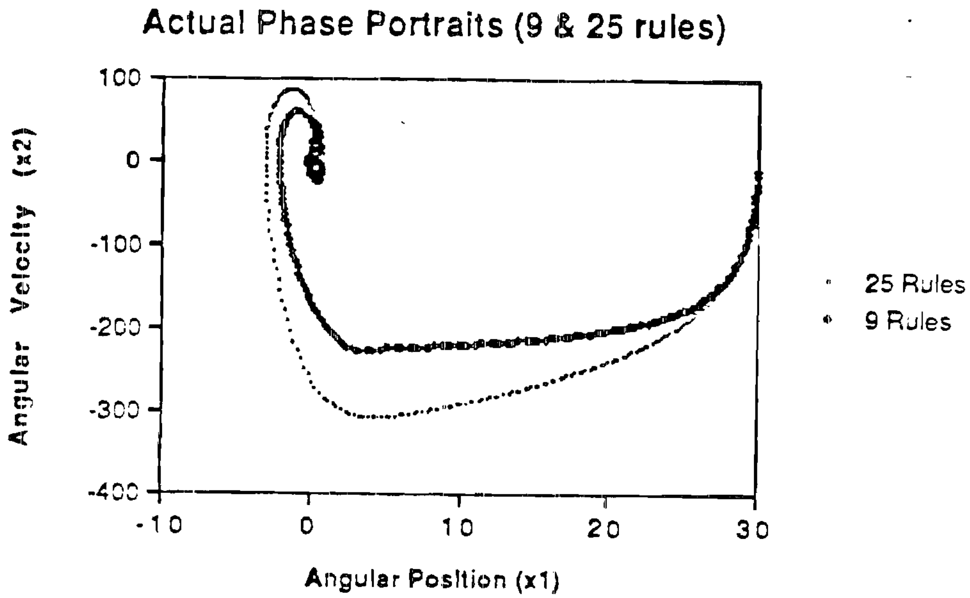

Figure 9: Actual phase portrait of the single-link robot arm

Simulation results show that fast convergence, asymptotic stability, and robustness with respect to disturbances are achieved. Although P^2A^2 may requires a large amount of fuzzy information for a system of higher dimension, it is easy to expand the P^2A^2 concept by using the n-dimensional vector fields. Fuzzy systems need more samples for the decision making process compared with analytical methods, however it handles unreliable information by using linguistic variables and fuzzy relations. In the simulations, an infinite-valued logic is applied to fuzzy sets, but, in the practical sense, simplified fuzzy relational matrices can be used if we use multi-valued logics of the quantized domain. A fuzzy controller can be implemented via a parallel computer architecture as shown in Figure 10 since the rules are disjunctive.

Some important aspects of the fuzzy rulebased controllers are as follows: First, the fuzzy dynamic controller $\dot{u}(t) = r_u(x(t), u(t), t)$ is a substitute for the fuzzy regulator $u = r(x(t))$. However, limit cycles are experienced probably caused by the double integrator in the closed-loop system. Second, there is a proportional relationship between the size of the fuzzy rules and the convergence rates. Finally, robustness properties of the fuzzy controller can be enhanced by adding convergent learning algorithms that, in other words, are the static rules that change the fuzzy rulebase itself which is dynamic.

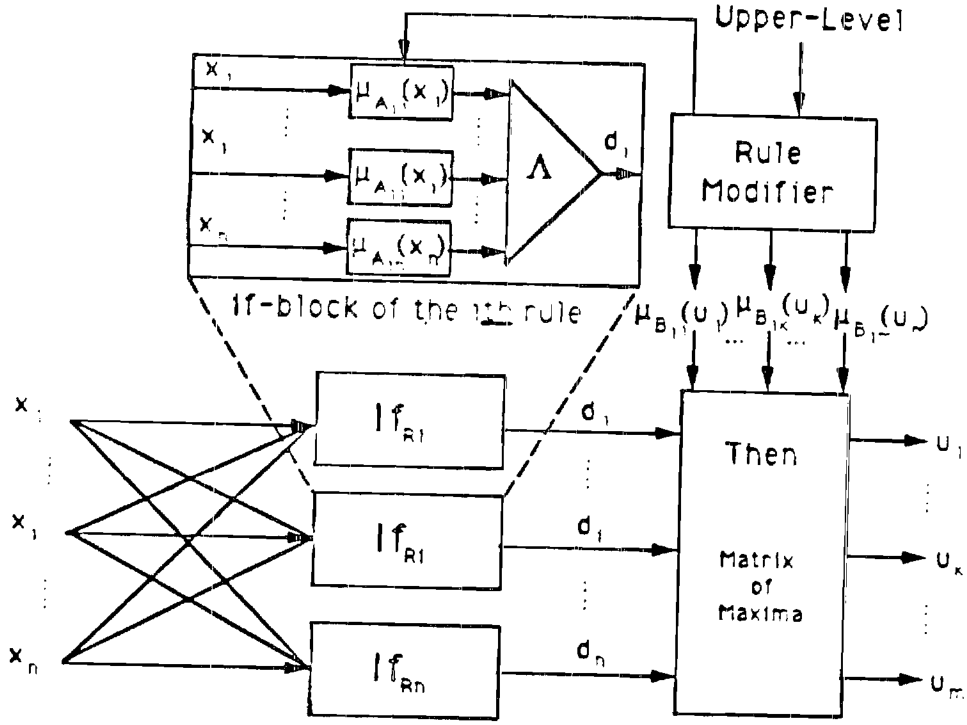

Figure 10: The parallel computer architecture of the fuzzy rulebased systems

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List of Figures

1	Block diagram of a fuzzy-logic feedback system	3
2	S, Π, Z membership functions	7
3	Typical cells and the vector fields of the P^2A^2	10
4	Single-link robot arm	16
5	The cells (a) and the phase portrait (b) of $x_1(t)$ and $x_2(t)$ from the P^2A^2	18
6	Angular position of the single-link robot arm	19
7	Angular velocity of the single-link robot arm	20
8	Fuzzy control torque input applied to the single-link robot arm	20
9	Actual phase portrait of the single-link robot arm	21
10	The parallel computer architecture of the fuzzy rulebased systems	22

Model Reference Fuzzy Control

by

Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

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Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering, Georgia Institute of Technology

Atlanta, Georgia 30332

Abstract

This paper is concerned with a fuzzy logic controller for linear systems. A fuzzy rulebase is employed to stabilize the closed-loop system consisting of a linguistic controller and a process. By utilizing a reference model, impulse-like control inputs can be avoided. A generic guideline for building a control rulebase is developed and membership functions for linguistic variables are chosen to be S, Π, and Z functions. The rules guarantee asymptotic stability. Simulation results show robustness and convergence of the proposed symbolic controller.

1 Introduction

Since the introduction of fuzzy sets and possibility measures by Zadeh [1,2] to cope with the approximate estimation of uncertainty, many learning and adaptation schemes have been developed for various applications. A fuzzy set is defined as a class of objects with a continuum of grades of membership using certainty or confidence factors in order to handle uncertainties or complexity in a particular domain of knowledge [3].

Compared with a probabilistic representation of systems caused by random phenomena, a fuzzy system is an algebraic relation derived from possibility measure theory and it can be represented either by the input-output property or by a transition of states in which case it is called a fuzzy dynamic system. When only imprecise or indefinite information about a process's dynamic behavior is available, then fuzzy sets and linguistic variables may be used to build better models for a process. Several definitions and properties of fuzzy sets and fuzzy numbers are included in Appendix.

A fuzzy number can be described by linguistic terms ('large', 'medium', 'small', 'very small', etc.) whose fuzziness provides many degrees of freedom in dealing with uncertainty by using non-uniform possibility distributions [3]. For any presumption level α $(0 < \alpha < 1)$, the confidence interval represents information that reduces the uncertainty by using lower and upper bounds. From an objective point of view, it may be a confidence interval between two measured data as defined in statistics; from a subjective viewpoint, it may be interpreted as the available expertise about the process. As α increases, the confidence interval decreases in most cases [4].

It is possible to apply fuzzy sets to a classical tracking control problem. Performance criteria of energy functions or least-squares error approaches are suitable for control purposes and rulebases should be constructed to obey an energy-dissipating property. The objective of this paper is to construct a rulebase having the properties of asymptotic stability and robustness under uncertainties such as bounded disturbances, parametric disturbances, and unmodeled dynamics. By providing sufficient conditions for the stabilizing rules, a prototype guideline for the control update is proposed. Simple SISO linear plants are used as examples.

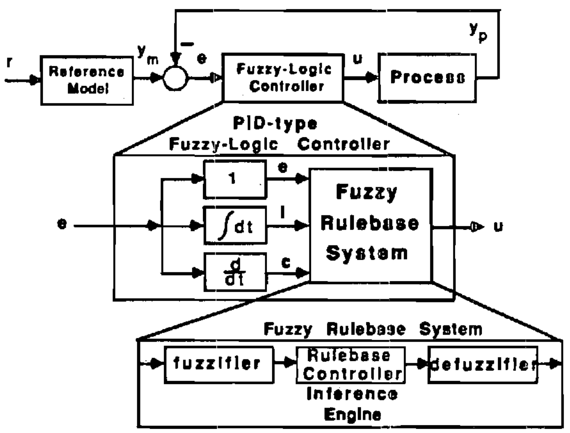

Figure 2.1: Block diagram of the MRFC structure

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2 Model Reference Fuzzy Control

In this section, an expert control system is proposed which consists of a fuzzy rule-based controller and a linear process. A model reference fuzzy control (MRFC) concept is introduced in order to guarantee the model-following property of a fuzzy controller. The mean of maxima procedure is applied to the inference scheme that estimates the control feedback.

2.1 Structure of MRFC

A rule set represents a control objective in fuzzy relations. From measured data such as the speed of a dc motor, the joint angle or velocity of a robot arm, aircraft elevator deflection, and so on, a rulebased controller interprets such data via a fuzzy decision maker and provides control signals which satisfy the performance criterion.

A reference model is implemented in parallel with the process so that the tracking error between the model and the process prevents impulse-like signals in the inferred control value. Figure 2.1 shows the block diagram of the proposed MRFC. The control input $u[k] = u(kt_s)$ can be represented by

$$u[k] = u[k-1] + \delta u[k] \tag{2.1}$$

where t_s is a sampling period and $\delta u(t) = \delta u[k]$ is the update for the control input $u[k]$ from the rulebase. The tracking error $e(t) = e[k]$ is defined by

$$e(t) = y_m(t) - y_p(t) \tag{2.2}$$

where $y_m(t)$ and $y_p(t)$ are the model output and the process output, respectively. The error $e(t)$ is fuzzified in the linguistic controller. The reference model is chosen to be stable and it may be a 1st-order SISO linear system.

Proportional-integral-derivative (PID) and proportional-derivative (PD) control concepts are used as fuzzy inputs to the symbolic controller. The derivative and integral of the tracking error are also fuzzified in the rulebased controller and are defined as follows:

$$c(t) = e[k] - e[k-1] \cong t_s\dot{e}(t) \tag{2.3}$$

$$i(t) = \int_0^t e(\tau)d\tau \tag{2.4}$$

where $c(t)$ and $i(t)$ are the change in error and the integral error, respectively. $\delta u(t), e(t), c(t)$, and $i(t)$ are fuzzified next by specifying the universe of discourse and by assigning appropriate membership functions for each variable as described in the following section.

The rulebased controller may be interpreted as

$$\mathcal{F} : e \Longrightarrow \delta u \quad \text{or} \quad \delta u = F(e) \tag{2.5}$$

where $\mathcal{F}$ and $F(\cdot)$ represent a nonlinear mapping and the corresponding nonlinear function, respectively, that describes a fuzzy relationship including a fuzzifier and a defuzzifier. Let the associated universe of discourse be $\Delta U, E, C, I$ for $\delta u, e, c, i$, respectively, then $\mathcal{F} : E \times C \times I \Rightarrow \Delta U$ for PID control and $\mathcal{F} : E \times C \Rightarrow \Delta U$ for PD control. If the Jacobian of $F(\cdot)$ is defined by

$$J_F = \frac{\partial F}{\partial e} \tag{2.6}$$

then $\dot{u} = \frac{du}{dt}$ may be written as

$$\dot{u} = J_F\dot{e}. \tag{2.7}$$

From (2.1) and (2.3), $\delta u = t_s\dot{u}$ and $c = t_s\dot{e}$, and, therefore, we obtain

$$\delta u = J_F c. \tag{2.8}$$

The Jacobian system for the process is defined by

$$\delta y_p = J_P \delta u. \tag{2.9}$$

Definition 2.1 *The model-following property is stated as follows:*

$$\delta y_m > 0 \Longrightarrow \delta y_p > 0 \quad \text{or} \quad \delta y_m < 0 \Longrightarrow \delta y_p < 0 \quad \forall t > 0, \tag{2.10}$$

or equivalently,

$$\delta y_p \delta y_m > 0 \quad \forall t > 0. \tag{2.11}$$

Consider the following lemma that describes the Jacobian system (2.8).

Lemma 2.1 *Consider a nonlinear system from y_m to y_p represented by*

$$\mathcal{H} : y_m \Rightarrow y_p \tag{2.12}$$

where $y_m, y_p \in \mathcal{R}^1$ and the associated Jacobian system $J_H : \delta y_m \Rightarrow \delta y_p$ defined by

$$\delta y_p = J_H \delta y_m. \tag{2.13}$$

The model-following property holds if and only if $0 < |J_H| < \infty$, and $J_H > 0 \ \forall t > 0$ where

$$J_H = J_P[I + J_F J_P]^{-1} J_F \tag{2.14}$$

where J_P is the Jacobian system for the process.

<u>Proof</u>: sufficiency — Since $\delta y_p = J_H \delta y_m$, and from the model-following property,

$$\delta y_p \delta y_m = J_H \delta y_m^2 > 0. \tag{2.15}$$

Therefore, $J_H > 0$.
necessity — Let $J_H > 0$ and J_H is finite then it is obvious that $\delta y_p \delta y_m > 0$, $\forall t > 0$.
<u>Remark</u>: In fact, using the model-following property, we can say that y_p has the same trend of increasing or decreasing y_m.

A sufficient condition for the model-following property can be found by the following theorem.

Theorem 2.1 *Let $J_P > 0$ in (2.9) or (2.14), then the model-following property holds if there exists $\alpha > 0$ such that $J_H > 0 \ \forall t > 0$, i.e.,*

$$J_F = \alpha J_P^T \quad \forall t > 0 \tag{2.16}$$

<u>Proof</u>: As $J_F = \alpha J_P^T$, substituting it into (2.14), $J_H > 0$, $\forall t > 0$.
<u>Remark</u>: The above theorem says that the Jacobian system for the fuzzy rulebase, J_F should be positive for the model-following property of the system $\mathcal{H}$ if J_P is positive.

2.2 Design of Membership Functions

The design of membership functions is an important step in the development of the fuzzy controller and is closely linked to the possibility distribution of linguistic variables. Fuzzy membership functions should satisfy the properties of convexity and normality in usual cases.

Membership functions for $e(t), \dot{e}(t)$, and $\delta u(t)$ make use of S, Z, and Π curves as proposed by Zadeh [2]. The parameters of these functions should be carefully determined according to the scope of each linguistic variable and they are closely related to the mean and the variance of a probability distribution. It is noted that fuzzy sets do not accumulate every available evidence since the operators involved are minimum or maximum procedures. We define S, Π, and Z membership functions for the MRFC as follows:

$$S(x;\alpha,\beta,\gamma) = \begin{cases} 0 & x < \alpha \\ 2(\frac{x-\alpha}{\gamma-\alpha})^2 & \alpha < x < \beta \\ 1 - 2(\frac{x-\gamma}{\alpha-\gamma})^2 & \beta < x < \gamma \\ 1 & x > \gamma \end{cases} \tag{2.17}$$

$$\Pi(x:\beta,\gamma) = \begin{cases} S(x;\gamma-\beta,\gamma-\frac{\beta}{2},\gamma) & x < \gamma \\ S(x;\gamma,\gamma+\frac{\beta}{2},\gamma+\beta) & x > \gamma \end{cases} \tag{2.18}$$

$$Z(x;\alpha,\beta,\gamma) = 1 - S(x;\alpha,\beta,\gamma) \tag{2.19}$$

The parameters of the membership functions are α, β, and γ. The transition points defined as the value of the universe of discourse for which the grade of membership is equal to $\frac{1}{2}$ are $x = \beta$ for S, Z; $x = \gamma \pm \frac{\beta}{2}$ for Π. The maximum grade of 1 occurs at $x = \gamma$ for all these functions and the minimum value of 0 at $x = \alpha$ for S, Z; $x = \gamma \pm \beta$ for Π.

2.3 Stability of The Closed-Loop Fuzzy System

Stability of the closed-loop fuzzy feedback system is an important issue. Kickert et al.[5] showed that a nonlinear fuzzy-logic controller for a specific simple case is identical to a multi-level relay which can be analyzed via a describing function technique and they anticipated the autonomous oscillation of the system. Ray et al.[6] connected a fuzzy-logic controller to the concept of L_2-stability by using a compensated system with Nyquist criteria. Kiszka et al.[7] introduced the concept of energistic stability for a class of fuzzy dynamic systems.

There are few mathematical properties describing the relationship between membership functions and the inferred control actions. Kania and his co-workers [8] proposed an α-stability property, α-decision stability property, $\alpha - \beta$ strong decision stability property, a good mapping property, a γ-good mapping property, and associated conditions for the above properties as defined in Appendix.

In this paper, we propose a symbolic controller that guarantees global asymptotic stability. Consider the fuzzy relations R_F given by the statements: let the size of a rulebase be s and the number of fuzzified variables m.

- if A_1 then B_1
  ⋮
- if A_i then B_i
  ⋮
- if A_s then B_s

where $A_i = \bigwedge_{j=1}^m A_{ij}$. R_F may be described by

$$R_F = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s). \tag{2.20}$$

For example, A_{11} ='e is positive large', A_{12} ='$\dot{e}$ is negative small', and B_1 ='δu is negative small'. The membership functions for each case are described by $\mu_{A_{11}}(e), \mu_{A_{12}}(\dot{e})$, and $\mu_{B_1}(\delta u)$, respectively. The above fuzzy relations imply the well-known compositional rule of inference [3] (the inference scheme of modus ponens).

In order to guarantee stability of the closed-loop system, the strategic rules should be constructed so as to meet the passivity or the energy dissipating property for a given performance index. The following is a set of guidelines for building a control rule set for stabilizing fuzzy relations:

1. Choose a characteristic function from a referential subset (choose a feasible subset) and select a membership function that follows the α-decision stability property for that feasible subset of a linguistic variable.

2. Navigate the whole set of possible linguistic rules, applying the chosen membership functions to each linguistic variable.

3. For every rule, the γ-good mapping property should hold ($\gamma \neq 0$). That is to say, if the premise part of the i-th rule, A_i, consists of several sub-premises or fuzzy subsets, A_{ij}, such as $A_i = \bigwedge_{j=1}^m A_{ij}$, then, for any j and for some presumption level, $\alpha_1 (0 < \alpha_1 < \alpha)$, there should exist at least one A_{kj} such that $A_{ij} \wedge A_{kj} \neq \emptyset$ for $i \neq k$. This is a necessary condition for the existence of the solution of the modified mean of maxima scheme.

4. In the closed-loop feedback system, if the above conditions hold and the strategic rules are built so that every fuzzy variable has the energy dissipating property, then the fuzzy feedback system is stable and converges. A rule satisfying stability may be deleted since $\delta u = 0$.

The modified mean of maxima procedure is one approach to the defuzzification of fuzzy control variables. As long as the above guidelines are satisfied, the change in control input $\delta u(t)$ can be found by

$$\delta u(t) = \frac{\sum_{i=1}^s \delta u_i^* f_i(t)}{\sum_{i=1}^s f_i(t)} \tag{2.21}$$

where $f_i(t)$ is the degree of fulfilment defined by

$$f_i(t) = \bigwedge_{j=1}^m \mu_{A_{ij}}(x_j(t)) \tag{2.22}$$

with $x_j \in X_j$ (for example, $x_1 = e$, $x_2 = \dot{e}$), and δu_i^* is defined by

$$\delta u_i^* = \{\bar{v} | \mu_{B_i}(v) = \max_v [\mu_{B_i}(v)]; v, \bar{v} \in \Delta U\} \tag{2.23}$$

where ΔU is the universe of discourse of δu. From (2.8), the relationship between the Jacobian J_F and the defuzzifier is described by

$$\delta u = J_F c = \frac{\sum_{i=1}^s \delta u_i^* f_i}{\sum_{i=1}^s f_i}. \tag{2.24}$$

The error becomes

$$e = y_m - \int_0^t h_p(t-\tau) \int_0^\tau \frac{\sum_{i=1}^s \delta u_i^* f_i(\xi)}{\sum_{i=1}^s f_i(\xi)} d\xi d\tau \tag{2.25}$$

where $h_p(\cdot)$ represents the impulse function of the process. Let the impulse response can be separated as

$$h_p(t-\tau) = h_1(t)h_2(\tau) \tag{2.26}$$

Then the change in error $c = t_s\dot{e}$ is

$$c = t_s[J_m\dot{r} - \dot{h}_1\int_0^t h_2(\tau)u(\tau)d\tau - h_p u]. \tag{2.27}$$

It is required that the update must satisfy $ec < 0$ in some region for asymptotic stability.

2.4 A Rulebase for Asymptotic Stability

Let $E(x,t)$ be a candidate energy function then Lyapunov's direct approach can be stated as follows: if $E(x,t)$ is bounded and its derivative $\dot{E}$ exists, the following conditions should hold for asymptotic stability [9]:

$$E(x,t) > 0 \quad \forall x \neq 0 \tag{2.28}$$

$$\dot{E}(x,t) < 0. \tag{2.29}$$

The Lyapunov function candidate is defined by (2.30),

$$E(e,t) = e^T Pe > 0 \tag{2.30}$$

$$\dot{E}(e,t) = \frac{1}{t_s}(c^T Pe + e^T Pc) \tag{2.31}$$

where P is a symmetric positive definite matrix. From (2.31), it is obvious that the signs of e and c should be opposite in order to satisfy $\dot{E}(e,t) < 0$. One possible c may be described by $c = De$ where D is a negative definite matrix.

Assumption 2.1 *Assume that the sign of the dc gain in the transfer function of a linear system is known (i.e., positive).*

The above assumption is minimal in the sense that the degree of the transfer function, relative degree, and exact system delay are not required as in the cases of analytical methods such as linear quadratic regulator (LQR), model reference adaptive control (MRAC), etc.

The following are 9 rules which guarantee global asymptotic convergence in compliance with Lyapunov stability. Here, we use abbreviations for various linguistic descriptions: 'PL' for 'positive large', 'PS' for 'positive small', 'SZ' for 'small near zero', 'NS' for 'negative small', and 'NL' for 'negative large'. Membership functions for e, c, and δu using S, Z, and Π curves are shown in Figure 2.2. A priori information about the process is necessary such as the trends in the impulse response, the approximate time delay, etc. The following is a PD rule set for the fuzzy rulebased controller.

1. if (e is PL) and (c is PL) then (δu is PL)
2. if (e is PL) and (c is SZ) then (δu is PS)
3. if (e is PL) and (c is NL) then (δu is SZ)
4. if (e is SZ) and (c is PL) then (δu is PS)
5. if (e is SZ) and (c is SZ) then (δu is SZ)
6. if (e is SZ) and (c is NL) then (δu is NS)
7. if (e is NL) and (c is PL) then (δu is SZ)
8. if (e is NL) and (c is SZ) then (δu is NS)
9. if (e is NL) and (c is NL) then (δu is NL)

The above rule set represents a PD control scheme for asymptotic stability. Rules 3,5,7 obey Lyapunov stability conditions; Rules 1,4,6,9 satisfy the model-following property of the Jacobian system J_P; and the component of $\delta u(t)$ contributed by the remaining rules 2,8 is chosen to reduce tracking errors in such a way that the process output $y_p(t)$ follows the reference model output $y_m(t)$ which is closely related to the above assumption. In the case of PID control, $i(t)$ is added to the condition part making the size of the rulebase larger. By adding i to the rule set, it is possible to increase the rate of convergence.

Theorem 2.2 *Let the sign of the dc gain in the transfer function and the Jacobian J_P of a process be known (dc gain> 0, $J_P > 0$) and let the suggested guidelines be satisfied for the chosen membership functions of the linguistic variables. Let the PD rule set satisfies the following relations:*

- *case 1: if $ec < 0$ then δu is zero. (Lyapunov stability)*
- *case 2: if $ec \geq 0$ ($c \neq 0$) then $\delta u = J_P c$ with $J_P > 0$. (the model-following property)*
- *case 3: if $ec = 0$ ($e \neq 0$) then δu has the same sign as e. (known dc gain of the process)*

Then asymptotic stability is achieved and the convergence of the tracking error is guaranteed.

Proof: (case1) $ec < 0$ — The vector field of (e, c) directs the state trajectory toward the origin. (case 2) $ec \geq 0$ — From the model-following property, if $\delta y_m \to 0$ then $\delta y_p \to 0$ since $J_B > 0$. Thus, the system error converges to some nominal value. (case 3) In this case, $c = 0$ and the error biases. Therefore, e needs to be adjusted according to the dc gain of the process.

It is noted that, if the sign of J_P is positive, then $J_P > 0$ is a sufficient condition for the model-following property. Figure 2.3 shows the state trajectory and the regions by the 9 rules in a typical case.

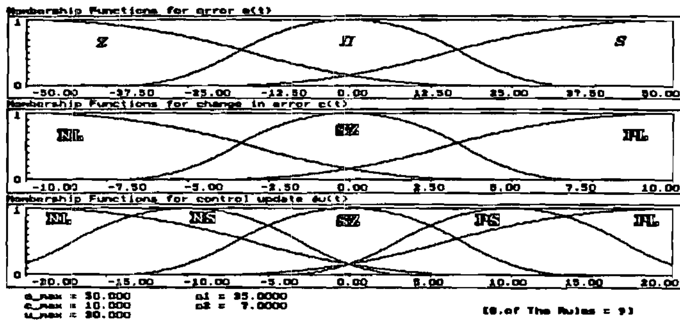

Figure 2.2: The membership functions of e, c, and δu

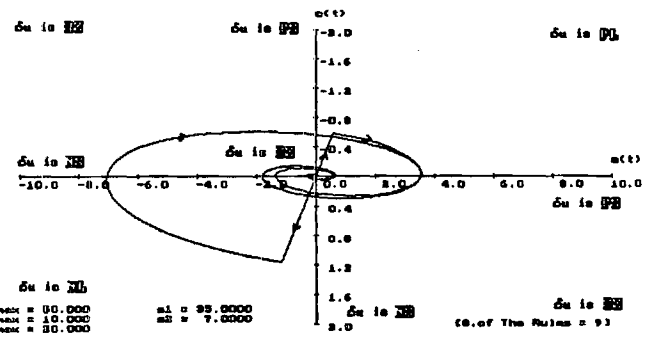

Figure 2.3: The regions and the state trajectory by the 9 rules

3 Simulation Results

A 2nd order single-input single-output (SISO) plant transfer function $H_p(s)$ and the unmodeled version of its transfer function $H_{up}(s)$ are chosen to show robustness of the proposed fuzzy controller,

$$H_p(s) = \frac{2}{s+2} \tag{3.1}$$

$$H_{up}(s) = \frac{40}{(s+2)(s+15)} \tag{3.2}$$

where $H_{up}(s)$ contains unmodeled dynamics of $H_p(s)$ multiplied by $\frac{20}{s+15}$. A reference model $H_m(s)$ is represented by

$$H_m(s) = \frac{3}{s+3}. \tag{3.3}$$

A reference input signal $r(t)$ is of a staircase type: $r(t) = 10$ at $t = 1$ sec, $r(t) = 20$ at $t = 5$ sec, and then $r(t) = 0$ at $t = 9$ sec.

In the fuzzy rulebased controller, the parameters of the membership functions are the maximum values in the region of interest such as $e_{max}, c_{max}, i_{max}$, and u_{max}; mean values of linguistic variables; the sizes of the feasible sets are related to s_e, s_c, and s_i. (s_e is the size of the feasible region between $\mu_{A_i}(x) = 1$ and $\mu_{A_i}(x) = 0.5$ in S, Π, Z functions)

- **case1: unmodeled dynamics**

  Using the above $H_{up}(s)$ as a process and $H_m(s)$ as a reference model, the fuzzy rulebased controller is applied to a tracking problem and the size of the rulebase varies from 9 to 12 to 25 or finally 27 rules. As the number of rules increases, the convergence rate improves as shown in Figure 3.1. The integral term $i(t)$ is included when the number of rules is 27. Subsequent cases are results of 27 rules with unmodeled dynamics. If the sign of the dc gain is negative, the rules should be changed in order to meet asymptotic stability of the closed-loop system.

- **case2: bounded disturbances**

  A uniform additive noise is considered to be corrupting the process output $y_p(t)$ and it is bounded between -0.5 and 0.5 as shown in Figure 3.2. Simulation results show robustness for bounded output disturbances.

- **case3: parameter variations**

  The poles of the process are changed from $t = 3$ sec to $t = 7$ sec and the transfer function $H'_{up}(s)$ in that case is described by

  $$H'_{up}(s) = \frac{60}{(s+4)(s+20)}.$$

  A change in the error dynamics and no significant changes in the process output are observed in Figure 3.3.

- **case4: unstable system**

  The process transfer function becomes unstable and is given by

  $$H_{up}(s) = \frac{4}{s-2}.$$

  Note that the control input $u(t)$ is flipped over in Figure 3.4. Since the steady-state values in the closed-loop system exist, it is possible for the fuzzy controller to find a control input satisfying the stability property.

4 Conclusions and Discussion

Simulation results indicate that the proposed fuzzy control scheme exhibits robustness and stability properties with respect to parametric disturbances, bounded noise, unstable systems, and unmodeled dynamics. The design parameters of the fuzzy controller are less sensitive than those of MRACs or LQRs.

Self-organizing techniques are required for the autonomous addition and deletion of rules and changes in parameters according to some strategic performance indices. As shown in the simulation results, corrective or learning schemes refine the structure of rulebased controllers. In the case of fuzzy relational matrices, some techniques are suggested in [10,11]. Usually, the size of the rulebase increases after a number of iterations of the learning process.

In order to apply the fuzzy linguistic system, expert knowledge is needed for a special domain application. Use of performance criteria in designing a rulebased controller is recommended and it is suggested to describe the plant dynamics as fully and completely as possible.

Some advantages and disadvantages of the rulebased control approach include [12]: Advantages are robustness compared with analytic methods, flexibility in modification of the domain, and improvement in the man-machine interface by providing reasoning behind a particular control decision. Disadvantages are ambiguity in completeness of the rulebase, difficulties in updating and calibrating the rulebased controllers, exhaustive use of the linguistic rules, and superfluous memory requirements.

In general, fuzzy systems need more samples for the decision making process compared with analytical methods but handle unreliable information by using linguistic variables and fuzzy relations. A fuzzy controller can be implemented by a parallel computer architecture since the rules are disjunctive.

5 Appendix: Mathematical Properties and Notations of Fuzzy Sets

Some definitions relating to fuzzy sets and fuzzy numbers are as follows [4]:

Definition 5.1 *An ordinary subset, A, of a referential set, $\mathcal{E}$, is defined by its 'characteristic function'* $\mu_A(x) \in \{0, 1\}, \quad \forall x \in \mathcal{E}$.

A feasible set $F(Y)$ may be described by a characteristic function. A linguistic description in Y such as 'large' resides in a feasible subset $F(Y) \in Y$.

Definition 5.2 *A fuzzy subset, A, from the referential set, $\mathcal{E}$ is defined by its characteristic function* $\mu_A(x) \in [0,1], \ \forall x \in \mathcal{E}$, *which is called the 'membership function'.*

A membership function may represent a non-uniform possibility distribution for the associated linguistic variable described by the characteristic function. The next two definitions are necessary and sufficient conditions for good membership functions in general. A fuzzy set is an invariant set as the presumption level increases.

Definition 5.3 *A fuzzy subset A on $\mathcal{X}$ is 'convex' if and only if* $\forall x_1, x_2 \in \mathcal{R}, \forall \lambda \in [0,1]$:

$$\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \mu_A(x_1) \wedge \mu_A(x_2).$$

Definition 5.4 *A fuzzy subset A on $\mathcal{X}$ is 'normal' if and only if* $\forall x \in \mathcal{R} : \max_x \mu_A(x) = 1$.

The formulas of some fuzzy number arithmetic operators are defined as follows [3,13]:

Definition 5.5 *A 'fuzzy implication' is a mapping* $S : A_i \Longrightarrow B_i$ *and the membership function for S is* $\mu_S(y,x) = \min[\mu_{A_i}(x), \mu_{B_i}(y)]$ *where* $x \in X$ *and* $y \in Y$.

Definition 5.6 *The 'compositional rule of inference' is as follows: for a given fuzzy implication* $S : A_i \Longrightarrow B_i$, *the fuzzy set* B'_i *inferred by a given fuzzy set* A'_i *has the membership function defined by* $\mu_{B'_i}(y) = \max_x \min[\mu_{A'_i}(x), \mu_S(y,x)]$.

Given the following mappings:

$$\mathcal{X} = \{(x, \mu(x))\}; \ \ \mu : X \longrightarrow [0,1]; \ \ x \in X$$

$$\mathcal{Y} = \{(y, \mu(y))\}; \ \ \mu : Y \longrightarrow [0,1]; \ \ y \in Y$$

$$R : \mathcal{X} \longrightarrow \mathcal{Y}$$

where $\mathcal{X}$ is a family of fuzzy sets defined on X, $\mathcal{Y}$ is a family of fuzzy sets defined on Y, and R is a fuzzy relation which can be interpreted as a mapping with its domain in $\mathcal{X}$ and a space of values constrained in $\mathcal{Y}$. Let $U \in \mathcal{X}$ and $V \in \mathcal{Y}$ then a formal fuzziness system is defined as

$$U \circ R = V. \tag{5.1}$$

Some definitions and mathematical properties of the formal fuzziness system are shown as follows [8,14]:

Definition 5.7 *The formal fuzziness system (5.1) has α-stability property with respect to some family $\mathcal{A}$ and some feasible subset* $F(Y) \in Y$ *if* $\mu_V(y) \leq \alpha \ \forall y \in Y - F(Y)$ *for every fuzzy set* $U \in \mathcal{A}$ *of* $U \circ R = V(*)$ *where ∘ is the operator for the compositional rule of inference.*

Theorem 5.1 *Consider the system where* $R = A \Rightarrow B$ *is given. Then* $\bigvee_{i=1}^{n} \mu_A(x_i) \leq \alpha$ *or* $\mu_B(y_j) \leq \alpha$ *for all* $y_j \in Y - F(Y)$ *iff the system (5.1) has the α-stability property with respect to a family $\mathcal{A}$.*

Definition 5.8 *Let* $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ *be given. Then R has the good mapping (GM) property iff* $A_i \circ R = B_i$ *for every* $i = 1, ..., s$.

Theorem 5.2 *Let* $F(Y) \in Y$ *be the feasible set. Then the system (5.1) with the relation defined above has the α-stability property with respect to a family $\mathcal{X}$ of all fuzzy sets defined on X if* $\bigvee_{j=1}^{s} \mu_{B_j}(y) \leq \alpha, \ \forall y \in Y - F(Y)$.

Theorem 5.3 *Let* $A_1, \cdots, A_s$ *be fuzzy sets defined on* $X, \Sigma_{i=1}^{s} \mu_{A_i}(x) = 1$ *for any* $x \in X$, *and let* $B_1, \cdots, B_s$ *be fuzzy sets defined on Y. Let* $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$. *Then* $\mu_{A_k \circ R}(y_j) = \mu_{B_k}(y_j)$ *if for some* $k \in \{1, \cdots, s\}$ *and some* $j \in \{1, \cdots, m\}$ *we have* $\mu_{B_k}(y_j) \leq \max_X \mu_{A_k}(x)$ *and* $\mu_{B_k}(y_j) \geq \mu_{B_l}(y_j), l \neq k$.

Definition 5.9 *A system (5.1) has the α-decision stability property with respect to some family $\mathcal{A}$ of fuzzy sets defined on $\mathcal{X}$ if the system has α-stability with respect to $\mathcal{A}$ and* $\max_{Y-F(Y)} \mu_{U \circ R}(y) < \max_{F(Y)} \mu_{U \circ R}(y)$.

Definition 5.10 *A system (5.1) has the α − β strong decision stability property with respect to some family $\mathcal{A}$ iff the system (5.1) has the α-decision stability property with respect to $\mathcal{A}$ and* $\bigvee_{y \in Y} \mu_{U \circ R}(y) \geq \beta > \alpha$.

Definition 5.11 *Let* $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ *be given where* $A_i \in \mathcal{X}$ *and* $B_i \in \mathcal{Y}$ *for* $i = 1, \cdots, s$. *Then a fuzzy relation R has the γ-good mapping (γ-GM) property in the weaker sense iff* $A_i \circ R = \tilde{B}_i$ *for every* $i = 1, ..., s$ *where* $B_i \subseteq \tilde{B}_i$, *and* $\mu_{\tilde{B}_i} = \mu_{B_i}$ *for* $\mu_{B_i} \geq \gamma, 0 \leq \gamma \leq 1$.

Note that, when $\gamma = 0$, the γ-GM property reduces to the GM property as defined above [14].

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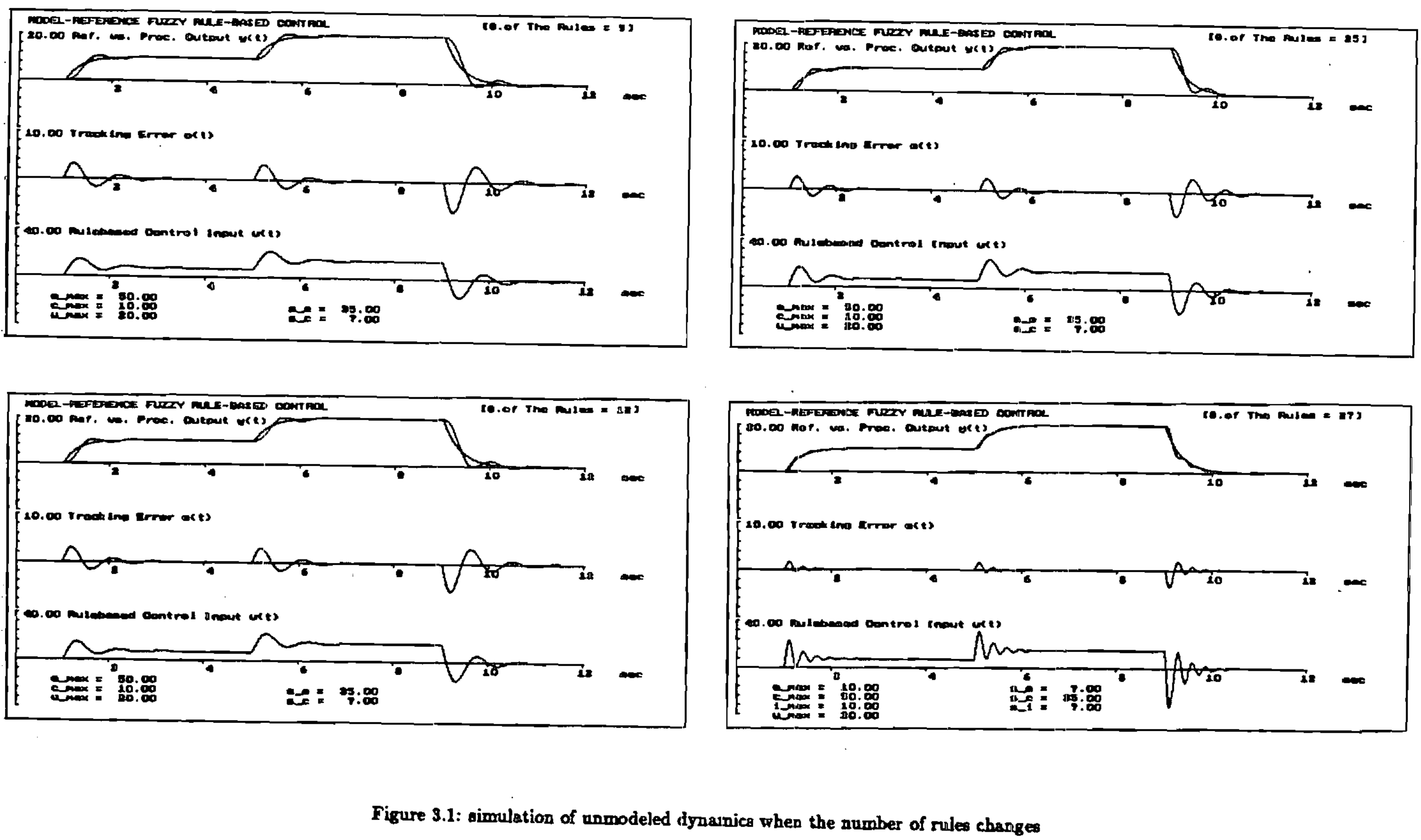

Figure 3.1: simulation of unmodeled dynamics when the number of rules changes

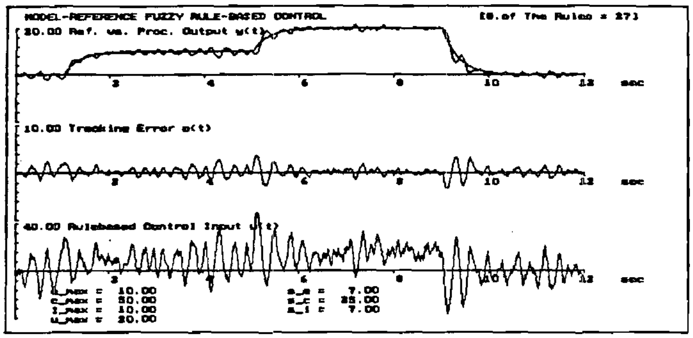

Figure 3.2: simulation of a bounded disturbance

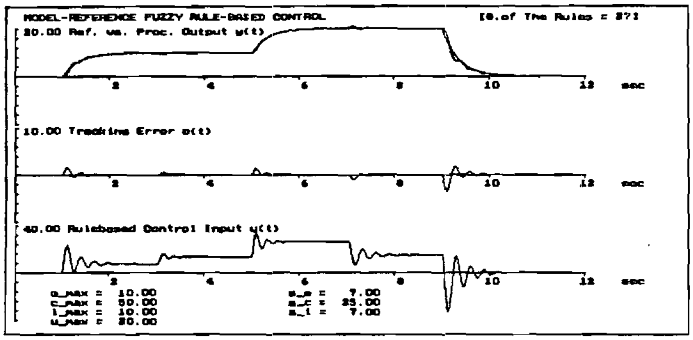

Figure 3.3: simulation of a parametric disturbance

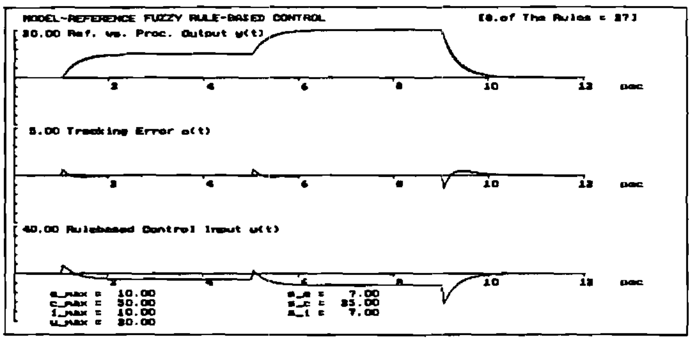

Figure 3.4: simulation of an unstable system

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

ELEPHONE: (404) 894- 6252

November 27, 1989

Dr. James G. Smith (Code 1211)
Program Manager, Applied Research &
Technology Directorate
Office of Naval Research
Arlington, Virginia 22217-5000

Dear Dr. Smith:

Enclosed please find a summary report on research performed under ONR contract number N00014-89-3113 titled "A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design."

Since research activity on this project was initiated in September 1989, this report covers the reporting period from September 1, 1989 to December 1, 1989 and details some preliminary results while pointing out future research directions.

I will be pleased to provide you with any additional information required.

Very Sincerely,

George Vachtsevanos
Principal Investigator
Professor
School of Electrical Engineering

Enclosure

Annual Letter Report for FY 89

A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design

(Contract No: N00014-89-J-3113)

Sponsor: **Office of Naval Research**
Applied Research and Technology Directorate

Principal Investigator: **Dr. George Vachtsevanos**
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Tel: (404)894-6252

December 2, 1989

A HYBRID ANALYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN

CONTRACT NO: N00014-89-J-3113

PI: Dr. George Vachtsevanos
Georgia Institute of Technology

In September 1989, the Office of Naval Research awarded contract No. N00014- 89-J-3113 to the Georgia Institute of Technology to develop fault-tolerant control strategies for large scale dynamical systems. Specifically, the technical issues under investigation are: Development of a structure-based modeling methodology of large scale systems which possesses features of structure flexibility, i.e. a component or subsystem may be deleted in a simple way that does not substantially disturb the global control strategy; development of robust failure detection and fault identification algorithms for single and multiple faults that are maximally sensitive to true failure conditions but insensitive to noise, thus reducing the possibility of false alarms; a technique to restructure the system dynamics by isolating the faulty components; a method that will lead to a structural control law which reconfigures the original system controller in order to meet the primary performance objective of guaranteed stability while the system is operating in a degraded mode; finally, these algorithmic developments are to be demonstrated on an actual physical system to be designated jointly by ONR and Georgia Tech.

The technical approach pursued to meet these objectives is outlined as follows: The underlying factor of the fault-tolerant methodology capitalizes upon structural features of the large scale system and employes a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. In the modeling area, the topological description of many complex dynamical processes may be cast in the form of structurally interconnected subsystems. The large scale system is composed of a number of linearly interconnected subsystems. In every subsystem, a control law is applied which consists of a local and a global feedback term, thus resulting in a two-level hierarchical control strategy. We examine first large scale systems which are described by linear state equations and exhibit this two-level hierarchical structure. The analysis will be extended eventually to nonlinear systems of a similar structure. A methodology for fault diagnosis is introduced which is based upon a combination of signal redundancy and detection/estimation procedures. An expert system, consisting of a multivalued rule base and an appropriate inferencing mechanism, assesses the aggregate of fault symptoms and determines the sensitivity of a failure condition. An innovative approach to multiple failure detection uses qualitative simulation techniques to decide on the impact of a component failure on other (healthy) system components. The proposed fault detection and identification scheme incorporates such additional functionalities as fault trending and the estimation of the "best" value of critical variables and parameters under failure states. Assume that the presence of a failure has been detected by the FDI algorithm and means for isolating the faulty or potentially faulty components are available, then the system restructuring function is undertaken by a supervisory controller which modifies the original (healthy state) description of the system by changing the appropriate entries of the local subsystem and interconnection matrices.

Finally, we propose the development of a two-level structural dynamic hierarchical approach to address the control reconfiguration problem of a faulted large scale system. The approach uses a structural state model, the Block Arrow Structure, which consists of N independent linear subsystems interconnected with a control common linear dynamic system. The objective is to find a time-invariant decentralized feedback controller which minimizes a quadratic cost functional and has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

A HYBRID ANALYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN
CONTRACT NO: N00014-89-J-3113
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Research Accomplishments

During this reporting period 9/1/89 - 12/1/89, the research team focused attention on the issues of modeling and fault detection and identification of dynamical Large Scale Systems(LSS).

1 LSS modeling

In the modeling area, a review of available techniques was completed and such schemes as aggregation and perturbation analysis, hierarchical and decentralized approaches and the descriptive variable method were assessed as to their suitability in fault tolerant control. The basic objective here is to develop mathematical modeling tools for large scale systems that possess attributes of modularity and structural flexibility so that a failed component or subsystem may be easily isolated without seriously affecting the global behavior of the system. More specifically, the failed system is to be restructured and the control actions reconfigured so that stability(survivability) is assured for the duration of the emergency.

We are developing a two-level structural dynamic hierarchical approach to address the control reconfiguration problem. We seek a structural control law and, by effectively combining information with control action, to move closer to create a structural intelligent control concept. We outline below the innovative features of this approach.

Suppose that the large scale system consists of l interconnected nonlinear subsystems of the form

$$\dot{z}_i = f_i(z_i, t) + g_i(z_1, \ldots, z_l, t) \qquad i = 1, 2, \ldots, l \tag{1}$$

where

$$z_i \in R^{n_i} \text{ is the state vector of the } i\text{th subsystem}$$

$$f_i : R^{n_1} \times J \to R^{n_1},$$

$$g_i : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_l} \times J \to R^{n_i}$$

Each subsystem, when isolated, is given by

$$z_i = f_i(z_i, t) \tag{2}$$

The function $g_i(z_1, \ldots, z_l, t)$ represents the interconnections of the ith subsystem with the remaining subsystems.

In every subsystem of the form (1), a control law is applied which consists of a local and a global feedback term. Specifically, for the ith subsystem (1), the control input is

$$u_i(t) = u_i^l(t) + u_i^g(t) \tag{3}$$

If the controllers are linear, then they have the form:

$$u_i^l(t) = k_{ii} z_i(t) \tag{4}$$

$$u_i^g(t) = \sum_{j=1, j \neq i}^{l} k_{ij} z_j(t) \tag{5}$$

where $k_{ij} \in R^{m_i \times m_j}$ and $u_i(t) \in R^{m_i}$.

A two-level hierarchical control is obtained by assigning proper interconnections and feedback loops to the large scale system. At the lower level we have l subsystems, S_i, $i = 1, 2, \ldots, l$, which are described in state space by the equations:

$$S_i : \dot{z}_i(t) = f_i(z_i, t) + h_i(u_i, t) + f_{i0}(z_0), \quad i = 1, 2, \ldots, l \tag{6}$$

where

$$h_i : R^{m_i} \times J \rightarrow R^{n_i}$$

$$f_{i0} : R^{n_0} \times J \rightarrow R^{n_i}$$

and $z_0 \in R_{n0}$ is the state vector of the coordinator system in the upper level. The upper level is occupied by the coordinator system S_0 described by the state space model:

$$S_0 : \dot{z}_0(t) = f_0(z_0, t) + \sum_{i=1}^{l} f_{0i}(z_i, t) + h_0(u_0, t) \tag{7}$$

where $u_0(t)$ is the control law of the coordinator and

$$h_0 : R^{m_0} \times J \rightarrow R^{n_0}$$

The structural implications of the two-level model on fault-tolerant control become evident when we consider a linearized version of the system dynamics. Here, we use a structural state model which consists of N independent linear subsystems $S_1, S_2, \ldots, S_N$ interconnected with a control common linear dynamic subsystem S_0. The mathematical model of the time- invariant large scale system, in a decomposed form, is

$$S_i : \dot{z}(t) = A_i z_i(t) + B_i u_i(t) + A_{i0} z_0(t) \tag{8}$$

$$\text{with} \qquad z_i(t_0) = z_{i0}, \quad z_0(t_0) = z_{0_0}$$

$$S_0 : \dot{z}_0(t) = A_0 z_0(t) + B_0 u_0(t) + \sum_{i=1}^{N} A_{0i} z_i(t) \tag{9}$$

where $z_i(t) \in R^{n_i}, z_0(t) \in R^{n_0}$ and $u_i(t) \in R^{r_i}, u_0(t) \in R^{r_0}$ are the state and control vectors for subsystems S_i and S_0, respectively.

The standard overall description of (8),(9) is

$$\dot{z}(t) = Az(t) + Bu(t), \qquad z(t_0) = z_0 \tag{10}$$

$$A = \begin{bmatrix} A_1 & 0 & \dots & A_{10} \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & A_l & A_{l0} \\ A_{01} & \cdots & A_{0l} & A_0 \end{bmatrix} \in R^{n \times n}, \qquad B = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & B_l & 0 \\ 0 & & 0 & B_0 \end{bmatrix} \in R^{n \times r}$$

Since the dynamics matrix (10) consists of Block elements arranged in an Arrow Structure, the system (10) is called a Block Arrow Structure(BAS) decentralized large scale system.

By appropriately defining a quadratic cost functional, a time-invariant BAS decentralized feedback controller may be designed which has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

We are currently investigating critical properties of LSS, from the fault- tolerant control point of view, that include restructurability and reconfigurability. Specifically, we have identified the following key properties for restructurable/reconfigurable systems:

- Flexibility: based upon the strength of subsystem interconnections (interlevel or intralevel).
- Stabilizability: in terms of structural controllability and structural observability concepts.

Our investigations thus far indicate that the modeling approach pursued is representative of a large class of engineered systems, that, because of critical performance requirements, require some degree of fault tolerance in the control design.

2 Fault Detection and Identification

A major effort has been undertaken, during this initial phase of the project, to develop an integrated methodology to fault detection and identification that capitalizes upon a combination of conventional techniques and artificial intelligence. New and innovative concepts are introduced in the FDI approach that maximize symptomatic evidence, account for sensor and system uncertainties and combine failure data even when the latter are of a conflicting nature.

FDI techniques are considered for sensor, actuator and component failures. Both single and multiple faults are treated. The research objective is to develop a systematic and thorough FDI procedure that is maximally sensitive to failures while avoiding false alarms. The approach is systematic because it relies on a modular architecture to (a) trigger efficiently the FDI routines, (b) validate sensor data, (c) combine failure evidence from such diverce sources as analytic redundancy, dectection/estimation theory and limit checking, (d) utilize expert system tools and Dempster-Shafer evidential theory to manage uncertainty and assess the symptomatic evidence, i.e. detect and identify faulty components, and (e) finally, provide trending information as well as the best value of critical variables and parameters for monitoring and control purposes.

This research team has previously developed FDI procedures based upon parity space by utilizing analytic redundancy. We will highlight below only some recent developments in detection/estimation and evidential theory. Symptoms derived from this technique will augment the failure evidence available from the parity space procedure to assure a robust FDI approach.

2.1 New Directions in FDI

Decision strategies for FDI based on a Bayesian approach were reviewed. Bayes' estimation theory using minimum mean squared estimation and joint detection/estimation(JDE) procedures as well as test methods such as the M-hypothesis, generalized likelihood and probability ratio tests were critically assessed as to their effectiveness in FDI and their computational cost and complexity. Trade-offs were considered and robustness issues were addressed. Such Bayesian methods as the multiple model approach were examined in detail using a general linear discrete-time multiple dynamic model. Here, failure modes are modeled as switching parameters and it is assumed that the number of failure models is finite and compact. The ith model of the linear process is represented by

$$x_i(t+1) = A_i(q,\theta_i)x_i(t) + B_i(q,\theta_i)u(t) + w_i(t) \tag{11}$$

$$y(t) = H_i(q,\theta_i)x_i(t) + v_i(t) \tag{12}$$

where $u(t), y(t)$ are the actuator input and the measured output, respectively; $x_i(t)$ is the state of the ith model; A_i, B_i, and H_i are the associated system matrices; θ_i is the parameter vector of the process which may be time-varying; q is the shift operator defined by $q^{-1}x(t+1) = x(t)$; $w_i(t) \propto N[0, Q_i]$ is the white Gaussian process noise; $v_i(t) \propto N[0, R_i]$ is the white Gaussian measurement noise.

The minimum mean squared estimate(MMSE) can be represented by

$$\hat{x}^{MS}(t|t) = E(x(t)|Y(t)] = \arg\min_{x(t)} E(x(t) - \hat{x}(t|t))(x(t) - \hat{x}(t|t))^T$$

$$\text{where } Y(t) = [y(t), y(t-1), \ldots, y(t-n)]^T$$

The ith Kalman filter equations are easily derived from these assumptions. The computational shortcomings of both optimal and suboptimal procedures following these Bayesian methods are well documented in the techniical literature. The statistical ratio test paradigms may be circumvented by using a Markov chain with an associated transition probability matrix. Consider a joint detection/estimation algorithm based on this approach. Each state of the model may be described by an integer index $s(t)$. We define a windowed Markov chain state sequence $I_i(t)$ by

$$I_i(t) = \{s(t) = i, s(t-1), \cdots, s(t-n+1)\} \tag{13}$$

where the window size is n, $s(t) \in \{1, 2, \cdots, M\}$, and M is the total number of stochastic models. $s(t)$ determines or identifies the structure of the failure model at time t.

Suppose that, given a stochastic process $s(t)$ with a finite number of possible integers, the associated Markov chain exists such that

$$p(s(t)|s(t-1), \cdots, s(0)) = p(s(t)|s(t-1)) = \pi(s(t), s(t-1)) \tag{14}$$

$$p(t) = \Pi p(t-1) \tag{15}$$

where $p(\cdot) \in R^M$ stands for probability of the state sequence, $\Pi = \{\pi(i, j)\} \in R^{M \times M}$ for all states $\{s(0), \cdots, s(t)\}$ and for all $t \geq 0$. $\pi(i, j)$ is referred to as the transition probality from the model state j to the next state of the model i. Now, we can consider state estimation and structural detection for this partitioned systems.

Given the probability density function $f(\cdot)$ of the state $x_i(t)$, the estimate of the ith model is

$$\hat{x}(t|t) = E[x_i(t)|Y(t), I_i(t)] \tag{16}$$

and the detection/estimation approach can be summarized as follows:

- **state estimation**:

$$\hat{x}(t|t) = \sum_{i=1}^{M} \hat{x}_i(t|t)p(I_i(t)|Y(t)) \tag{17}$$

$$\text{where } p(I_i(t)|Y(t)) = \frac{f(Y(t)|I_i(t))p(I_i(t))}{\sum_{j=1}^{M} f(Y(t)|I_j(t))p(I_j(t))} \tag{18}$$

- **structural detection**

$$p(i|Y(t)) = \sum_{\forall I_i(t)} p(I_i(t)|Y(t)) \tag{19}$$

where $f(\cdot|\cdot)$ is a conditional pdf of $Y(t)$ given the sequence $I_i(t)$.

It is noted that this state estimation approach requires a bank of Kalman filters for the fixed window and the number of states must be the same in each failure model. These problems may be alleviated by employing an alternate approach which is based on multiple parametric models. The proposed approach relies on the error equation of auto-regressive moving average (ARMA) models. The ith model of the system is described by

$$A_i(q,\theta) = q^{-d}B_i(q,\theta)u(t) \tag{20}$$

where $A_i(q,\theta)$ is a monic polynomial, $B_i(q,\theta)$ is a stable polynomial, and d is the system delay. If the process is considered to be linear, then from the Kalman filter interpretation for the time-varying system, the parameter estimate $\hat{\theta}_i(t)$ can be calculated as follows:

$$\hat{\theta}_i(t) = E[\theta_i(t)|Y(t)] = \hat{\theta}_i(t-1) + K_i(t)\{y(t) - \phi_i(t)^T\hat{\theta}_i(t-1)\} \tag{21}$$

where $\hat{\theta}_i(t) = [a_1, \cdots, a_n; b_1, \cdots, b_m]^T$, $\phi_i(t) = [-y(t-1), \cdots, -y(t-n); u(t-1), \cdots, u(t-m)]^T$, and $K_i(t), P_i(t)$ are the familiar Kalman gain and covariance matrices, respectively.

Now, parameter estimation and structural detection may be accomplished by using the following computations:

- **parameter estimation**:

$$\hat{\theta}(t) = \sum_{i=1}^{M} \hat{\theta}_i(t)p(I_i(t)|Y(t)) \tag{22}$$

$$\text{where } p(I_i(t)|Y(t)) = \frac{f(Y(t)|I_i(t))p(I_i(t))}{\sum_{j=1}^{M} f(Y(t)|I_j(t))p(I_j(t))} \tag{23}$$

- **structural detection**

$$p(i|Y(t)) = \sum_{\forall I_i(t)} p(I_i(t)|Y(t)) \tag{24}$$

The generalized likelihood ratio (GLR) test is shown to be appropriate here to execute the detection and estimation strategy.

A new strategy to the detection/estimation problem is based on possibilistic methods and is implemented using expert system tools. A new failure detection scheme is obtained by utilizing Zadeh's fuzzy set theory in constructing a decision-making rule base. Two possible approaches are considered: The first one involves the design of membership functions for the a posteriori probability or likelihood ratio of each failure model followed by a determination of the fuzzy relational mapping for the rules. The second refers to the collection of all measures of the failure modes in evidence, which may be functions of standardized normal variables, to evaluate statistical ratio tests for them, and finally, to fire those rules that satisfy a particular failure mode.

For, the fuzzy failure detection scheme, some appropriate measure such as the a posteriori probability or likelihood of each failure model is used. To illustrate the point, the fuzzy rulebase consists of rules of the form:

- If 'p1 is large' and 'p2 is small', then 'prob model is 1' or
- If 'r1 is large' and 'r2 is small', then 'likely model is 1'

where p1 and p2, r1 and r2 are the a posteriori probabilities and likelihood ratios for models 1 and 2, respectively. The goal here is to find the fuzzy relational mapping or fuzzy relational matrix for failure detection. The advantages of this methodology are a significant reduction in decision errors by using linguistic variables and a considerable saving in computational time through parallel (disjunctive) rule implementation.

The coupling of the new FDI scheme to the multiple parametric model approach is achieved as follows: For each kth component of the parameter vector θ_i, the parametric space is divided into several crisp cells that represent some specific symptoms or failure evidence in the associated parameter. These cells can be chosen according to probabilistic specifications such as mean, variance, etc. Next, under real time parameter estimation conditions, these values for each model in a particular cell can be used to determine the failure mode at hand and infer structural detection through the output of the fuzzy rulebase.

Failure detection may also be accomplished using production rules. In this case, the decision depends on a description of crisp failure situations under the assumption that all possible failure modes are completely specified.

A typical rule in the production rulebase for failure detection of impulse lines is

- If 'reading is full-scale', then 'line is blocked'.

This last methodology holds a great deal of promise when all possible failure modes are considered.

Presently, simple dynamical systems are modeled and both Bayesian and possibilistic approaches are simulated with the intent of arriving at the optimum detection/estimation algorithm that satisfies the performance criteria and yet is computationally efficient.

The final thrust of our current research activity is concerned with the study of the Dampster-Schafer mathematical theory of evidence and its possible implications to the FDI problem. Dempster-Schafer theory provides an excellent tool for combining several pieces of evidence from a knowledge source. We intend to use this tool in order to combine failure symptomatic evidence resulting from the application of parity space, possibilistic detection/estimation and limit checking techniques. The power of D-S theory is in the representation of subjective(as opposed to probabilistic) uncertainty, mainly by virtue of its expression of ignorance. The groundwork for applying D-S theory to the FDI problem has been laid and we expect to report on this task in the near future.

In summary, substantial progress has been made in conceptualizing and developing new and innovative techniques to address the FDI and modeling problems of dynamical large scale systems that undergo component or sensor failures. The distinguishing charasteristic of our approach is a blend of analytical and symbolic representations. Computational tools are used to develop, simulate and demonstrate the power and effectiveness of these techniques. For the remainder of this fiscal year, we will continue to pursue this research direction so that we may produce a viable and generic package of FDI routines that can be applied to any complex physical system.

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Presentations and Technical Contributions

1. In September 1989, a presentation was given to the staff of NASA's MSFC at Huntsville, Alabama by Drs. Vachtsevanos and Arkin on possible applications of fault-tolerant control technology to the space station program.
2. On 11/9/89, Dr. G. Vachtsevanos was invited to present a seminar to the technical staff of Rockwell International Corp., the Science Center at Thousand Oaks, California on fault-tolerant control techniques. He held technical discussions with Rockwell personnel on issues of fuzzy logic and the application of fault-tolerant control techniques to aerospace systems.
3. In November 1989, discussions were held and a statement on fault- tolerant control was mailed to Nothrop Corp.
4. Interest on our fault-tolerant control activities has been expressed by the Federal Aviation Administration. A position paper on this subject was mailed to FAA research staff.
5. In June 1989, Dr. Vachtsevanos chaired an invited session on Intelligent Control at the 1989 Automatic Control Conference and presented a paper entitled "Fault Tolerant Control of Dynamical Large Scale Systems". A copy of the paper is attatched to this report.
6. In December 1989, Dr. Vachtsevanos will present a paper, co-authored by Dr. H. Kang, on "Model Reference Fuzzy Control" at the 28th IEEE Conference on Decision and Control, Tampa, Florida.
7. A paper entitled "Stability and Robustness of a Nonlinear Fuzzy Controller Based on the Phase Portrait Assignment Algorithm", co-authored with H. Kang, has been submitted for publication to the IEEE Trans. on Systems, Man and Cybernetics.

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Project Participants

The parincipal Investigator of this project is

Dr. George Vachtsevanos, Professor of Electrical Engineering at the Georgia Institute of Technology, Atlanta, Georgia.

Dr. Ronald Arkin, Assistant Professor at the School of Information and Computer Science, the Georgia Institute of Technology, serves as a co- investigator to the project.

Dr. Hoon Kang, a recent Ph.D. graduate at the Georgia Institute of Technology, is participating in project activities as a post-doctoral fellow.

Three graduate students in the School of Electrical Engineering at Georgia Tech are currently receiving partial support from this project.

FAULT-TOLERANT CONTROL OF DYNAMICAL LARGE SCALE SYSTEMS

George Vachtsevanos & Young-Tae Kim
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250

Manolis Christodoulou
Technical University of Crete
Chania, Greece

ABSTRACT

A fault-tolerant design procedure for large scale engineered systems is introduced. The proposed methodology is based on a hybrid approach that capitalizes upon beneficial attributes of conventional systems and control-theoretic techniques, as well as methods of artificial intelligence. The paper focuses on a fault propagation algorithm that assesses quickly the impact of a failure on other healthy components and subsystems. Qualitative reasoning techniques are employed to determine the state transitions. An example from a major subsystem of the space station is used to illustrate the algorithmic developments.

1. INTRODUCTION

Modern dynamical systems, such as aircraft and space vehicles, nuclear power plants, and many industrial and manufacturing processes, are complex systems that place a heavy burden on performance monitoring, status evaluation, and control strategies. As the technological sophistication of these systems increases, they are required to be more fault tolerant so that the system availability is kept high at the maximum possible rate.

The problem of designing fault-tolerant control systems has been addressed in the recent past in terms of two general and fundamental approaches [1-5]. The first one relies upon modern control-theoretic techniques to recast the fault tolerant system design into a classical control problem, while the second is based on decision-theoretic concepts related to the field of artificial intelligence in order to provide partial answers to the problem. A critical review of both AI-based and control-theoretic techniques for fault-tolerant control reveals that there is no unified methodology to integrate those diverse issues of system modeling, fault detection and isolation, fault propagation, system restructuring, and controller reconfiguration.

This need is addressed by introducing a problem-focused system integration philosophy that capitalizes upon a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. AI techniques allow a marked reduction of the set of hypotheses, thus permitting traditional control-theoretic approaches to be applied efficiently.

When one or more failures or abrupt parameter changes are detected, the system enters an emergency mode. The proposed fault-tolerant control system design procedure takes over in that event and entails the following tasks (Figure 1):

- Fault Detection and Identification (FDI)
- Fault Propagation (FP)
- System Restructuring
- Controller Reconfiguration.

A methodology for fault diagnosis of large scale dynamical systems is introduced which is based upon a combination of signal redundancy techniques and fuzzy logic.

Suppose that a fault or parameter deviation occurs in some subsystem of a large scale process. Then the deviation propagates to adjacent subsystems before the faulty component is isolated. The fault might propagate through an adjacent subsystem without causing any further fault, it might be absorbed in this subsystem (depending on its transition pattern), or it might cause this subsystem to fail. When a faulty behavior or a marked deviation in the system's behavior is observed, then it will be necessary to track the current behavior pattern of the system and its major subsystems and take fault-tolerant action before a permanent failure occurs and cripples the system. This paper addresses this issue by recognizing the unconventional nature of the problem. For a fault propagation model to be effective, it must be capable of predicting the impact of a failure on other system components well in advance before the actual effect is realized. Only then may the system have opportunity to restructure itself by isolating failed or potential faulty subsystems so that overall system survivability is assured.

2. FAULT PROPAGATION

A fault propagation model is defined here to be a representation of the propagation of input system variable deviations that originate at the faulty unit to other system components. A fault is initiated in a unit which by definition is unhealthy, propagates through a set of units which are generally healthy (although in some units, an additional enabling fault is a condition for further propagation), and terminates in a unit or units which are thereby rendered unhealthy.

Fault Tree Analysis (FTA) [6] has been classically used to discover the paths to failure in complex systems.

In a different approach [7], it is possible to model the subsystems of the large scale system as nonvariant stochastic automata with a transition matrix between the input and output states.

The proposed approach to modeling of fault propagation begins with a system structural diagram and decomposes it into its constituent units or components; it then represents the system as a causal network via a qualitative simulation.

Qualitative Reasoning (QR) has been recently an area of intense research activity within the artificial intelligence and cognitive sciences [8-11]. Investigators in these two fields have studied qualitative causal reasoning models of physical systems or mechanisms.

Considering the nature and complexity of large scale dynamical systems, Kuipers' approach to qualitative simulation seems to be the most appropriate. We propose to augment his original approach by including explicitly constraints that represent conservation of energy via scalar Lyapunov functions. This addition eliminates most spurious behaviors and it is hoped that will lead to predicting the single correct behavior of the faulted system.

2.1 QUALITATIVE SIMULATION

The goal of qualitative physics is to provide a theoretical framework for understanding the behavior of physical systems.

From the qualitative structure of the physical system, Qualitative Simulation (QSIM) uses a local propagation strategy to predict the qualitative behavior of a mechanism. The qualitative structure of a mechanism is described by a set of qualitative constraints. Each constraint specifies a relationship that holds between two or more functions of the mechanism. Each function has a qualitative state consisting of a qualitative value and direction of change. A qualitative value can be a landmark value such as 0 or it can be an open interval between two landmark values. At first, QSIM determines a complete initial state by propagating known qualitative values and directions of change (given the initial condition) through the given constraints before proceeding to predict possible behaviors. With a complete set at a distinguished time point, QSIM uses a strategy of proposing and filtering sets of qualitative transitions to predict all possible subsequent states in the following open time interval. A number of real valued parameters, which vary continuously over time, characterize a physical system. Each parameter can be considered as a function $f:[a,b] \to R^*$, where $R^* = [-\infty,\infty]$, the extended real number line. The qualitative behavior of f on [a,b] is the sequence of qualitative states of f:

$$QS(f,t_0), QS(f,t_0,t_1), QS(f,t_1), QS(f,t_1,t_2), \ldots, QS(f,t_{n-1},t_n), QS(f,t_n)$$

Constraints holding between parameters in the structural description of the system serve to limit the possible combinations of qualitative behavior. Constraints are expressed as predicates rather than as functions for two reasons: First, they will be used as predicates in the QSIM algorithm to test the consistency of sets of qualitative values. Second, if a constraint were to be treated as a function, it is unclear how to define precisely the function's range. At large there are two types of constraints which are used in QSIM:

- Arithmetic constraints
- Qualitative function constraints.

An example of an arithmetic constraint is:

- ADD(f,g,h): iff $f(t) + g(t) = h(t)$ for every $t \in [a,b]$.

Qualitative function constraints are defined as follows:

- M^+ is a two-place predicate on reasonable functions $f,g:[a,b] \to R^*$.

$M^+(f,g)$ is true, iff $f(t) = H(g(t))$ for all $t \in [a,b]$, where H is a function with domain $g([a,b])$ and range $f([a,b])$, differentiable with $dH(x)/dt > 0$ for all x in the interior of the domain. Similarly, M^- can be defined except that $dH(x)/dt < 0$.

We propose additional constraints related to energy functions. A scalar Lyapunov function, associated with a subsystem of the engineered large scale system, is viewed as an energy function.

Let V(x) be a candidate Lyapunov function with $\dot{V}$ its time derivative. Qualitative constraints may be imposed by asserting the following conditions:

(1) If $\dot{V} < 0$, the system tends to be stable and thus its energy decreases.

(2) If $\dot{V} = 0$, the system is marginally stable and its energy remains constant.

(3) If $\dot{V} > 0$, the system may be unstable, thus its energy may be increasing.

2.2 FROM STATE SPACE TO QUALITATIVE SPACE

A large scale engineered system is considered to be composed of a number of linearly interconnected linear or nonlinear subsystems. A control law is applied to each subsystem which consists of a local and a global feedback term. Let us focus attention first to a typical subsystem without regard to interconnections. We will define some useful concepts for linear systems only although the extension to the nonlinear case is straightforward except for the number and the nature of the qualitative constraints.

Consider the state space model of a controllable system described as:

$$\dot{x} = Ax + Bu \qquad y = Cx \tag{1}$$

where x is an n-dimensional state vector and A,B,C are matrices with appropriate dimensions.

With the controllability assumption, the system equations may be written in the controller canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u \quad (2)$$

Or, more explicitly as:

$$\begin{aligned} x_1 &= \dot{x} \\ x_2 &= \dot{x}_1 \\ &\vdots \\ x_n &= \dot{x}_{n-1} \\ \dot{x}_n &= -a_1 x_n - a_2 x_{n-1} - \cdots - a_n x_1 + u \end{aligned} \quad (3)$$

Equations (3) define n+1 qualitative variables and their derivatives. We seek the qualitative behavior of these variables starting from some given initial values. The qualitative value of a variable is either a symbolic or numeric value and its direction of change, i.e. increasing, descreasing, or steady.

A set of constraint equations defined on the basis of constituent component models and their interconnections (the state equations), as well as functional and global (energy) relations, specify the transition from state space to qualitative space. The qualitative operations do not define a mathematical field. Only a few qualitative solutions or combinations are consistent with the equations. The set of qualitative constraints is intended to limit the behavioral modes or interpretations of the system to the correct ones.

From the above relations, an appropriate set of qualitative constraints may be derived as:

$\mathrm{DERIV}(x_1,x_2), \mathrm{DERIV}(x_2,x_3), \ldots, \mathrm{DERIV}(x_{n-1},x_n)$.

The last equation of the set is expressed qualitatively as:

$$\mathrm{DERIV}(x_n, K)$$

where K is composed of addition constaints as depicted in Figure 2a. A total of n-derivative, n-multiplicative, and n-additive constraints are required to convert from the state space model to the qualitative model, as shown in Figure 2b.

The tree of behavioral modes can be pruned by the consideration of functional and energy-type constraints. A transformation of the state model to a Schwartz matrix form provides the most appropriate setting for the scalar Lyapunov function as a qualitative constraint criterion.

Similar qualitative state transition diagrams can be derived for multivariable systems.

2.3 FAULT PROPAGATION TO OTHER MAJOR SUBSYSTEMS OF THE LSS

When a system undergoes structural perturbations (faults), the interactions between constituent subsystems are expected to have major effects on stability. In assessing the impact of propagating faults on other (healthy) subsystems, we will use the following assertion: If the subsystems are stable to a degree larger than the strength of the interconnection, then the composite system is stable.

Consider a large scale unforced system described by:

$$\dot{x} = f(x,t) \quad (4)$$

which is assumed to consist of N-subsystems:

$$\dot{x}_i = f_i(x_i,t) + g_i(x,t) \quad , \quad i = 1,\ldots,N \quad (5)$$

and assume that the origin e = 0 is an equilibrium state.

When the i-th subsystem is completely decoupled, then:

$$\dot{x}_i = f_i(x_i,t) \quad (6)$$

i.e. the i-th subsystem is totally isolated from the rest of the system.

Assume a Lyapunov function $v_i(x_i,t)$ for the isolated subsystems (6). Use a weighted sum:

$$r(x,t) = \sum_{i=1}^{N} c_i v_i(x_i,t) \quad (7)$$

as a potential Lyapunov function for the large scale system. The c_i's are positive constants.

In stability studies of large scale systems, we define a positive definite function $w_i(x_i)$ and we check for:

$$\dot{v}_i(x_i,t) \overset{?}{\leq} -a_i[w_i(x_i)]^2 \quad (8)$$

The constant a_i is considered as the degree of stability in view of the fact that it gives a lower bound for the decrease in $\dot{v}$ with respect to $w_i^2(x_i)$. Next we check the interconnection terms:

$$|g_i(x,t)| \overset{?}{\leq} \sum_{j=1}^{N} b_{ij} w_j(x_j) \quad (9)$$

where the nonnegative constants b_{ij} indicate the interconnection strength in the sense that they provide an upper bound with respect to $w_j^2(x_j)$. Therefore, a qualitative measure of the impact of a faculty j-th subsystem on the behavior of the i-th subsystem may be obtained by comparing the interconnection strength:

$$|g_i(x,t)| \text{ to } \dot{v}_i(x_i,t) \ .$$

In terms of the constants a_i and b_{ij}, we may form the N x N matrix $E = (e_{ij})$, where $e_{ii} = a_i - b_{ii}$, $e_{ij} = -b_{ij}$, $i \neq j$, and compute the leading principle minors of E; a check on their sign and relative magnitude will determine again the fault propagation impact. The evolution of the time derivatives of the Lyapunov functions and their relative values compared to the

interconnections strength over the first distinguished time points after the occurrence of a fault will provide the information required to eliminate spurious behaviors.

3. AN EXAMPLE

The proposed qualitative simulation approach is demonstrated, in outline form, with an example. The test system is the Thermal Control System of the space station's common module. Jointly with the Boeing Aerospace Company, a hierarchical control algorithm and fault diagnostic routines for this major large scale system of the space station have been developed [12]. The algorithms have been implemented and successfully tested on a subscale laboratory prototype constructed by Boeing. In our example, a fault condition is hypothesized and propagated through the system's major components. If certain threshold or tolerance limits are exceeded, then the affected subsystems are isolated, before the hierarchical controller is reconfigured, in order to preserve the system integrity. A simplified schematic of the Thermal Control System is shown in Figure 3.

The qualitative simulation begins with a structural description which consists of a set of constraints holding among time varying, real valued parameters. The behavioral description consists of a finite set of time points representing the qualitative distinct states of the system and values for each parameter at each time point. This description consists of the ordinal relations (i.e. >, <, and =) holding among the different values in the behavioral description. Certain values are thermal distinguished or landmark values and play a special role in the qualitative simulation. Beginning with a set of assertions about the initial state of the system, the process takes place through the propagation/prediction cycle.

Algebraic relations of the system may, therefore, be represented as qualitative constraints. Let us demonstrate the procedure by considering the example of component HX-2 (Figure 4a). The algebraic equations for HX-2 are:

$$\begin{aligned} Q_2 &= \dot{m}_2 C_v \Delta T_2 \\ \Delta T_2 &= T_{o2} - T_{i2} \\ \dot{m}_2 &= (1-\beta)\,\dot{m}_0 \\ T_{i3} &= T_{o2} - (1-\beta)\,\Delta T_2 \end{aligned} \qquad (10)$$

where β denotes the bypass control valve variable (i.e. $\beta = 0$ means closed valve and $\beta = 1$ means fully opened valve). And constraints for HX-2 may be written as:

$$(C1)\,\Delta T_2 < 5^\circ F \quad , \quad (C2)\,T_{i2} = (70 \pm 2.5)^\circ F \qquad (11)$$

We assume that C_v is constant. Qualitative constraints relations for HX-2 are as follows:

$$\begin{aligned} &(QC1)\,M^-(\beta,\dot{m}_2) \quad , \quad (QC2)\,MULT(\dot{m}_2,\Delta T_2,Q_2) \\ &(QC3)\,MULT(1-\beta,\Delta T_2,T) \quad , \quad (QC4)\,ADD(T_{i2},T,T_{i3}) \end{aligned} \qquad (12)$$

A qualitative constraints model for HX-2 is shown in Figure 4b. We can begin the process with active states at the distinguished time $t = (t_0,t_1)$. We assume that the qualitative states of each variable at time t are as follows:

$$QS(\beta,t_0,t_1) = [(0,1),dec] \qquad (q1)$$

$$QS(\dot{m}_2,t_0,t_1) = [0,\dot{m}_0),inc] \qquad (q2)$$

$$QS(Q_2,t_0,t_1) = [(0,Q_2^*),inc] \qquad (q3)$$

$$QS(\Delta T_2,t_0,t_1) = [(0,5),inc] \qquad (q4)$$

$$QS(T,t_0,t_1) = [(a,b),inc] \qquad (q5)$$

$$QS(T_{i2},t_0,t_1) = [67.5,72.5),std] \qquad (q6)$$

At the distinguished time $t = (t_1,t_2)$, these states undergo the following transitions:

$$QS(\beta,t_1,t_2) = \begin{bmatrix}(0,\ 1),dec\\(0,\ 1),inc\\(0,\ 1),std\end{bmatrix} \quad \begin{matrix}(q1\text{-}1)\\(q1\text{-}2)\\(q1\text{-}3)\end{matrix}$$

$$QS(\dot{m}_2,t_1,t_2) = \begin{matrix}[(0,\dot{m}_0),dec]\\ [(0,\dot{m}_0),inc]\\ [(0,\dot{m}_0),std]\end{matrix} \quad \begin{matrix}(q2\text{-}1)\\(q2\text{-}2)\\(q2\text{-}3)\end{matrix}$$

$$QS(Q_2,t_1,t_2) = \begin{matrix}[(0,Q_2^*),dec]\\ [(0,Q_2^*),ind]\\ [(0,Q_2^*),std]\end{matrix} \quad \begin{matrix}(q3\text{-}1)\\(q3\text{-}2)\\(q3\text{-}3)\end{matrix}$$

$$QS(\Delta T_2,t_1,t_2) = \begin{bmatrix}(0,\ 5),dec\\(0,\ 5),inc\\(0,\ 5),std\end{bmatrix} \quad \begin{matrix}(q4\text{-}1)\\(q4\text{-}2)\\(q4\text{-}3)\end{matrix}$$

The assumed initial qualitative state at time (t_0,t_1) has three possible transitions at time (t_1,t_2). Let us further assume that a failure has occurred at t_1 and the faculty component was identified to be a valve stuck in one position. A stuck valve means that β = const(std) (i.e. (q1) can undergo a transition to only (q1-3) at time (t_1,t_2)). Let us investigate next how this fault propagates through the system by checking the qualitative constraints under possible state transitions. Here we will check only for (QC1) and (QC2) to show how the search is conducted and the effect of the failure event on other components is determined.

$$(QC1)\,M^-(\beta,\dot{m}_2) \; ,$$

from the assumption that β = const

(q1-3)(q2-1) : not consistent
(q1-3)(q2-2) : not consistent
(q1-3)(q2-3) : consistent

$$(QC2)\,MULT(\dot{m}_2,\Delta T_2,Q_2) \; ,$$

by checking (QC1) we get (q2-3) for $QS(\dot{m}_2,t_1,t_2)$.

Among nine possible transitions, we determine readily that there are three consistent ones as: (q2-3)(q4-1)(q3-1),

(q2-3)(q4-2)(q3-2), or (q2-3)(q4-3)(q3-3). Therefore, failure of the valve causes a variation of the temperature difference dependent on the absorbed heat. Similarly, we can check the other qualitative constraints (QC3,QC4) related to this particular system. Energy function related constraints will also be used to narrow the behavioral interpretations and optimize the search algorithm.

The fault propagation model thus provides the information sought as to the impact of this fault condition on other system components.

4. CONCLUSIONS

The proposed fault propagation procedure proceeds as follows:

(1) Construct the qualitative model of the physical system.

(2) Estimate the initial qualitative state of the faulty system.

(3) Using qualitative simulation, find all possible behaviors.

(4) Construct the constraints related to the total energy function of the system.

(5) Using the constraints in (4), eliminate spurious behaviors.

(6) Determine the genuine behavior caused by the failure.

The key difficulty in employing such a qualitative technique to assess the impact of fault propagation is due to the generation of behavioral modes that are not consistent with the physical behavior of the system. Functional and energy related constraints that augment arithmetic qualitative constraints seem to effectively reduce spurious behaviors, thus leading to the correct interpretation.

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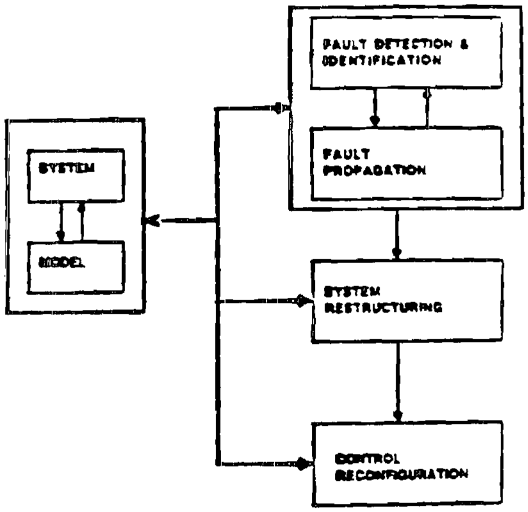

Figure 1. Block Diagram of Proposed Fault Tolerant Control Philosophy

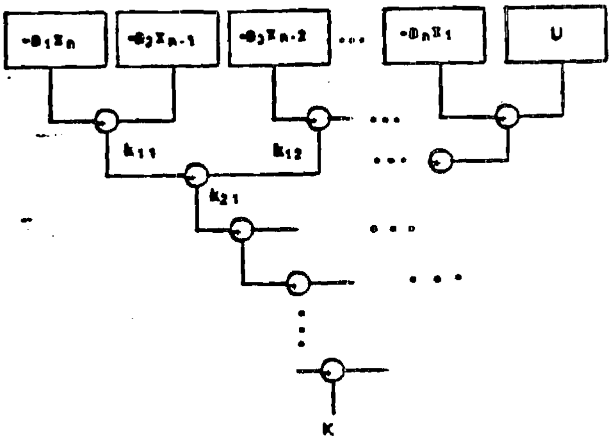

Figure 2a. Qualitative Constraints from $\dot{x}_n = -a_1x_n - a_2x_{n-1} - \ldots$

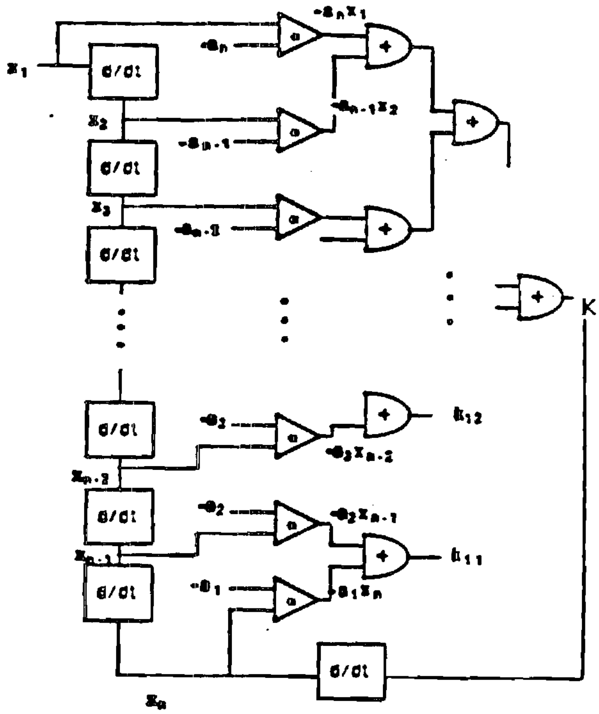

Figure 2b. Block Diagram of Qualitative Constraints Model

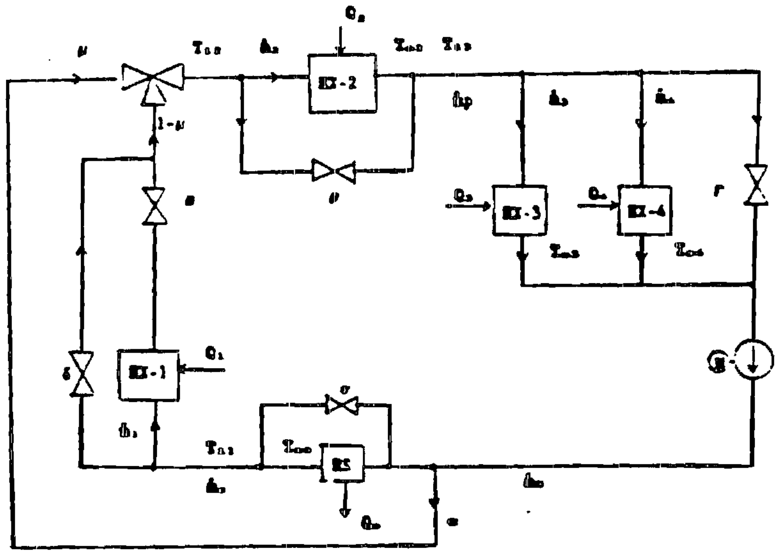

Figure 3. A Schematic of the Thermal Control System

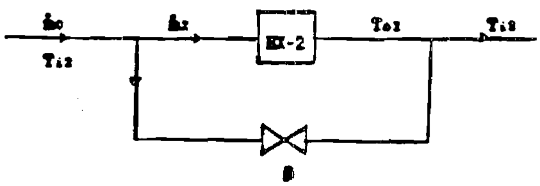

Figure 4a. Block Diagram of Heat Exchanger 2

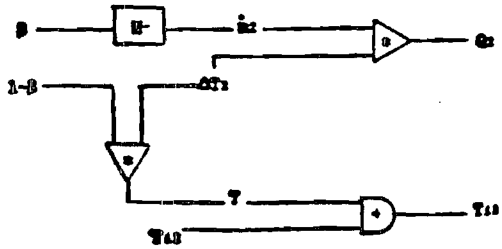

Figure 4b. Qualitative Constraints Model of HX-2

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Stability and Robustness of A Nonlinear Fuzzy Controller Based on The Phase Portrait Assignment Algorithm

by

Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering

Georgia Institute of Technology

Atlanta, Georgia 30332

(404)-894-6252 (404)-894-3834

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Abstract

This paper presents a new design technique and the associated stability analysis of fuzzy logic controllers for a class of nonlinear systems. The design approach is called 'the phase portrait assignment algorithm' (P^2A^2), and it establishes a stabilizing rulebase by utilizing fuzzy measures as introduced by Zadeh. This method can substitute for classical control paradigms when the latter suffer from dynamic, parametric, and stochastic uncertainties. The P^2A^2 technique not only provides a bridge between qualitative reasoning and asymptotic stability of nonlinear feedback control systems, but also gives an insight into the dynamical system behavior. A single-link robot arm is used to illustrated the performance and advantages of the proposed fuzzy control mechanism.

Keywords: fuzzy-logic controller, nonlinear feedback system, stability analysis

1 Introduction

The introduction of fuzzy sets and possibility measures by Zadeh [1,2] made it possible to cope with the approximate estimation and control problem under large-grain uncertainty. A fuzzy set is defined as a class of objects with a continuum of grades of membership using certainty or confidence factors in order to handle uncertainties or complexity in a particular domain of knowledge [3] and a fuzzy number can be described by linguistic terms such as 'large', 'medium', 'small', 'very small', and so on, whose fuzziness provides many degrees of freedom in dealing with uncertainty as a non-uniform possibility distribution [3]. Furthermore, a fuzzy confidence interval represents information that reduces (expands) the uncertainty by using lower and upper bounds as the presumption level or the α-cut increases (decreases). The characteristic functions of linguistic variables represent the associated crisp subsets by using uniform distributions of the feasible subset (possibility = 1) and the non-feasible subset (possibility = 0). Referring to the characteristic functions, membership functions are determined by using non-uniform possibility distributions.

Since Zadeh's introduction of fuzzy mapping and control [4], fuzzy concepts can be applied to the control of dynamic systems whereas stability of a fuzzy feedback control system has been an important issue. Mamdani [5] and Tong[6] suggested means for the application of fuzzy systems to control engineering. Kickert et al.[7] approximated a nonlinear fuzzy-logic controller with a multi-level relay in a simple case which was analyzed by the describing function method since an oscillation was anticipated. Tong [8] showed some results of asymptotic properties of fuzzy feedback systems. Mamdani et al.[9] introduced a self-organizing control system of a fuzzy rule modifier by using a performance table. The concept of L_2-stability was examined in [10] by using Nyquist criteria of a compensated system and a fuzzy-logic controller. Kania et al.[11] introduced the concept of energistic stability for fuzzy dynamic systems. Chen et al.[12] applied Hsu's cell-to-cell mapping theory [13] to the global behavior of the cell state space implicitly by grouping the cells. Finally, Kang et al.[14] emphasized evidential aspects of the nonlinear system dynamics and the transformation from crisp relations to fuzzy relations by using the vector field of the phase plane approach.

The objective of this paper is to provide a general method for constructing rule sets for fuzzy feedback controllers so that global asymptotic stability may be guaranteed with respect to a class of nonlinear systems. This is based upon 'the principle of increasing precision with decreasing intelligence' [15] where the basic idea is to relate evidence of the asymptotic system behavior to crisp subsets by using vector fields, explicitly, and then, to fuzzify each crisp subset (non-fuzzy cell). Periodic behavior of the closed-loop system may be avoided by applying the proposed algorithm. To preview the organization of the paper, Section 2 deals with the design procedure and stability analysis of the proposed scheme whereas convergence and robustness are also considered. A single-link robot arm is used to show the performance of the fuzzy regulator in Section 3 of this paper. Finally, Section 4 contains conclusions and a discussion

of the contributions.

2 Nonlinear Fuzzy Control

A design procedure for a fuzzy expert control mechanism, 'the phase portrait assignment algorithm' (P^2A^2), is proposed for a closed-loop feedback system consisting of a multi-input nonlinear process and a fuzzy rulebase controller as shown in Figure 1. The compositional rule of inference and the modified mean-of-maxima defuzzifier are used for deciding the control feedback input.

2.1 Nonlinear Fuzzy-Logic Feedback Control Systems

Let a nonlinear system be described by a vector differential equation of the form [16]

$$\dot{x}(t) = f(x(t), u(t), t) \qquad t \geq t_0 \tag{1}$$

where $x(t) \in \mathcal{R}^n$ represents the state vector with $x_0 = x(t_0)$; $u(t) \in \mathcal{R}^m$ is the control input vector; and $f(\cdot,\cdot,\cdot)$ denotes a nonlinear function. In brief,

$$x(\cdot) : \mathcal{R}_+ \longrightarrow \mathcal{R}^n$$

$$u(\cdot) : \mathcal{R}_+ \longrightarrow \mathcal{R}^m$$

$$f(\cdot,\cdot,\cdot) : \mathcal{R}^n \times \mathcal{R}^m \times \mathcal{R}_+ \longrightarrow \mathcal{R}^n \tag{2}$$

where $\mathcal{R}_+$ is a subset of $\mathcal{R}^1$ such that $\mathcal{R}_+ = [t_0, \infty)$. A general method of constructing a fuzzy rule set is necessary so that the rulebase updates the control input $u(t)$ to ensure global asymptotic stability with respect to a given nonlinear system. The rulebase can be represented as follows:

- rule 1: if (x_1 is $L^1_{x_1}$) and $\cdots$ and (x_n is $L^1_{x_n}$) then (u_1 is $L^1_{u_1}$) and $\cdots$ and (u_m is $L^1_{u_m}$).

$$\vdots \qquad\qquad \vdots \tag{3}$$

- rule s: if (x_1 is $L^s_{x_1}$) and $\cdots$ and (x_n is $L^s_{x_n}$) then (u_1 is $L^s_{u_1}$) and $\cdots$ and (u_m is $L^s_{u_m}$).

where x_j ($j = 1..n$) and u_k ($k = 1..m$) are fuzzy representations for $x_j(t)$ and $u_k(t)$, the elements of $x(t)$ and $u(t)$, respectively; $L^i_{x_j}$ and $L^i_{u_k}$ are linguistic variables such as 'positive large', 'negative small', and so on. The number of rules 's' depends on the number of possible linguistic variables 'ℓ', and $s = \ell^n$ when all possible combinations are considered but this number is finally determined by the P^2A^2. Reduction in the number of rules is possible only when the control $u(t)$ has no influence on the system behavior for some rules [17].

Consider the constraints of the control inputs and the states.

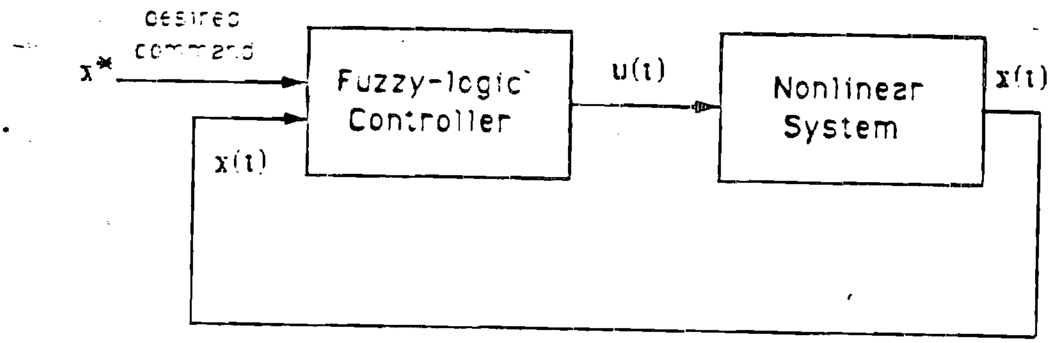

Figure 1: Block diagram of a fuzzy-logic feedback system

Assumption 2.1 (admissible controls) : *The control input $u(t)$ is bounded within some range*

$$u_{k\,\min} < u_k(t) < u_{k\,\max} \qquad \text{for } \; k = 1..m \tag{4}$$

implying that the maximum and minimum values, $u_{k\,\max}$ and $u_{k\,\min}$, of the actuator are bounded. If the set of admissible controls is defined by

$$\mathcal{U}(t) = \{u_k(t) : t \in [t_0, t_1], k = 1..m\} \in \mathcal{R}^m \tag{5}$$

then the above constraints may be described by

$$\mathcal{U}_c(t) = \{u(t) : \|u(t) - \bar{u}\|_p < M_u, t \in [t_0, t_1]\} \tag{6}$$

for some fixed vector $\bar{u} = \{\bar{u}_k\}$ where M_u is a finite positive real number; $\bar{u}_k$ may be $\frac{1}{2}(u_{k\,\min} + u_{k\,\max})$; and $\|\cdot\|_p$ represents p-norm.

Assumption 2.2 (the domain-of-interest in the state space) : *The domain-of-interest (DOI) in the phase plane is considered to provide global asymptotic stability for a local positive definite function. The state $x(t)$ is bounded as follows:*

$$x_{j\,\min} < x_j(t) < x_{j\,\max} \qquad \text{for } \; j = 1..n \tag{7}$$

where $x_{j\,\max}, x_{j\,\min}$ are real numbers. The set of allowable state trajectories is denoted by

$$\mathcal{X}_s(t) = \{x_j(t) : t \in [t_0, t_1], j = 1..n\} \in \mathcal{R}^n \tag{8}$$

and the DOI may be described by

$$\mathcal{X}_c(t) = \{\|x(t) - \bar{x}\|_p < M_x, t \in [t_0, t_1]\} \tag{9}$$

for some fixed vector $\bar{x} = \{\bar{x}_j\}$, where M_x is a finite positive real number and $\bar{x}_j$ may be $\frac{1}{2}(x_{j\,\min} + x_{j\,\max})$.

Note that $\|x(t)\|_\infty$ is related to the Cartesian coordinates while $\|x(t)\|_2$ is related to polar or spherical coordinates.

Definition 2.1 (extended reachability) *: Let S_u and S_x be subsets of $\mathcal{U}_c(t)$ and $\mathcal{X}_c(t)$, respectively, such that*

$$S_u = \{u(t) : \|u(t) - u^*\|_p < \delta_u\} \tag{10}$$

$$S_x = \{x(t) : \|x(t) - x^*\|_p < \delta_x\} \tag{11}$$

for fixed vectors $u^* \in \mathcal{R}^m$, $x^* \in \mathcal{R}^n$, and for small positive real numbers δ_u, δ_x. Extended reachability is stated as follows:

For the nonlinear system (1) with the initial state $x_0 = x(t_0)$, there exists S_u such that S_x is reachable (accessible) from x_0 at $t = t_0$ with respect to S_u for some finite time $t = T$.

Remark : Roughly speaking, extended reachability implies that $u(t)$, within a δ_u-neighborhood of u^*, leads $\mathcal{X}_s(t)$ to the δ_x-neighborhood of x^* in a finite time. In short, if $u(t) \in S_u$ for $t \in [t_0, T]$,

$$\|x_0 + \int_{t_0}^{T} f(x(\tau), u(\tau), \tau) d\tau - x^*\|_p < \delta_x. \tag{12}$$

Extended reachability relaxes the existing reachability condition by accommodating the fuzzified subsets of $x(t)$ and $u(t)$.

Some definitions relating to fuzzy sets and fuzzy numbers are appropriate for a better understanding of the theoretical developments in this paper (see [18]):

Definition 2.2 *An ordinary subset, A, of a referential set, X, is defined by its 'characteristic function' $\mu_A(x) \in \{0, 1\}, \quad \forall x \in X$.*

A feasible set $F(X)$ is a crisp set described by a characteristic function. A linguistic description in X such as 'large' resides in a feasible subset $F(X) \subset X$ such that $\mu_{F(X)}(x) = 1$ if $x \in F(X)$ and $\mu_{F(X)}(x) = 0$ otherwise.

Definition 2.3 *A fuzzy subset, A, from the universe of discourse, X, is defined by its characteristic function $\mu_A(x) \in [0, 1], \quad \forall x \in X$, which is called the 'membership function'.*

Now consider the following mappings of the fuzzy controller in (3):

$$\mathcal{X} = \{(x, \mu(x))\}; \quad \mu : X \longrightarrow [0,1]; \quad x \in X$$

$$\mathcal{U} = \{(u, \mu(u))\}; \quad \mu : U \longrightarrow [0,1]; \quad u \in U$$

$$R_F : \mathcal{X} \longrightarrow \mathcal{U} \tag{13}$$

where $\mathcal{X}, \mathcal{U}$ are families of fuzzy sets defined on X, U, respectively, and R_F is a fuzzy relation which can be interpreted as a mapping with its domain in $\mathcal{X}$ and a space of values constrained in $\mathcal{U}$. Let $A \in \mathcal{X}$ and $B \in \mathcal{U}$, then a formal fuzziness system is defined as

$$A \circ R_F = B \tag{14}$$

where '$\circ$' denotes the operator for the compositional rule of inference. A membership function may represent a non-uniform possibility distribution for the associated linguistic variable described by the characteristic function, and moreover, a fuzzy set is an invariant and consonant set as the presumption level increases.

Consider a controller structure of the fuzzy relational system represented by

$$u(t) = r(x(t)) \qquad t \geq t_0 \tag{15}$$

where $r(\cdot) : \mathcal{R}^n \longrightarrow \mathcal{R}^m$ is a memoryless nonlinear mapping described by the compositional rule of inference (3). Now the fuzzy feedback system can be represented by

$$\dot{x}(t) = f(x(t), r(x(t)), t) \triangleq g(x(t), t). \tag{16}$$

Our goal is to find a fuzzy controller $r(\cdot)$ that stabilizes the nonlinear feedback system $\dot{x}(t) = g(x(t), t)$, and to provide a generic procedure of constructing the stabilizing rule set by using fuzzy logic. If we denote the fuzzy assignments (fuzzy sets) by

$$A_{ij} = x_j \text{ is } L^i_{x_j} = L^i_{x_j}(x_j) \qquad (i = 1..s; j = 1..n) \tag{17}$$

$$B_{ik} = u_k \text{ is } L^i_{u_k} = L^i_{u_k}(u_k) \qquad (i = 1..s; k = 1..m) \tag{18}$$

then the fuzzy relation R_F of the parallel implications in (3) can be obtained as [3]

$$R_F = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s) = \bigvee_{i=1}^{s} (A_i \Rightarrow B_i) \tag{19}$$

where $A_i = \bigwedge_{j=1}^{n} A_{ij}$ and $B_i = \bigwedge_{k=1}^{m} B_{ik}$ with $\bigwedge$ and $\bigvee$ denoting the minimum and maximum operators, respectively.

Definition 2.4 *A 'fuzzy implication' is a mapping $S_i : A_i \Longrightarrow B_i$ and the membership function for S_i is $\mu_{S_i}(u,x) = \mu_{A_i}(x) \bigwedge \mu_{B_i}(u)$ where $x \in X$ and $u \in U$.*

If we consider a fuzzy relational matrix, the quantization of the support in X determines the size of the matrix, and the element of R_F is

$$R_F(p,q) = \mu_{A_i}(x_p) \bigwedge \mu_{B_i}(u_q) \tag{20}$$

where p, q are integers; x_p, u_q are the quantized representatives in X, U, respectively.

Definition 2.5 *The compositional rule of inference can be stated as follows [7]: let the i-th fuzzy implication be $\Sigma_i : A_i \Longrightarrow B_i$ whose membership function is*

$$\mu_{\Sigma_i}(y,x) = \mu_{A_i}(x) \wedge \mu_{B_i}(y) \tag{21}$$

then, for any implication Σ_i^* such that $\Sigma_i^* : A_i^* \Longrightarrow B_i^*$, the membership function for B_i^* is defined by

$$\mu_{B_i^*}(y) = \vee_x \{\mu_{A_i^*}(x) \wedge \mu_{\Sigma_i}(y,x)\} \tag{22}$$

and R_F may be represented by

$$R_F = \cup_{i=1}^{s} \Sigma_i. \tag{23}$$

The design of membership functions is an important issue closely linked to the possibility distribution of linguistic variables. Fuzzy membership functions should satisfy the properties of convexity and normality [18]. One approach to the design of membership functions is to make use of the S, Π, and Z curves as proposed by Zadeh [2]. The parameters of these functions should be carefully determined according to the scope of each linguistic variable [17]. We define the S, Π, and Z membership functions as follows:

$$S(x;a,b,c) = \begin{cases} 0 & x < a \\ 2(\frac{x-a}{c-a})^2 & a < x < b \\ 1 - 2(\frac{x-c}{a-c})^2 & b < x < c \\ 1 & x > c \end{cases} \tag{24}$$

$$\Pi(x;b,c) = \begin{cases} S(x; c-b, c-\frac{b}{2}, c) & x < c \\ S(x; c, c+\frac{b}{2}, c+b) & x > c \end{cases} \tag{25}$$

$$Z(x;a,b,c) = 1 - S(x;a,b,c) \tag{26}$$

For example, $A_{11} =$ 'x_1 is positive large' can be described by

$$\mu_{A_{11}}(x_1(t)) = S(x_1(t);a,b,c). \tag{27}$$

The parameters of the membership functions are a, b, and c. The transition points, defined as the value of the universe of discourse for which the grade of membership is equal to $\frac{1}{2}$, are $x = b$ for S, Z; $x = c \pm \frac{b}{2}$ for Π. The maximum grade of 1 occurs at $x = c$ for all these functions and the minimum value of 0 at $x = a$ for S, Z; $x = c \pm b$ for Π. These design parameters are chosen to include the state trajectories of $x(t)$ for all x_0 in the DOI [19]. Typical S, Π, Z curves are shown in Figure 2. Other kinds of membership functions are triangle, trapezoid, arctangent, normal distribution functions, and so forth.

A number of mathematical properties describe the relationship between membership functions and the

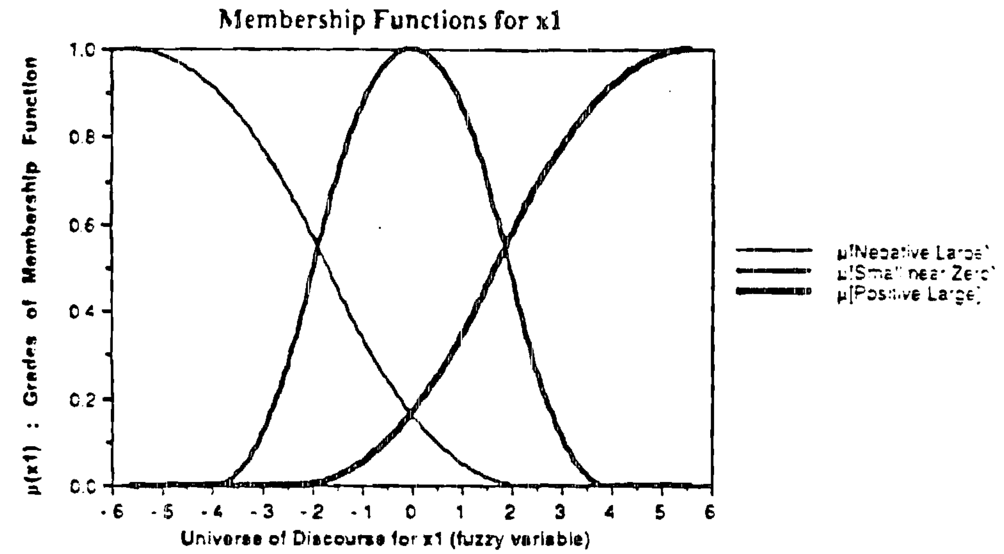

Figure 2: S, Π, Z membership functions

inferred control action. Kania and his co-workers [20] proposed an α-stability property, an α-decision stability property, an α-β strong decision stability property, a good mapping property, a γ-good mapping property, and the associated conditions. Some definitions and mathematical properties of the formal fuzziness system are shown as follows [20,21]:

Definition 2.6 *The formal fuzziness system (14) has the α-stability (α-S) property with respect to some family $\mathcal{A}$ and some feasible subset $F(U) \in U$ if $\mu_B(u) \leq \alpha \;\; \forall u \in U - F(U)$ for every fuzzy set $A \in \mathcal{A}$ of $A \circ R = B$ where the operator $\circ$ is defined in (14).*

Theorem 2.1 *Consider the system where $R = A \Rightarrow B$ is given. Then $\bigvee_{i=1}^{n} \mu_A(x_i) \leq \alpha$ or $\mu_B(u_j) \leq \alpha$ $\forall u_j \in U - F(U)$ if and only if the system (14) has the α-S property with respect to a family $\mathcal{A}$.*

Definition 2.7 *A system* (14) *has the α-decision stability (α-DS) property with respect to some family $\mathcal{A}$ of fuzzy sets defined on X if the system has the α-S property with respect to $\mathcal{A}$ and* $\max_{U-F(U)} \mu_{A \circ R}(u) < \max_{F(U)} \mu_{A \circ R}(u)$.

The α-DS property assures that the grade of membership in the feasible subset $F(U)$ is greater than that in the nonfeasible subset $U - F(U)$. The next definition is a stronger version of the α-DS property [20]:

Definition 2.8 *A system* (14) *has the α-β strong decision stability (α-β SDS) property with respect to some family $\mathcal{A}$ if and only if the system* (14) *has the α-DS property with respect to $\mathcal{A}$ and* $\bigvee_{u \in U} \mu_{A \circ R}(u) \geq \beta > \alpha$.

A membership function should follow the α-DS ($\alpha - \beta$ SDS) property for the feasible subset of a linguistic variable, and for every rule, the γ-good mapping property should hold.

Definition 2.9 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given. Then R has the good mapping (GM) property if and only if $A_i \circ R = B_i$ for every $i = 1, ..., s$.*

Definition 2.10 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given where $A_i \in \mathcal{X}$ and $B_i \in \mathcal{U}$ for $i = 1, \cdots, s$. Then a fuzzy relation R has the γ-good mapping (γ-GM) property in the weaker sense if and only if $A_i \circ R = \tilde{B}_i$ for every $i = 1, ..., s$ where $B_i \subseteq \tilde{B}_i$, and $\mu_{\tilde{B}_i} = \mu_{B_i}$ for $\mu_{B_i} \geq \gamma, 0 \leq \gamma \leq 1$.*

Remark: Note that, when $\gamma = 0$, the γ-GM property reduces to the GM property as defined above [21]. It is recommended that from the γ-GM property, at the boundary of each crisp subset, overlapping of the associated adjacent membership functions takes place where the grade of membership is $\gamma = 0.5$ [22]. This overlapping concept is important in the sense that we can minimize the decision errors by overlapping adjacent membership functions where $\mu_A(x_i) = 0.5$.

2.2 Design: The Phase Portrait Assignment Algorithm (P^2A^2)

Let us consider the P^2A^2 for a nonlinear fuzzy controller described by (3) or (15). The following steps describe how to establish the fuzzy rulebase that guarantees aymptotic stability of the closed-loop system consisting of (1) and (15). The key point here is that the procedure entails three basic steps: first, the classification of *integral* manifolds according to the evidence in the phase plane, second, the selection of nonfuzzy cells from the crisp relations, and finally, the transformation of each crisp relation into a fuzzy relation.

- (i) Find $u^* \in S_u$ with which $x(t)$ converges to or passes through S_x in a finite time T, such that $x(T) \in S_x$. There may be several u^*'s for some x_0's that satisfy the extended reachability condition.

These trajectories are called 'the primary-attracting manifolds' (the PAM's). This concept is closely related to the switching curves [23,24].

- (ii) Divide an admissible control element $u_k(t)$ into m_k regions resulting in $\bar{u}_{kr}$'s ($r = 1..m_k$) in such a way that some u_k^*'s $\in \{\bar{u}_{kr}\}$ where $\bar{u}_{kr}$ represents the mean of $u_k(t)$ in the r-th region. A linguistic variable $L^i_{u_k}$ is defined and is assigned to each $\bar{u}_{kr}$. The maximally assignable controls are mm_k constant $\bar{u}_{kr}$'s.

- (iii) Plot the state trajectories $C_t(x_0, \bar{u}_{kr}) \in \mathcal{X}_s(t)$ for every x_0 in the DOI and for each $\bar{u}_{kr}$. The PAM's are denoted by $C_t^*(x_0, u_k^*)$. Define 'the primary-attracting cells' (the PAC's) R^*'s $\in \mathcal{R}^n$ that cover the PAM's $C_t^*(x_0, u_k^*)$ in the DOI and assign u_k^* to R^*. The size of R^* partially determines the number of fuzzy rules as the size of the DOI is fixed.

- (iv) Plot the state trajectories $C_t(x_0, \bar{u}_{kr})$ for every x_0 and for some $\bar{u}_{kr}$'s ($\neq u_k^*$). If such $\bar{u}_{kr}$'s lead to or intersect with $C_t^*(x_0, u_k^*)$ either directly or indirectly (via another $C_t(x_0, \bar{u}_{kr})$) in a finite time, then these $C_t(x_0, \bar{u}_{kr})$'s are defined as 'the secondary-attracting manifolds' (the SAM's) $C_t^\dagger(x_0, u_k^\dagger)$ while these $\bar{u}_{kr}$'s are denoted by $u_k^\dagger$'s.

- (v) Define 'the secondary-attracting cells' (the SAC's) $R^\dagger \in \mathcal{R}^n$ that include $C_t^\dagger(x_0, u_k^\dagger)$'s by assigning $u_k^\dagger$ to each $R^\dagger$. $R^\dagger$'s are chosen in a such way that $C_t^\dagger(x_0, u_k^\dagger)$'s may have the shortest path, if possible, to $C_t^*(x_0, u_k^*)$ or to nearby $C_t^\dagger(x_0, u_k^\dagger)$ that eventually leads to $C_t^*(x_0, u_k^*)$ and so that $u_k^\dagger$'s may not change abruptly between $R^\dagger$'s. Also, $C_t^\dagger(x_0, u_k^\dagger)$'s may not form a closed-cycle.
  Remark : Steps (iv) and (v) may be difficult since there are many choices for assigning the SAC's. But there should be only one u_k^* or $u_k^\dagger$ for each R^* or $R^\dagger$, respectively.

- (vi) Repeat steps (iv) and (v) until the whole DOI is filled with R^*'s and $R^\dagger$'s. The rest of the manifolds $C_t(x_0, \bar{u}_{kr})$ not satisfying $C_t^*(x_0, u_k^*)$ or $C_t^\dagger(x_0, u_k^\dagger)$ are defined as 'the diverging manifolds' (the DVM's) $C_t^\Delta(x_0, \bar{u}_{kr})$ and these x_0's and $\bar{u}_{kr}$'s should be avoided. Similarly, 'the diverging cells' (the DVC's) that include $C_t^\Delta(x_0, \bar{u}_{kr})$'s are defined as R^Δ's.

- (vii) The final attracting cells, R^*'s (PAC's) and $R^\dagger$'s (SAC's), constitute the cells of the P^2A^2. The total number of cells is equal to the total number of rules. On the boundary of the DOI, all the manifolds $C_t^\dagger(x_0, u_k^\dagger)$ and $C_t^*(x_0, u_k^*)$ should be directed toward the inner regions.
  Remark : $C_t(x_0, \bar{u}_{kr})$ can be obtained analytically by finding the n-dimensional vector fields for the nonlinear system as described above; heuristically by exploiting the expertise of experts; or experimentally for unknown complex systems. Figure 3 shows an example of the attracting cells, R^*'s and $R^\dagger$'s, and the DVC's R^Δ's for a simple second-order system.

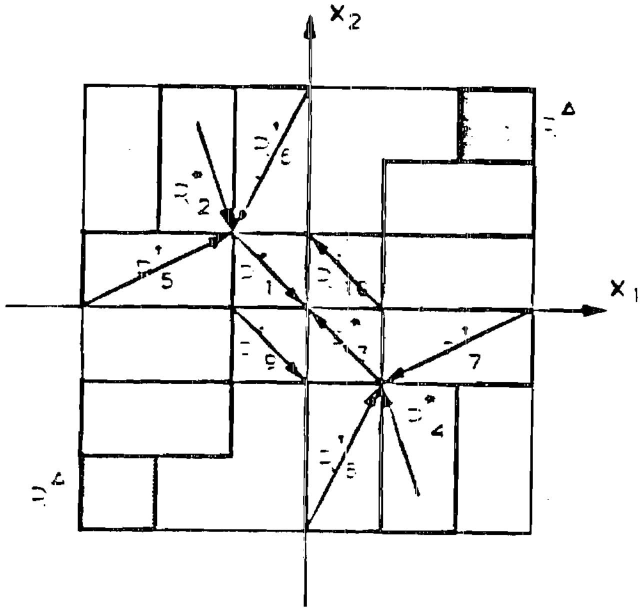

Figure 3: Typical cells and the vector fields of the P^2A^2

- (viii) If there are s cells in the DOI, the i-th cell is defined by the pair $c\{R[i], \bar{u}[i]\}$ where $\bar{u}[i] \in \mathcal{R}^m$ is a control vector of the chosen m constant elements among $\{u_k^*, u_k^\dagger; k = 1..m\}$ for $i = 1..s$, and for each cell, a linguistic characterization is performed by assigning n $L_{x_j}^i$'s (for $j = 1..n$) to each $R[i] \in \{R^*, R^\dagger\}$ in the DOI. Every fuzzy assignment $\{A_{ij}, B_{ik}\}$ in the i-th rule is determined according to the above steps, since each admissible control $\bar{u}_{kr}$ has its own linguistic description $L_{u_k}^i$ for $k = 1..m$ for each $R[i]$. Each cell $R[i]$ in the DOI is assigned to its particular linguistic characterizations $\{L_{x_j}^i : j = 1..n\}$ by using the connectivity of the i-th cell $c\{R[i], \bar{u}[i]\}$. The membership functions are described by $\mu_{A_{ij}}(x_j(t))$ for $A_{ij} =$ 'x_j is $L_{x_j}^i$' and $\mu_{B_{ik}}(u_k(t))$ for $B_{ik} =$ 'u_k is $L_{u_k}^i$'.

- (ix) The control input $u_k(t)$ in (15) is determined via defuzzification by

$$u_k(t) = \frac{\sum_{i=1}^{s} d_i(x(t))\bar{u}_k[i]}{\sum_{i=1}^{s} d_i(x(t))} \qquad \text{for } k = 1..m \tag{28}$$

where $\bar{u}_k[i]$ is the k-th element of $\bar{u}[i]$ and $d_i(x(t))$ is the degree of fulfilment for the i-th rule defined by

$$d_i(x(t)) = \Lambda_{j=1}^{n} \mu_{A_{ij}}(x_j(t)) \qquad \text{for } i = 1..s \tag{29}$$

$$d_i(\cdot) : \mathcal{R}^n \longrightarrow [0,1] \tag{30}$$

and $\Sigma_{i=1}^{s} d_i(x(t)) \neq 0$ is required for the existence of a solution of the defuzzification scheme. It is noted that

$$r_k(\cdot) = \frac{\Sigma_{i=1}^{s} \bar{u}_k[i] d_i(\cdot)}{\Sigma_{i=1}^{s} d_i(\cdot)} \tag{31}$$

where $r_k(\cdot)$ represents the k-the element of $r(\cdot)$ in (15).

Remark : The number of cells in the DOI is the same as the number of rules in the fuzzy rule set. When the fuzzy characterization functions are mapped into the fuzzy membership functions, α-DS property [20] should hold for the feasible subsets. Moreover, for each linguistic variable, overlapping of the adjacent membership functions is necessary to ensure that $\Sigma_{i=1}^{s} d_i(x(t)) \neq 0$ for any $x(t)$ in the DOI (the γ-GM property) [17]. Note that the DOI is decomposed into two kinds of cells, PAC's and SAC's. The crisp relational mapping R_C may be represented using the crisp cells as

$$R_C = \cup_{i=1}^{s} c\{R[i], \bar{u}[i]\} \tag{32}$$

where $R_C : \mathcal{R}^n \longrightarrow \mathcal{R}^m$. If we introduce the fuzziness transformation $\mathcal{F}$, the fuzzy relational mapping (19) may be defined by

$$R_F = \mathcal{F}[R_C] = \mathcal{F}[\cup_{i=1}^{s} c\{R[i], \bar{u}[i]\}]. \tag{33}$$

Lemma 2.1 *If the γ-GM property and the α-DS property (or the α-β SDS property) hold, then there exists a $\delta_u[i] > 0$ for all $x(t) \in R[i]$ such that*

$$|u_k(t) - \bar{u}_k[i]| < \delta_{u_k}[i] \qquad \text{for } k = 1..m \tag{34}$$

or

$$\|u(t) - \bar{u}[i]\| < \delta_u[i] \tag{35}$$

where $\bar{u}[i] = \{\bar{u}_k[i] : k = 1..m\}$ and $\delta_u[i] = \max_k\{\delta_{u_k}[i]\}$.

Proof : Let overlapping (the γ-GM property) occur at $\gamma = \alpha$ for the design, then there exists only one $d_i(x(t))$ such that $d_p(x(t)) > \alpha$ if $i = p$, that is, the p-th rule is the most contributing fuzzy rule at time t.

$$d_p(x(t)) = \wedge_{j=1}^{n} \mu_{A_{pj}}(x_j(t)) = \alpha_p > \alpha \qquad \text{if } i = p \tag{36}$$

$$d_i(x(t)) = \wedge_{j=1}^{n} \mu_{A_{ij}}(x_j(t)) = \alpha_i \ll \alpha \qquad \text{if } i \neq p \tag{37}$$

and from the α-DS property

$$\Sigma_{\substack{i \neq p \\ i=1}}^{s} \alpha_i \ll \alpha_p \tag{38}$$

is satisfied by approximation subject to the minimum operators. From (28),

$$u_k(t) = \frac{\alpha_p \bar{u}_k[p] + \Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}$$

$$= \frac{\alpha_p}{\Sigma_{i=1}^{s} \alpha_i} \bar{u}_k[p] + \frac{\Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}. \tag{39}$$

Since $\alpha_p \cong \Sigma_{i=1}^{s} \alpha_i$ from (38),

$$u_k(t) = \bar{u}_k[p] + \Delta_k[p] \tag{40}$$

where $\Delta_k[p] \triangleq \frac{\Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}$. Therefore,

$$|u_k(t) - \bar{u}_k[p]| < \delta_{u_k}[p] \tag{41}$$

where $\delta_{u_k}[p] = |\Delta_k[p]|$ for $k = 1..m$.

<u>Remark</u> : If the granularity of the fuzzy rulebase is dense enough, (38) is strongly confirmed, and therefore, $\delta_{u_k}[p]$ in (41) is reduced. Moreover, reduction in the fuzzy rules is possible as long as the system behavior is not affected by deleting some rules.

Theorem 2.2 (global convergence) *Let the extended reachability condition hold for a nonlinear system described by (1) and let the P^2A^2 be applied to the control input $u(t)$. Then, for any x_0 in the DOI of the state space,*

$$||x(t) - x^*||_p < \epsilon \tag{42}$$

is satisfied for $t \geq T$ where $\epsilon > 0$ and $x(t)$ is globally convergent to an ϵ-neighborhood of x^*.

<u>Proof</u> : Let the index 'i' of the most contributing rule be omitted for simplicity. The proof is divided into two parts.

case (i) $x_0 \in R^*$: $x(t)$ is found by

$$x(t) = x_0 + \int_{t_0}^{t} f(x(\tau), u(\tau), \tau) d\tau. \tag{43}$$

In the worst case, $u(\tau) = u^* + \Delta^* \in S_u$, and from Lemma 2.1 of the P^2A^2, $x(t)$ continues to remain in R^*'s reaching S_x in a finite time $t_x = t_1 - t_0$. From (12),

$$||x(t_1) - x^*||_p = ||x_0 + \int_{t_0}^{t_1} f(x(\tau), u^* + \Delta^*, \tau) d\tau - x^*||_p < \delta_x \tag{44}$$

at $t = t_1 > t_0$.

case (ii) $x_0 \in R^\dagger$: There exists $t = t_2 > t_0$ such that $x(t)$ is transferred from $R^\dagger$ to R^* by applying the worst-case control $u(t) = u^\dagger + \Delta^\dagger$ from Lemma 2.1 of the P^2A^2 such that

$$x(t_2) = x_0 + \int_{t_0}^{t_2} f(x(\tau), u^\dagger + \Delta^\dagger, \tau) d\tau \in R^* \tag{45}$$

and then, similarly from (i), we can use $u(t) = u^* + \Delta^* \in S_u$, resulting in $x(t) \in S_x$ at time $T = t_2 + t_x$. Therefore,

$$\|x(t) - x^*\|_p < \epsilon \tag{46}$$

in a finite time $t \geq T$ where $\epsilon = \delta_x > 0$.

2.3 Analysis: Stability of The Nonlinear Fuzzy-logic Controller

The fuzzy-logic control system is analyzed by using Lyapunov's direct method [25] in this section. Since the P^2A^2 forces the vector field of the dynamic equation (1) to be directed toward S_x by using the fuzzy-logic rulebase control $u(t)$, the following proposition is considered for stability analysis of nonlinear fuzzy control systems considering the vector fields.

Proposition 2.1 *Let the extended reachability condition hold and let the P^2A^2 be applied to (1). If $x(t) \in R^*$, then for some x^*,*

$$\dot{x}(t) = \Psi(t)(x(t) - x^*) + \xi_x \tag{47}$$

is satisfied $\forall t \in \mathcal{R}_+$ where the eigenvalues of $\Psi(t)$ are negative, i.e., $\lambda_i[\Psi(t)] < 0$ for i=1..n, $\Psi(t) = \Psi(x(t), u(t), t) \in \mathcal{R}^{n \times n}$; and $\xi_x \in \mathcal{R}^n$ is a point in the σ-neighborhood of $\dot{x}(t)$ when $x(t) = x^*$ such that

$$\|\dot{x}(t) - \xi_x\|_p|_{x(t)=x^*} < \sigma. \tag{48}$$

<u>Proof</u> : Since $x(t) \in R^*$, the vector field of $\dot{x}(t) = f(x(t), u^*, t)$ is directed toward $x^* \in S_x$ by applying the fuzzy control input $u(t) = u^* + \Delta^* \in S_u$ from the P^2A^2. If $x_i(t), \xi_{xi}, x_i^*$ are the elements of $x(t), \xi_x, x^*$, respectively, then the following is satisfied.

$$\frac{\dot{x}_i(t) - \xi_{xi}}{x_i(t) - x_i^*} < 0 \qquad \forall t \in [t_0, t_1] \tag{49}$$

for $i = 1..n$. Thus, for some function of time $\Psi_i(t) < 0, \forall t \in \mathcal{R}_+$.

$$\dot{x}_i(t) = \Psi_i(t)(x_i(t) - x_i^*) + \xi_{xi} \tag{50}$$

where $\Psi_i(t)$ may be a diagonal element of $\Psi(t)$. Therefore, in general,

$$\dot{x}(t) = \Psi(t)(x(t) - x^*) + \xi_x \tag{51}$$

where $\lambda_i[\Psi(t)] < 0$ for $i = 1..n$ and $\forall t \in [t_0, t_1]$. t_1 can be extended to infinity by considering piecewise time-scale analysis in the different R^*'s.

Remark : It is noted that (49) is related to a Lipschitz condition of (1) when $u(t) = u^*$ in such a way that the uniqueness of the solution to (1) is guaranteed if $\Psi_i(t)$ is finite $\forall t \in \mathcal{R}_+$ [16].

Theorem 2.3 *Let a local positive definite function $v(t)$ in R^* be defined by*

$$v(t) = (x(t) - x^*)^T \mathcal{P}(x(t) - x^*) \tag{52}$$

where $\mathcal{P}$ is a symmetric positive definite matrix. If the P^2A^2 is applied to (1) with the extended reachability condition, x^* is asymptotically stable with a ball attractor S_z for all $x_0 \in R^*$, $\forall t \in [t_0, \infty)$.

Proof : Considering $\dot{v}(t)$.

$$\dot{v}(t) = \dot{x}(t)^T \mathcal{P}(x(t) - x^*) + (x(t) - x^*)^T \mathcal{P}\dot{x}(t) \tag{53}$$

and if we define $z(t) = x(t) - x^*$ then from (47) in Proposition 2.1,

$$\dot{v}(t) = (\Psi(t)z(t) + \xi_x)^T \mathcal{P}z(t) + z(t)^T \mathcal{P}(\Psi(t)z(t) + \xi_x)$$

$$= -z(t)^T \mathcal{Q}(t)z(t) + 2\xi_x^T \mathcal{P}z(t) < 0 \tag{54}$$

is required for asymptotic stability where $\mathcal{Q}(t) \in \mathcal{R}^{n\times n}$ is a positive definite matrix defined by

$$\mathcal{Q}(t) = -(\Psi(t)^T \mathcal{P} + \mathcal{P}\Psi(t)). \tag{55}$$

Therefore, for asymptotic stability,

$$z(t)^T \mathcal{Q}(t)z(t) > 2\xi_x^T \mathcal{P}z(t) \tag{56}$$

should be satisfied and this is implied by

$$||z(t)||_p > \frac{2\lambda_{max}[\mathcal{P}]||\xi_x||_p}{\inf_t \lambda_{min}[\mathcal{Q}(t)]} \tag{57}$$

where $\lambda_{max}[A], \lambda_{min}[A]$ represent the maximum and minimum eigenvalues of a matrix A. If $S_z \in \mathcal{R}^n$ is defined by

$$S_z = \{z(t) : ||z(t)||_p < \delta_x\} \tag{58}$$

where $\delta_z = \frac{2\lambda_{max}[\mathcal{P}]||\xi_z||_p}{\inf_t \lambda_{min}[\mathcal{Q}(t)]} > 0$, then S_z is a ball attractor outside of which $\dot{v}(t) < 0$ holds with asymptotic stability, and it is easy to realize $S_z = S_z$.

Remark : Note that $\lambda_{min}[\mathcal{Q}(t)]$ may change but $\inf_t \lambda_{min}[\mathcal{Q}(t)] < 0$ is satisfied by Proposition 2.1. If $||\xi_z||_p$ is small enough compared with $\inf_t \lambda_{min}[\mathcal{Q}(t)]$, the attractor (58) reduces to a point.

Corollary 2.1 *Let the extended reachability condition and the P^2A^2 be satisfied for a multi-input nonlinear system in (1). Then x^* is globally asymptotically stable with a ball attractor S_x for all x_0 in the DOI and $\forall t \in [t_0, \infty)$.*

Proof : (i) If $x(t) \in R^\dagger$, there exists a control in the δ_u-neighborhood of $u^\dagger$ that transfers $x(t)$ from $R^\dagger$ to R^* in a finite time by the P^2A^2.
(ii) If $x(t) \in R^*$, there exists a control in the δ_u-neighborhood of $u^* \in S_u$ that forces $x(t)$ to converge to S_x by applying Theorem 2.3 and x^* is asymptotically stable.
From (i) and (ii), for all x_0 in the DOI, global asymptotic stability is guaranteed.
Remark : (i) is referred to as 'transferring capability' since $x(t) \in R^\dagger$ is moved to $x(t) \in R^*$ in a finite time by using the P^2A^2. The corollary 2.1 can be applied to the fuzzy regulators for nonlinear systems.

Theorem 2.4 **(fuzzy robustness)** *Given the nonlinear system in (1), let the extended reachability condition hold and let the P^2A^2 be applied to (1). The size of the attractor in (58) can be reduced to the minimum-size attractor S_x if we can select $\xi_x^* = \min_x[\xi_x]$.*

Proof : The proof is obvious from Theorem 2.3 since $S_x(S_x)$ in (58) can be minimized if there exist a sequence of u^* so that the minimum Euclidean distance can be obtained by applying the P^2A^2 such that $\xi_x = \xi_x^*$ in (47).
Remark (fuzzy robustness) : As the proposed fuzzy controller deals with fuzzified cells in the DOI, the controller is robust against disturbances as long as dynamic and parametric uncertainties of nonlinear systems, or some external noise, are bounded within some allowable regions. It is asserted that any state $x(t)$ outside the ϵ-neighborhood of x^* is attracted to S_x in a finite time, and therefore, the bounded region can allow the corresponding magnitude of any disturbance with the same performance.

3 Example: Simulation of A Single-Link Robot Arm

As an illustrative example, consider a single-link robot arm shown in Figure 4 with a mass of 1.0 [Kg] at the end of the link of 1.0 [m] length. The dynamics can be derived by using the Lagrange-Euler equation of motion [26].

$$u(t) = m\ell^2\ddot{\theta}(t) - mg\ell\sin\theta(t) \tag{59}$$

and since $m = 1[Kg]$, $\ell = 1[m]$, and $g = 9.8[m/sec^2]$ if we define $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$, the following vector equations can be obtained

$$\dot{x}_1(t) = f_1(x(t), u(t), t) = x_2(t)$$

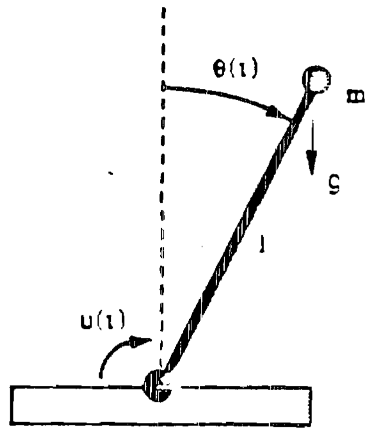

Figure 4: Single-link robot arm

$$\dot{x}_2(t) = f_2(x(t), u(t), t) = 9.8 \sin x_1(t) + u(t). \tag{60}$$

The direction of the vector field of (60) is found by

$$\varphi(t) = \tan_2^{-1}(f_1, f_2) \tag{61}$$

where $\tan_2^{-1}(\cdot,\cdot)$ can access any direction in the (x_1, x_2) coordinates. Making use of the vector field $\varphi(t)$ in the P^2A^2, the fuzzy rulebase for the control input $u(t)$ in (3) and (15) can be constructed to regulate $x_1(t)$ and $x_2(t)$ to zero ($x^* = 0$) as follows:

- R_1: if (x_1 is PL) and (x_2 is PL) then (u is NL)
- R_2: if (x_1 is PL) and (x_2 is SZ) then (u is NS)
- R_3: if (x_1 is PL) and (x_2 is NL) then (u is SZ)
- R_4: if (x_1 is SZ) and (x_2 is PL) then (u is NS)
- R_5: if (x_1 is SZ) and (x_2 is SZ) then (u is SZ)
- R_6: if (x_1 is SZ) and (x_2 is NL) then (u is PS)
- R_7: if (x_1 is NL) and (x_2 is PL) then (u is SZ)

- R_8: if (x_1 is NL) and (x_2 is SZ) then (u is PS)
- R_9: if (x_1 is NL) and (x_2 is NL) then (u is PL)

where we use abbreviations for various linguistic variables: 'PL' for 'positive large', 'PS' for 'positive small', 'SZ' for 'small near zero', 'NS' for 'negative small', and 'NL' for 'negative large'. Membership functions for the fuzzy variables x_1, x_2, and u, are the S, Π, and Z curves. The above 9 rules guarantee global asymptotic stability in the sense of Lyapunov.

In the simulations, the initial conditions are chosen to be $x_1(0) = 30$ [deg] and $x_2(0) = 0$. The maximum value for the control input $u(t)$ is $\pm 300[Nm](M_u)$ while the boundary values for $x(t)$ are $\pm\frac{\pi}{32}$ for $x_1(t)$; $\pm 2\pi$ for $x_2(t)$. In the case of 9 rules, Figure 5 (a) and (b) show the non-fuzzy cells and the phase portrait of the nonlinear system (60) for different $\bar{u}[i]$'s of the fuzzy rulebase established by the P^2A^2 utilizing the vector field where the PAC's and the SAC's are represented by R^*'s and $R^\dagger$'s, respectively. The results of 9 rules and those of 25 rules of the fuzzy-logic controller are compared in Figures 6, 7, and 8. Figure 9 shows actual state trajectories of 9 rules and 25 rules. As mentioned earlier, more rules usually imply higher convergence rates since the granularity of the DOI reduces the fuzziness in the phase plane. Moreover, an impact force of $100[N]$ is applied to the end of the link during $0.01[sec]$ starting at $t = 4.0[sec]$ to consider some uncertainty at the joint.

Although the simulations exclude the actuator dynamics, if the actuator modes are fast enough compared to the robot dynamics, the 9 rules can be applied without further modifications. If not, the number of states should be increased by adding the actuator dynamics to the fuzzy rulebase. For more information on joint flexibility in connection to actuators, see [27].

4 Conclusions and Open Problems

The research work reported in this paper supports a connection between artificial intelligence and analytic methods by linking the 'if-then' rules of the control input to a systematic design procedure called the P^2A^2 for nonlinear systems. Advantages of fuzzy-logic controllers include robustness and flexibility in modifying the particular domain by providing qualitative reasoning for a specific control decision. Also, we can reduce the decision errors by transforming crisp sets into fuzzy sets.

Global asymptotic stability of nonlinear fuzzy-logic control is considered by using Lyapunov's direct method. Some difficult problems associated with fuzzy control, such as ambiguity in completeness of the fuzzy rulebase, updating and calibrating the rulebase controllers, and exhaustive use of the linguistic rules, can be solved by applying the P^2A^2. We can easily change, add, or delete some rules according to the organization level.

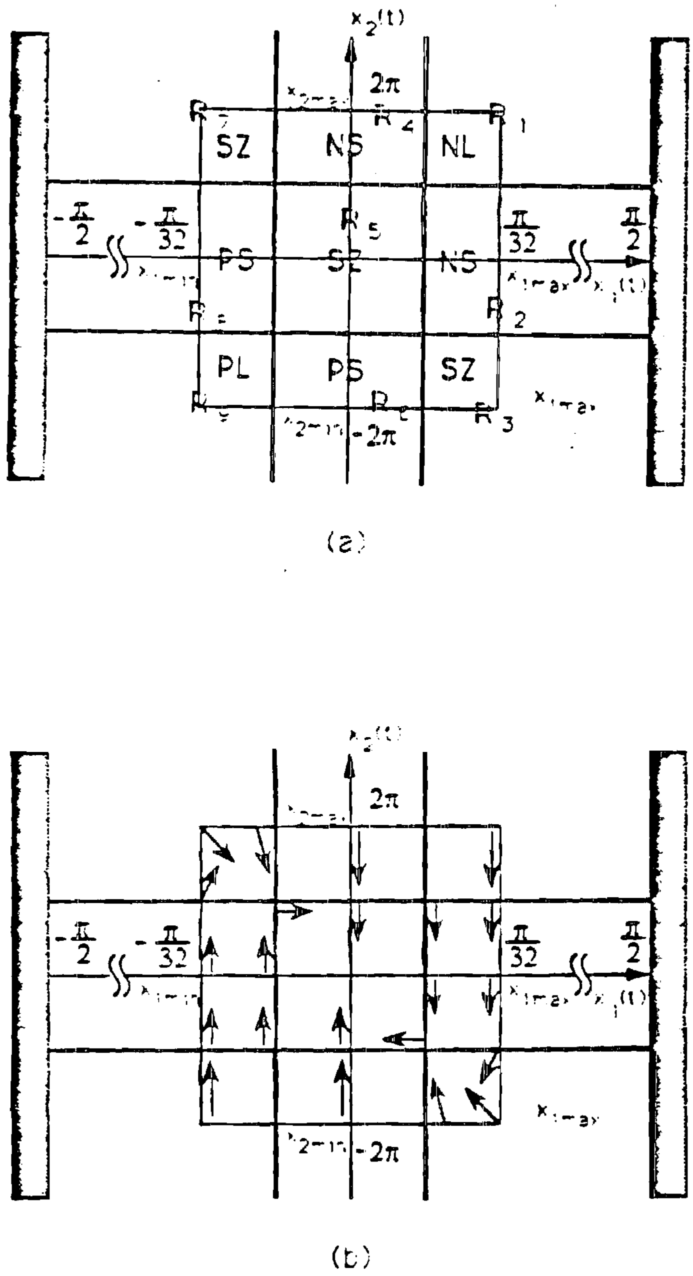

Figure 5: The cells (a) and the phase portrait (b) of $x_1(t)$ and $x_2(t)$ from the P^2A^2

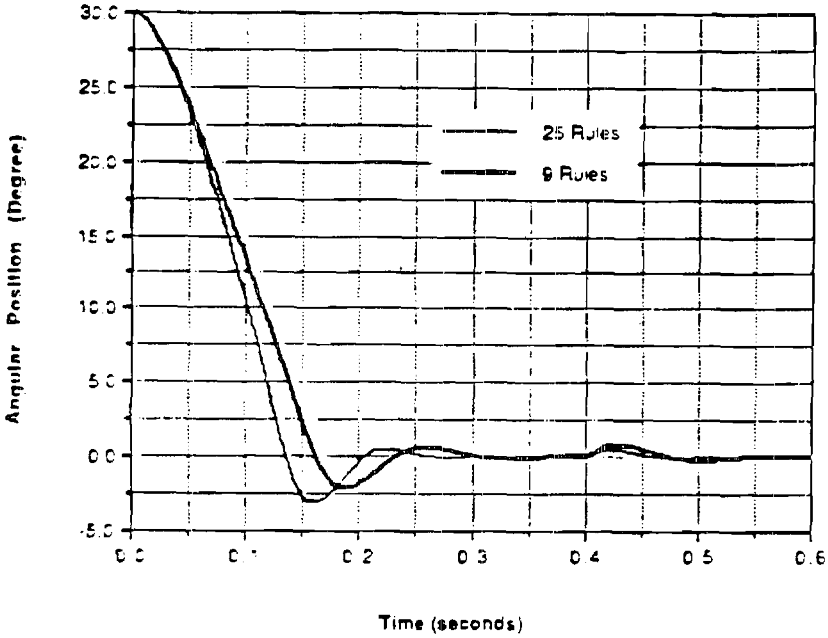

Figure 6: Angular position of the single-link robot arm

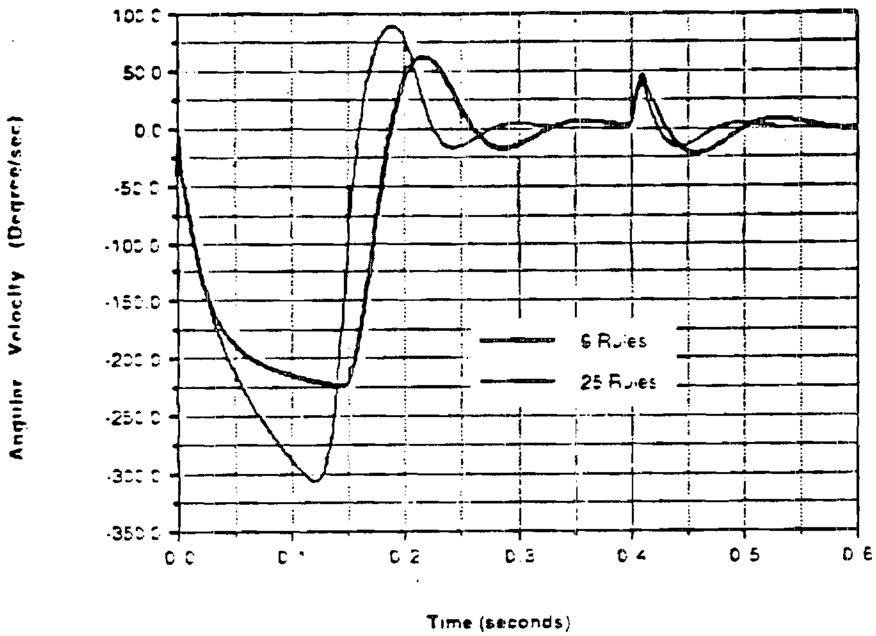

Figure 7: Angular velocity of the single-link robot arm

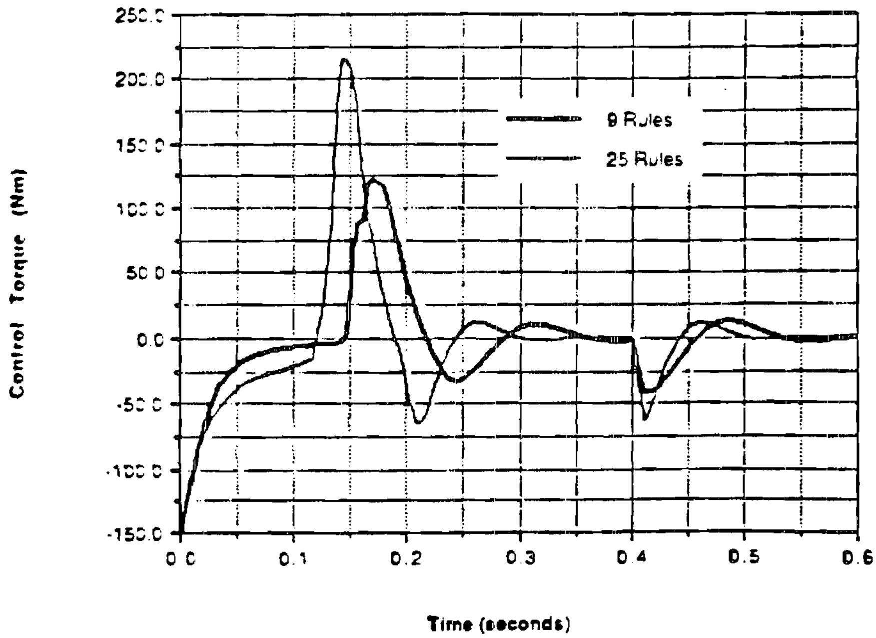

Figure 8: Fuzzy control torque input applied to the single-link robot arm

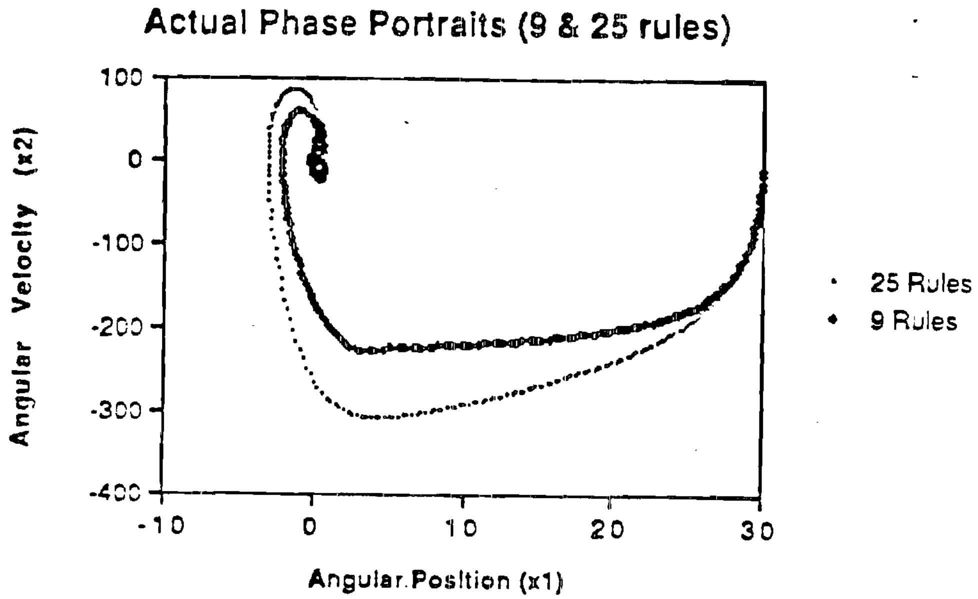

Figure 9: Actual phase portrait of the single-link robot arm

Simulation results show that fast convergence, asymptotic stability, and robustness with respect to disturbances are achieved. Although P^2A^2 may requires a large amount of fuzzy information for a system of higher dimension, it is easy to expand the P^2A^2 concept by using the n-dimensional vector fields. Fuzzy systems need more samples for the decision making process compared with analytical methods, however it handles unreliable information by using linguistic variables and fuzzy relations. In the simulations, an infinite-valued logic is applied to fuzzy sets, but, in the practical sense, simplified fuzzy relational matrices can be used if we use multi-valued logics of the quantized domain. A fuzzy controller can be implemented via a parallel computer architecture as shown in Figure 10 since the rules are disjunctive.

Some important aspects of the fuzzy rulebased controllers are as follows: First, the fuzzy dynamic controller $\dot{u}(t) = r_u(x(t), u(t), t)$ is a substitute for the fuzzy regulator $u = r(x(t))$. However, limit cycles are experienced probably caused by the double integrator in the closed-loop system. Second, there is a proportional relationship between the size of the fuzzy rules and the convergence rates. Finally, robustness properties of the fuzzy controller can be enhanced by adding convergent learning algorithms that, in other words, are the static rules that change the fuzzy rulebase itself which is dynamic.

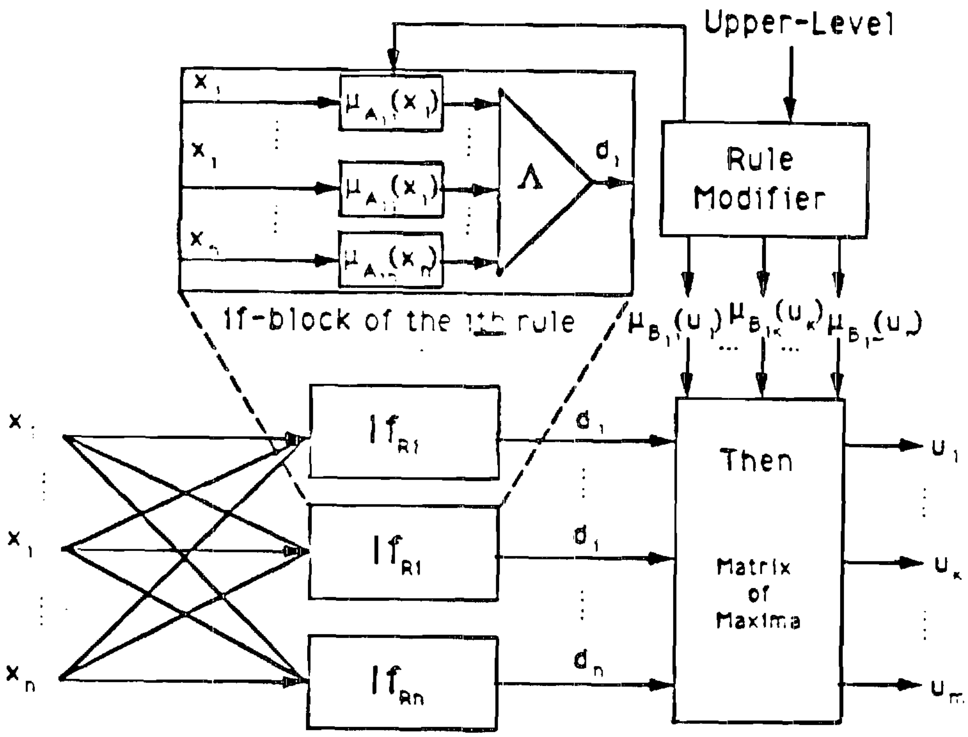

Figure 10: The parallel computer architecture of the fuzzy rulebased systems

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List of Figures

1 Block diagram of a fuzzy-logic feedback system 3
2 S, Π, Z membership functions 7
3 Typical cells and the vector fields of the P^2A^2 10
4 Single-link robot arm 16
5 The cells (a) and the phase portrait (b) of $x_1(t)$ and $x_2(t)$ from the P^2A^2 18
6 Angular position of the single-link robot arm 19
7 Angular velocity of the single-link robot arm 20
8 Fuzzy control torque input applied to the single-link robot arm 20
9 Actual phase portrait of the single-link robot arm 21
10 The parallel computer architecture of the fuzzy rulebased systems 22

Model Reference Fuzzy Control

by

Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

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Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering, Georgia Institute of Technology
Atlanta, Georgia 30332

Abstract

nis paper is concerned with a fuzzy logic controller for linear systems. A fuzzy lebase is employed to stabilize the closed-loop system consisting of a linguistic ntroller and a process. By utilizing a reference model, impulse-like control inputs n be avoided. A generic guideline for building a control rulebase is developed and embership functions for linguistic variables are chosen to be S, Π, and Z functions. he rules guarantee asymptotic stability. Simulation results show robustness and nvergence of the proposed symbolic controller.

Introduction

ince the introduction of fuzzy sets and possibility measures by Zadeh [1,2] to pe with the approximate estimation of uncertainty, many learning and adapta- on schemes have been developed for various applications. A fuzzy set is defined as class of objects with a continuum of grades of membership using certainty or con- dence factors in order to handle uncertainties or complexity in a particular domain f knowledge [3].

Compared with a probabilistic representation of systems caused by random phe- omena, a fuzzy system is an algebraic relation derived from possibility measure heory and it can be represented either by the input-output property or by a transi- ion of states in which case it is called a fuzzy dynamic system. When only imprecise r indefinite information about a process's dynamic behavior is available, then fuzzy ets and linguistic variables may be used to build better models for a process. Several efinitions and properties of fuzzy sets and fuzzy numbers are included in Appendix.

A fuzzy number can be described by linguistic terms ('large', 'medium', 'small', very small', etc.) whose fuzziness provides many degrees of freedom in dealing with ncertainty by using non-uniform possibility distributions [3]. For any presumption evel α ($0 < \alpha < 1$), the confidence interval represents information that reduces the ncertainty by using lower and upper bounds. From an objective point of view, it nay be a confidence interval between two measured data as defined in statistics; rom a subjective viewpoint, it may be interpreted as the available expertise about he process. As α increases, the confidence interval decreases in most cases [4].

It is possible to apply fuzzy sets to a classical tracking control problem. Perfor- nance criteria of energy functions or least-squares error approaches are suitable for ontrol purposes and rulebases should be constructed to obey an energy-dissipating roperty. The objective of this paper is to construct a rulebase having the properties f asymptotic stability and robustness under uncertainties such as bounded distur- ances, parametric disturbances, and unmodeled dynamics. By providing sufficient onditions for the stabilizing rules, a prototype guideline for the control update is roposed. Simple SISO linear plants are used as examples.

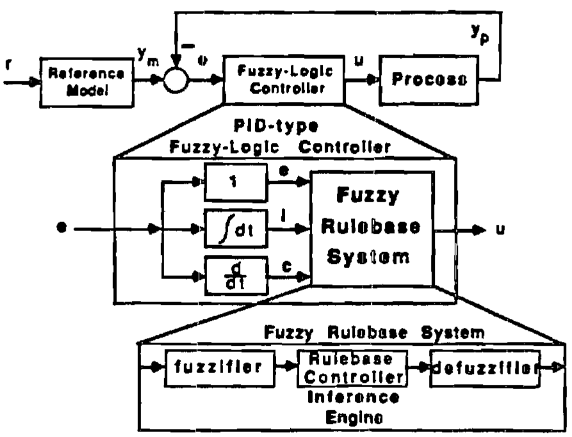

Figure 2.1: Block diagram of the MRFC structure

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2 Model Reference Fuzzy Control

In this section, an expert control system is proposed which consists of a fuzzy rule-based controller and a linear process. A model reference fuzzy control (MRFC) concept is introduced in order to guarantee the model-following property of a fuzzy controller. The mean of maxima procedure is applied to the inference scheme that estimates the control feedback.

2.1 Structure of MRFC

A rule set represents a control objective in fuzzy relations. From measured data such as the speed of a dc motor, the joint angle or velocity of a robot arm, aircraft elevator deflection, and so on, a rulebased controller interprets such data via a fuzzy decision maker and provides control signals which satisfy the performance criterion.

A reference model is implemented in parallel with the process so that the tracking error between the model and the process prevents impulse-like signals in the inferred control value. Figure 2.1 shows the block diagram of the proposed MRFC. The control input $u[k] = u(kt_s)$ can be represented by

$$u[k] = u[k-1] + \delta u[k] \tag{2.1}$$

where t_s is a sampling period and $\delta u(t) = \delta u[k]$ is the update for the control input $u[k]$ from the rulebase. The tracking error $e(t) = e[k]$ is defined by

$$e(t) = y_m(t) - y_p(t) \tag{2.2}$$

where $y_m(t)$ and $y_p(t)$ are the model output and the process output, respectively. The error $e(t)$ is fuzzified in the linguistic controller. The reference model is chosen to be stable and it may be a 1st-order SISO linear system.

Proportional-integral-derivative (PID) and proportional-derivative (PD) control concepts are used as fuzzy inputs to the symbolic controller. The derivative and integral of the tracking error are also fuzzified in the rulebased controller and are defined as follows:

$$c(t) = e[k] - e[k-1] \cong t_s \dot{e}(t) \tag{2.3}$$

$$i(t) = \int_0^t e(\tau) d\tau \tag{2.4}$$

where $c(t)$ and $i(t)$ are the change in error and the integral error, respectively. $\delta u(t), e(t), c(t)$, and $i(t)$ are fuzzified next by specifying the universe of discourse and by assigning appropriate membership functions for each variable as described in the following section.

The rulebased controller may be interpreted as

$$\mathcal{F} : e \Longrightarrow \delta u \quad \text{or} \quad \delta u = F(e) \tag{2.5}$$

where $\mathcal{F}$ and $F(\cdot)$ represent a nonlinear mapping and the corresponding nonlinear function, respectively, that describes a fuzzy relationship including a fuzzifier and a defuzzifier. Let the associated universe of discourse be $\Delta U, E, C, I$ for $\delta u, e, c, i$, respectively, then $\mathcal{F} : E \times C \times I \Rightarrow \Delta U$ for PID control and $\mathcal{F} : E \times C \Rightarrow \Delta U$ for PD control. If the Jacobian of $F(\cdot)$ is defined by

$$J_F = \frac{\partial F}{\partial e} \tag{2.6}$$

then $\dot{u} = \frac{du}{dt}$ may be written as

$$\dot{u} = J_F \dot{e}. \tag{2.7}$$

From (2.1) and (2.3), $\delta u = t_s \dot{u}$ and $c = t_s \dot{e}$, and, therefore, we obtain

$$\delta u = J_F c. \tag{2.8}$$

The Jacobian system for the process is defined by

$$\delta y_p = J_P \delta u. \tag{2.9}$$

Definition 2.1 ***The model-following property is stated as follows:***

$$\delta y_m > 0 \Longrightarrow \delta y_p > 0 \quad \text{or} \quad \delta y_m < 0 \Longrightarrow \delta y_p < 0 \quad \forall t > 0, \tag{2.10}$$

r equivalently,

$$\delta y_p \delta y_m > 0 \quad \forall t > 0. \tag{2.11}$$

onsider the following lemma that describes the Jacobian system (2.8).

emma 2.1 *Consider a nonlinear system from y_m to y_p represented by*

$$\mathcal{H} : y_m \Rightarrow y_p \tag{2.12}$$

here $y_m, y_p \in \mathcal{R}^1$ and the associated Jacobian system $J_H : \delta y_m \Rightarrow \delta y_p$ defined by

$$\delta y_p = J_H \delta y_m. \tag{2.13}$$

he model-following property holds if and only if $0 < |J_H| < \infty$, and $J_H > 0$ $\forall t > 0$ here

$$J_H = J_P[I + J_F J_P]^{-1} J_F \tag{2.14}$$

here J_P is the Jacobian system for the process.

roof: sufficiency — Since $\delta y_p = J_H \delta y_m$, and from the model-following property,

$$\delta y_p \delta y_m = J_H \delta y_m^2 > 0. \tag{2.15}$$

herefore, $J_H > 0$.

ecessity — Let $J_H > 0$ and J_H is finite then it is obvious that $\delta y_p \delta y_m > 0$, $\forall t > 0$.

emark: In fact, using the model-following property, we can say that y_p has the ame trend of increasing or decreasing y_m.

A sufficient condition for the model-following property can be found by the folowing theorem.

Theorem 2.1 *Let $J_P > 0$ in (2.9) or (2.14), then the model-following property holds f there exists $\alpha > 0$ such that $J_H > 0$ $\forall t > 0$, i.e.,*

$$J_F = \alpha J_P^T \quad \forall t > 0 \tag{2.16}$$

Proof: As $J_F = \alpha J_P^T$, substituting it into (2.14), $J_H > 0$, $\forall t > 0$.

Remark: The above theorem says that the Jacobian system for the fuzzy rulebase, J_F should be positive for the model-following property of the system $\mathcal{H}$ if J_P is positive.

2.2 Design of Membership Functions

The design of membership functions is an important step in the development of the fuzzy controller and is closely linked to the possibility distribution of linguistic variables. Fuzzy membership functions should satisfy the properties of convexity and normality in usual cases.

Membership functions for $e(t), c(t)$, and $\delta u(t)$ make use of S, Z, and Π curves as proposed by Zadeh [2]. The parameters of these functions should be carefully determined according to the scope of each linguistic variable and they are closely related to the mean and the variance of a probability distribution. It is noted that fuzzy sets do not accumulate every available evidence since the operators involved are minimum or maximum procedures. We define S, Π, and Z membership functions for the MRFC as follows:

$$S(x;\alpha,\beta,\gamma) = \begin{cases} 0 & x < \alpha \\ 2(\frac{x-\alpha}{\gamma-\alpha})^2 & \alpha < x < \beta \\ 1 - 2(\frac{x-\gamma}{\alpha-\gamma})^2 & \beta < x < \gamma \\ 1 & x > \gamma \end{cases} \tag{2.17}$$

$$\Pi(x:\beta,\gamma) = \begin{cases} S(x;\gamma-\beta,\gamma-\frac{\beta}{2},\gamma) & x < \gamma \\ S(x;\gamma,\gamma+\frac{\beta}{2},\gamma+\beta) & x > \gamma \end{cases} \tag{2.18}$$

$$Z(x;\alpha,\beta,\gamma) = 1 - S(x;\alpha,\beta,\gamma) \tag{2.19}$$

The parameters of the membership functions are α, β, and γ. The transition points defined as the value of the universe of discourse for which the grade of membership is equal to $\frac{1}{2}$ are $x = \beta$ for S, Z; $x = \gamma \pm \frac{\beta}{2}$ for Π. The maximum grade of 1 occurs at $x = \gamma$ for all these functions and the minimum value of 0 at $x = \alpha$ for S, Z; $x = \gamma \pm \beta$ for Π.

2.3 Stability of The Closed-Loop Fuzzy System

Stability of the closed-loop fuzzy feedback system is an important issue. Kickert et al.[5] showed that a nonlinear fuzzy-logic controller for a specific simple case is identical to a multi-level relay which can be analyzed via a describing function technique and they anticipated the autonomous oscillation of the system. Ray et al.[6] connected a fuzzy-logic controller to the concept of L_2-stability by using a compensated system with Nyquist criteria. Kiszka et al.[7] introduced the concept of energistic stability for a class of fuzzy dynamic systems.

There are few mathematical properties describing the relationship between membership functions and the inferred control actions. Kania and his co-workers [8] proposed an α-stability property, α-decision stability property, $\alpha - \beta$ strong decision stability property, a good mapping property, a γ-good mapping property, and associated conditions for the above properties as defined in Appendix.

In this paper, we propose a symbolic controller that guarantees global asymptotic stability. Consider the fuzzy relations R_F given by the statements: let the size of a rulebase be s and the number of fuzzified variables m.

- if A_1 then B_1

  ⋮
- if A_i then B_i

  ⋮
- if A_s then B_s

where $A_i = \bigwedge_{j=1}^{m} A_{ij}$. R_F may be described by

$$R_F = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s). \tag{2.20}$$

For example, A_{11} ='e is positive large', A_{12} ='c is negative small', and B_1 ='δu is negative small'. The membership functions for each case are described by $\mu_{A_{11}}(e), \mu_{A_{12}}(c)$, and $\mu_{B_1}(\delta u)$, respectively. The above fuzzy relations imply the well-known compositional rule of inference [3] (the inference scheme of modus ponens).

In order to guarantee stability of the closed-loop system, the strategic rules should be constructed so as to meet the passivity or the energy dissipating property for a given performance index. The following is a set of guidelines for building a control rule set for stabilizing fuzzy relations:

1. Choose a characteristic function from a referential subset (choose a feasible subset) and select a membership function that follows the α-decision stability property for that feasible subset of a linguistic variable.

2. Navigate the whole set of possible linguistic rules, applying the chosen membership functions to each linguistic variable.

3. For every rule, the γ-good mapping property should hold ($\gamma \neq 0$). That is to say, if the premise part of the i-th rule, A_i, consists of several sub-premises or fuzzy subsets, A_{ij}, such as $A_i = \bigwedge_{j=1}^{m} A_{ij}$, then, for any j and for some presumption level, $\alpha_1 (0 < \alpha_1 < \alpha)$, there should exist at least one A_{kj} such that $A_{ij} \wedge A_{kj} \neq \emptyset$ for $i \neq k$. This is a necessary condition for the existence of the solution of the modified mean of maxima scheme.

4. In the closed-loop feedback system, if the above conditions hold and the strategic rules are built so that every fuzzy variable has the energy dissipating property, then the fuzzy feedback system is stable and converges. A rule satisfying stability may be deleted since $\delta u = 0$.

The modified mean of maxima procedure is one approach to the defuzzification of fuzzy control variables. As long as the above guidelines are satisfied, the change in control input $\delta u(t)$ can be found by

$$\delta u(t) = \frac{\sum_{i=1}^{s} \delta u_i^* f_i(t)}{\sum_{i=1}^{s} f_i(t)} \tag{2.21}$$

where $f_i(t)$ is the degree of fulfilment defined by

$$f_i(t) = \bigwedge_{j=1}^{m} \mu_{A_{ij}}(x_j(t)) \tag{2.22}$$

with $x_j \in X_j$ (for example, $x_1 = e$, $x_2 = c$), and δu_i^* is defined by

$$\delta u_i^* = \{\vartheta | \mu_{B_i}(v) = \max_v [\mu_{B_i}(v)]; v, \vartheta \in \Delta U\} \tag{2.23}$$

where ΔU is the universe of discourse of δu. From (2.8), the relationship between the Jacobian J_P and the defuzzifier is described by

$$\delta u = J_F c = \frac{\sum_{i=1}^{s} \delta u_i^* f_i}{\sum_{i=1}^{s} f_i}. \tag{2.24}$$

The error becomes

$$e = y_m - \int_0^t h_p(t-\tau) \int_0^\tau \frac{\sum_{i=1}^{s} \delta u_i^* f_i(\xi)}{\sum_{i=1}^{s} f_i(\xi)} d\xi d\tau \tag{2.25}$$

ere $h_p(\cdot)$ represents the impulse function of the process. Let the impulse response ı be separated as

$$h_p(t-\tau) = h_1(t)h_2(\tau) \tag{2.26}$$

en the change in error $e = t_s\dot{e}$ is

$$c = t_s[J_m\dot{r} - \dot{h}_1 \int_0^t h_2(\tau)u(\tau)d\tau - h_p u]. \tag{2.27}$$

is required that the update must satisfy $ec < 0$ in some region for asymptotic ıbility.

4 A Rulebase for Asymptotic Stability

t $E(x,t)$ be a candidate energy function then Lyapunov's direct approach can be ıted as follows: If $E(x,t)$ is bounded and its derivative $\dot{E}$ exists, the following nditions should hold for asymptotic stability [9]:

$$E(x,t) > 0 \quad \forall x \neq 0 \tag{2.28}$$

$$\dot{E}(x,t) < 0. \tag{2.29}$$

he Lyapunov function candidate is defined by (2.30),

$$E(e,t) = e^T P e > 0 \tag{2.30}$$

$$\dot{E}(e,t) = \frac{1}{t_s}(c^T P e + e^T P c) \tag{2.31}$$

here P is a symmetric positive definite matrix. From (2.31), it is obvious that the gns of e and c should be opposite in order to satisfy $\dot{E}(e,t) < 0$. One possible c ıay be described by $c = De$ where D is a negative definite matrix.

ıssumption 2.1 *Assume that the sign of the dc gain in the transfer function of a near system is known (i.e., positive).*

'he above assumption is minimal in the sense that the degree of the transfer function, elative degree, and exact system delay are not required as in the cases of analytical ıethods such as linear quadratic regulator (LQR), model reference adaptive control MRAC), etc.

The following are 9 rules which guarantee global asymptotic convergence in comıliance with Lyapunov stability. Here, we use abbreviations for various linguistic lescriptions: 'PL' for 'positive large', 'PS' for 'positive small', 'SZ' for 'small near ero', 'NS' for 'negative small', and 'NL' for 'negative large'. Membership functions or e, c, and δu using S, Z, and Π curves are shown in Figure 2.2. A priori informaıion about the process is necessary such as the trends in the impulse response, the ıpproximate time delay, etc. The following is a PD rule set for the fuzzy rulebased ontroller.

1. If (e is PL) and (c is PL) then (δu is PL)
2. If (e is PL) and (c is SZ) then (δu is PS)
3. If (e is PL) and (c is NL) then (δu is SZ)
4. If (e is SZ) and (c is PL) then (δu is PS)
5. If (e is SZ) and (c is SZ) then (δu is SZ)
6. If (e is SZ) and (c is NL) then (δu is NS)
7. If (e is NL) and (c is PL) then (δu is SZ)
8. If (e is NL) and (c is SZ) then (δu is NS)
9. If (e is NL) and (c is NL) then (δu is NL)

'he above rule set represents a PD control scheme for asymptotic stability. Rules ,5,7 obey Lyapunov stability conditions; Rules 1,4,6,9 satisfy the model-following roperty of the Jacobian system J_P; and the component of $\delta u(t)$ contributed by the emaining rules 2,8 is chosen to reduce tracking errors in such a way that the process ıutput $y_p(t)$ follows the reference model output $y_m(t)$ which is closely related to the bove assumption. In the case of PID control, $i(t)$ is added to the condition part ıaking the size of the rulebase larger. By adding i to the rule set, it is possible to ncrease the rate of convergence.

'heorem 2.2 *Let the sign of the dc gain in the transfer function and the Jacobian J_P of a process be known (dc gain> 0, $J_P > 0$) and let the suggested guidelines be atisfied for the chosen membership functions of the linguistic variables. Let the PD ule set satisfies the following relations:*

- *case 1: if $ec < 0$ then δu is zero. (Lyapunov stability)*
- *case 2: if $ec \geq 0$ ($e \neq 0$) then $\delta u = J_P c$ with $J_P > 0$. (the model-following property)*
- *case 3: if $ec = 0$ ($e \neq 0$) then δu has the same sign as e. (known dc gain of the process)*

Then asymptotic stability is achieved and the convergence of the tracking error is guaranteed.

Proof: (case1) $ec < 0$ — The vector field of (e, c) directs the state trajectory toward the origin. (case 2) $ec \geq 0$ — From the model-following property, if $\delta y_m \to 0$ then $\delta y_p \to 0$ since $J_B > 0$. Thus, the system error converges to some nominal value. (case 3) In this case, $c = 0$ and the error biases. Therefore, e needs to be adjusted according to the dc gain of the process.

It is noted that, if the sign of J_P is positive, then $J_P > 0$ is a sufficient condition for the model-following property. Figure 2.3 shows the state trajectory and the regions by the 9 rules in a typical case.

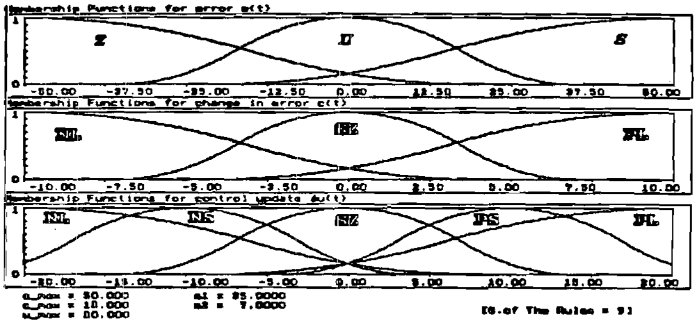

Figure 2.2: The membership functions of e, c, and δu

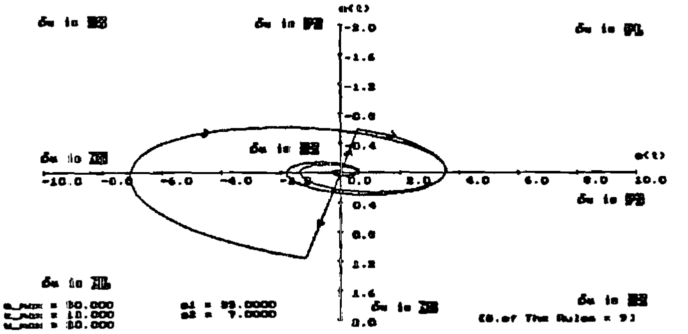

Figure 2.3: The regions and the state trajectory by the 9 rules

3 Simulation Results

A 2nd order single-input single-output (SISO) plant transfer function $H_p(s)$ and the unmodeled version of its transfer function $H_{up}(s)$ are chosen to show robustness of the proposed fuzzy controller,

$$H_p(s) = \frac{2}{s+2} \tag{3.1}$$

$$H_{up}(s) = \frac{40}{(s+2)(s+15)} \tag{3.2}$$

where $H_{up}(s)$ contains unmodeled dynamics of $H_p(s)$ multiplied by $\frac{20}{s+15}$. A reference model $H_m(s)$ is represented by

$$H_m(s) = \frac{3}{s+3}. \tag{3.3}$$

A reference input signal $r(t)$ is of a staircase type: $r(t) = 10$ at $t = 1$ sec, $r(t) = 20$ at $t = 5$ sec, and then $r(t) = 0$ at $t = 9$ sec.

In the fuzzy rulebased controller, the parameters of the membership functions are the maximum values in the region of interest such as $e_{max}, c_{max}, i_{max}$, and u_{max}; mean values of linguistic variables; the sizes of the feasible sets are related to s_e, s_c, and s_i. (s_e is the size of the feasible region between $\mu_{A_i}(x) = 1$ and $\mu_{A_i}(x) = 0.5$ in S, Π, Z functions)

- **case1: unmodeled dynamics**

  Using the above $H_{up}(s)$ as a process and $H_m(s)$ as a reference model, the fuzzy rulebased controller is applied to a tracking problem and the size of the rulebase varies from 9 to 12 to 25 or finally 27 rules. As the number of rules increases, the convergence rate improves as shown in Figure 3.1. The integral term $i(t)$ is included when the number of rules is 27. Subsequent cases are results of 27 rules with unmodeled dynamics. If the sign of the dc gain is negative, the rules should be changed in order to meet asymptotic stability of the closed-loop system.

- **case2: bounded disturbances**

  A uniform additive noise is considered to be corrupting the process output $y_p(t)$ and it is bounded between -0.5 and 0.5 as shown in Figure 3.2. Simulation results show robustness for bounded output disturbances.

- **case3: parameter variations**

  The poles of the process are changed from $t = 3$ sec to $t = 7$ sec and the transfer function $H'_{up}(s)$ in that case is described by

  $$H'_{up}(s) = \frac{60}{(s+4)(s+20)}.$$

  A change in the error dynamics and no significant changes in the process output are observed in Figure 3.3.

- **case4: unstable system**

  The process transfer function becomes unstable and is given by

  $$H_{up}(s) = \frac{4}{s-2}.$$

  Note that the control input $u(t)$ is flipped over in Figure 3.4. Since the steady-state values in the closed-loop system exist, it is possible for the fuzzy controller to find a control input satisfying the stability property.

4 Conclusions and Discussion

Simulation results indicate that the proposed fuzzy control scheme exhibits robustness and stability properties with respect to parametric disturbances, bounded noise, unstable systems, and unmodeled dynamics. The design parameters of the fuzzy controller are less sensitive than those of MRACs or LQRs.

Self-organizing techniques are required for the autonomous addition and deletion of rules and changes in parameters according to some strategic performance indices. As shown in the simulation results, corrective or learning schemes refine the structure of rulebased controllers. In the case of fuzzy relational matrices, some techniques are suggested in [10,11]. Usually, the size of the rulebase increases after a number of iterations of the learning process.

In order to apply the fuzzy linguistic system, expert knowledge is needed for a special domain application. Use of performance criteria in designing a rulebased controller is recommended and it is suggested to describe the plant dynamics as fully and completely as possible.

Some advantages and disadvantages of the rulebased control approach include [12]: Advantages are robustness compared with analytic methods, flexibility in modification of the domain, and improvement in the man-machine interface by providing reasoning behind a particular control decision. Disadvantages are ambiguity in completeness of the rulebase, difficulties in updating and calibrating the rulebased controllers, exhaustive use of the linguistic rules, and superfluous memory requirements.

In general, fuzzy systems need more samples for the decision making process compared with analytical methods but handle unreliable information by using linguistic variables and fuzzy relations. A fuzzy controller can be implemented by a parallel computer architecture since the rules are disjunctive.

5 Appendix: Mathematical Properties and Notations of Fuzzy Sets

Some definitions relating to fuzzy sets and fuzzy numbers are as follows [4]:

Definition 5.1 *An ordinary subset, A, of a referential set, $\mathcal{E}$, is defined by its 'characteristic function' $\mu_A(x) \in \{0,1\}$, $\forall x \in \mathcal{E}$.*

A feasible set $F(Y)$ may be described by a characteristic function. A linguistic description in Y such as 'large' resides in a feasible subset $F(Y) \in Y$.

Definition 5.2 *A fuzzy subset, A, from the referential set, $\mathcal{E}$ is defined by its characteristic function $\mu_A(x) \in [0,1]$, $\forall x \in \mathcal{E}$, which is called the 'membership function'.*

A membership function may represent a non-uniform possibility distribution for the associated linguistic variable described by the characteristic function. The next two definitions are necessary and sufficient conditions for good membership functions in general. A fuzzy set is an invariant set as the presumption level increases.

Definition 5.3 *A fuzzy subset A on $\mathcal{X}$ is 'convex' if and only if $\forall x_1, x_2 \in \mathcal{R}, \forall \lambda \in [0,1]$:*

$$\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \mu_A(x_1) \wedge \mu_A(x_2).$$

Definition 5.4 *A fuzzy subset A on $\mathcal{X}$ is 'normal' if and only if $\forall x \in \mathcal{R} : \max_x \mu_A(x) = 1$.*

The formulas of some fuzzy number arithmetic operators are defined as follows [3,13]:

Definition 5.5 *A 'fuzzy implication' is a mapping $S : A_i \Longrightarrow B_i$ and the membership function for S is $\mu_S(y,x) = \min[\mu_{A_i}(x), \mu_{B_i}(y)]$ where $x \in X$ and $y \in Y$.*

Definition 5.6 *The 'compositional rule of inference' is as follows: for a given fuzzy implication $S : A_i \Longrightarrow B_i$, the fuzzy set B'_i inferred by a given fuzzy set A'_i has the membership function defined by $\mu_{B'_i}(y) = \max_x \min[\mu_{A'_i}(x), \mu_S(y,x)]$.*

Given the following mappings:

$$\mathcal{X} = \{(x, \mu(x))\}; \quad \mu : X \longrightarrow [0,1]; \quad x \in X$$

$$\mathcal{Y} = \{(y, \mu(y))\}; \quad \mu : Y \longrightarrow [0,1]; \quad y \in Y$$

$$R : \mathcal{X} \longrightarrow \mathcal{Y}$$

where $\mathcal{X}$ is a family of fuzzy sets defined on X, $\mathcal{Y}$ is a family of fuzzy sets defined on Y, and R is a fuzzy relation which can be interpreted as a mapping with its domain in $\mathcal{X}$ and a space of values constrained in $\mathcal{Y}$. Let $U \in \mathcal{X}$ and $V \in \mathcal{Y}$ then a formal fuzziness system is defined as

$$U \circ R = V. \tag{5.1}$$

Some definitions and mathematical properties of the formal fuzziness system are shown as follows [8,14]:

Definition 5.7 *The formal fuzziness system (5.1) has α-stability property with respect to some family $\mathcal{A}$ and some feasible subset $F(Y) \in Y$ if $\mu_V(y) \leq \alpha$ $\forall y \in Y - F(Y)$ for every fuzzy set $U \in \mathcal{A}$ of $U \circ R = V(\cdot)$ where $\circ$ is the operator for the compositional rule of inference.*

Theorem 5.1 *Consider the system where $R = A \Rightarrow B$ is given. Then $\bigvee_{i=1}^{n} \mu_A(x_i) \leq \alpha$ or $\mu_B(y_j) \leq \alpha$ for all $y_j \in Y - F(Y)$ iff the system (5.1) has the α-stability property with respect to a family $\mathcal{A}$.*

Definition 5.8 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given. Then R has the good mapping (GM) property iff $A_i \circ R = B_i$ for every $i = 1, ..., s$.*

Theorem 5.2 *Let $F(Y) \in Y$ be the feasible set. Then the system (5.1) with the relation defined above has the α-stability property with respect to a family $\mathcal{X}$ of all fuzzy sets defined on X if $\bigvee_{j=1}^{s} \mu_{B_j}(y) \leq \alpha$, $\forall y \in Y - F(Y)$.*

Theorem 5.3 *Let $A_1, \cdots, A_s$ be fuzzy sets defined on X, $\Sigma_{i=1}^{s} \mu_{A_i}(x) = 1$ for any $x \in X$, and let $B_1, \cdots, B_s$ be fuzzy sets defined on Y. Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$. Then $\mu_{A_k \circ R}(y_j) = \mu_{B_k}(y_j)$ if for some $k \in \{1, \cdots, s\}$ and some $j \in \{1, \cdots, m\}$ we have $\mu_{B_k}(y_j) \leq \max_X \mu_{A_k}(x)$ and $\mu_{B_k}(y_j) \geq \mu_{B_l}(y_j), l \neq k$.*

Definition 5.9 *A system (5.1) has the α-decision stability property with respect to some family $\mathcal{A}$ of fuzzy sets defined on X if the system has α-stability with respect to $\mathcal{A}$ and $\max_{Y-F(Y)} \mu_{U \circ R}(y) < \max_{F(Y)} \mu_{U \circ R}(y)$.*

Definition 5.10 *A system (5.1) has the $\alpha - \beta$ strong decision stability property with respect to some family $\mathcal{A}$ iff the system (5.1) has the α-decision stability property with respect to $\mathcal{A}$ and $\bigvee_{y \in Y} \mu_{U \circ R}(y) \geq \beta > \alpha$.*

Definition 5.11 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given where $A_i \in \mathcal{X}$ and $B_i \in \mathcal{Y}$ for $i = 1, \cdots, s$. Then a fuzzy relation R has the γ-good mapping (γ-GM) property in the weaker sense iff $A_i \circ R = \tilde{B}_i$ for every $i = 1, ..., s$ where $B_i \subseteq \tilde{B}_i$, and $\mu_{\tilde{B}_i} = \mu_{B_i}$ for $\mu_{B_i} \geq \gamma, 0 \leq \gamma \leq 1$.*

Note that, when $\gamma = 0$, the γ-GM property reduces to the GM property as defined above [14].

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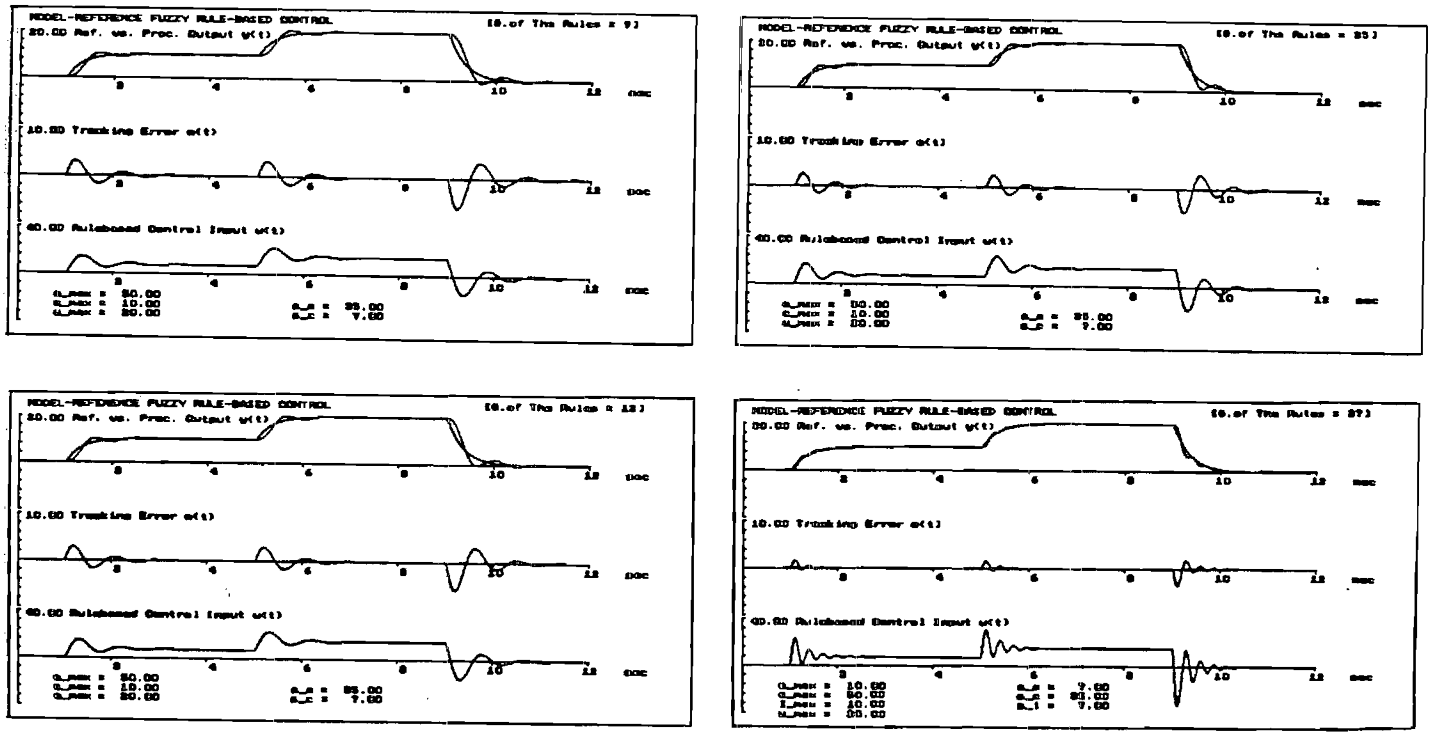

Figure 3.1: simulation of unmodeled dynamics when the number of rules changes

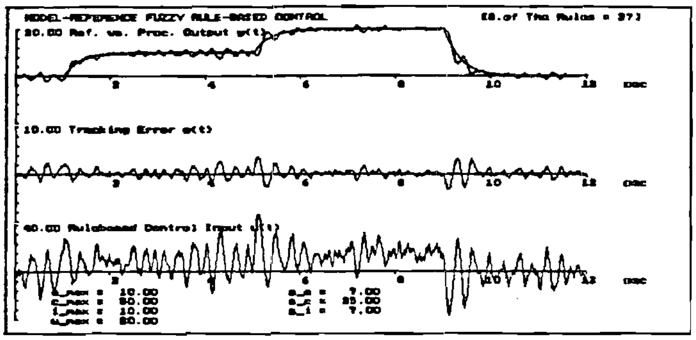

Figure 3.2: simulation of a bounded disturbance

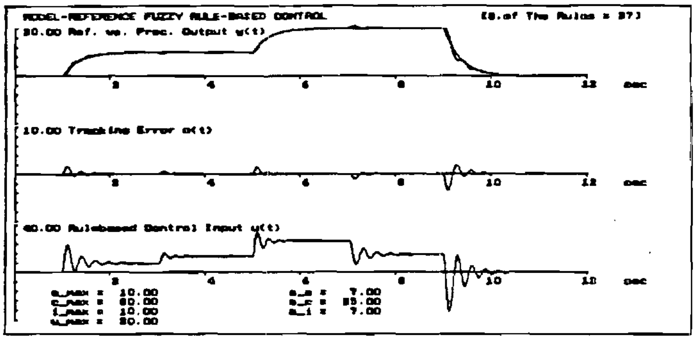

Figure 3.3: simulation of a parametric disturbance

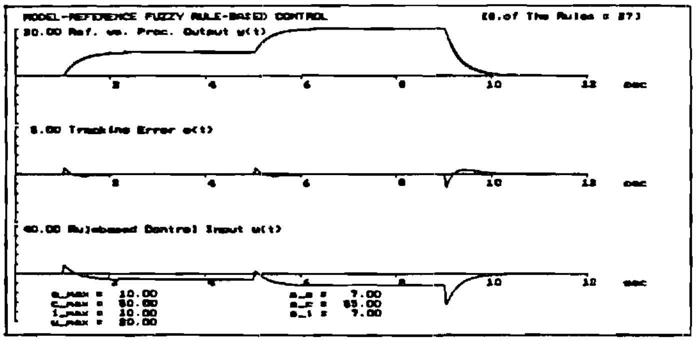

Figure 3.4: simulation of an unstable system

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

LEPHONE: (404) 894- 6252

November 27, 1989

Dr. James G. Smith (Code 1211)
Program Manager, Applied Research &
Technology Directorate
Office of Naval Research
Arlington, Virginia 22217-5000

Dear Dr. Smith:

Enclosed please find a summary report on research performed under ONR contract number N00014-89-3113 titled "A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design."

Since research activity on this project was initiated in September 1989, this report covers the reporting period from September 1, 1989 to December 1, 1989 and details some preliminary results while pointing out future research directions.

I will be pleased to provide you with any additional information required.

Very Sincerely,

George Vachtsevanos
Principal Investigator
Professor
School of Electrical Engineering

Enclosure

Annual Letter Report for FY 89

A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design

(Contract No: N00014-89-J-3113)

Sponsor: **Office of Naval Research**
Applied Research and Technology Directorate

Principal Investigator: **Dr. George Vachtsevanos**
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Tel: (404)894-6252

December 2, 1989

A HYBRID ANALYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN

CONTRACT NO: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

In September 1989, the Office of Naval Research awarded contract No. N00014- 89-J-3113 to the Georgia Institute of Technology to develop fault-tolerant control strategies for large scale dynamical systems. Specifically, the technical issues under investigation are: Development of a structure-based modeling methodology of large scale systems which possesses features of structure flexibility, i.e. a component or subsystem may be deleted in a simple way that does not substantially disturb the global control strategy; development of robust failure detection and fault identification algorithms for single and multiple faults that are maximally sensitive to true failure conditions but insensitive to noise, thus reducing the possibility of false alarms; a technique to restructure the system dynamics by isolating the faulty components; a method that will lead to a structural control law which reconfigures the original system controller in order to meet the primary performance objective of guaranteed stability while the system is operating in a degraded mode; finally, these algorithmic developments are to be demonstrated on an actual physical system to be designated jointly by ONR and Georgia Tech.

The technical approach pursued to meet these objectives is outlined as follows: The underlying factor of the fault-tolerant methodology capitalizes upon structural features of the large scale system and employes a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. In the modeling area, the topological description of many complex dynamical processes may be cast in the form of structurally interconnected subsystems. The large scale system is composed of a number of linearly interconnected subsystems. In every subsystem, a control law is applied which consists of a local and a global feedback term, thus resulting in a two-level hierarchical control strategy. We examine first large scale systems which are described by linear state equations and exhibit this two-level hierarchical structure. The analysis will be extended eventually to nonlinear systems of a similar structure. A methodology for fault diagnosis is introduced which is based upon a combination of signal redundancy and detection/estimation procedures. An expert system, consisting of a multivalued rule base and an appropriate inferencing mechanism, assesses the aggregate of fault symptoms and determines the sensitivity of a failure condition. An innovative approach to multiple failure detection uses qualitative simulation techniques to decide on the impact of a component failure on other (healthy) system components. The proposed fault detection and identification scheme incorporates such additional functionalities as fault trending and the estimation of the "best" value of critical variables and parameters under failure states. Assume that the presence of a failure has been detected by the FDI algorithm and means for isolating the faulty or potentially faulty components are available, then the system restructuring function is undertaken by a supervisory controller which modifies the original (healthy state) description of the system by changing the appropriate entries of the local subsystem and interconnection matrices.

Finally, we propose the development of a two-level structural dynamic hierarchical approach to address the control reconfiguration problem of a faulted large scale system. The approach uses a structural state model, the Block Arrow Structure, which consists of N independent linear subsystems interconnected with a control common linear dynamic system. The objective is to find a time-invariant decentralized feedback controller which minimizes a quadratic cost functional and has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

A HYBRID ANALYTICAL/INTELLIGENT
APPROACH TO FAULT-TOLERANT
CONTROL SYSTEM DESIGN
CONTRACT NO: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

Research Accomplishments

During this reporting period 9/1/89 - 12/1/89, the research team focused attention on the issues of modeling and fault detection and identification of dynamical Large Scale Systems(LSS).

1 LSS modeling

In the modeling area, a review of available techniques was completed and such schemes as aggregation and perturbation analysis, hierarchical and decentralized approaches and the descriptive variable method were assessed as to their suitability in fault tolerant control. The basic objective here is to develop mathematical modeling tools for large scale systems that possess attributes of modularity and structural flexibility so that a failed component or subsystem may be easily isolated without seriously affecting the global behavior of the system. More specifically, the failed system is to be restructured and the control actions reconfigured so that stability(survivability) is assured for the duration of the emergency.

We are developing a two-level structural dynamic hierarchical approach to address the control reconfiguration problem. We seek a structural control law and, by effectively combining information with control action, to move closer to create a structural intelligent control concept. We outline below the innovative features of this approach.

Suppose that the large scale system consists of l interconnected nonlinear subsystems of the form

$$\dot{z}_i = f_i(z_i, t) + g_i(z_1, \ldots, z_l, t) \qquad i = 1, 2, \ldots, l \tag{1}$$

where

$$z_i \in R^{n_i} \text{ is the state vector of the } i\text{th subsystem}$$

$$f_i : R^{n_1} \times J \to R^{n_1},$$

$$g_i : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_l} \times J \to R^{n_i}$$

Each subsystem, when isolated, is given by

$$z_i = f_i(z_i, t) \tag{2}$$

The function $g_i(z_1, \ldots, z_l, t)$ represents the interconnections of the ith subsystem with the remaining subsystems.

In every subsystem of the form (1), a control law is applied which consists of a local and a global feedback term. Specifically, for the ith subsystem (1), the control input is

$$u_i(t) = u_i^l(t) + u_i^g(t) \tag{3}$$

If the controllers are linear, then they have the form:

$$u_i^l(t) = k_{ii} z_i(t) \tag{4}$$

$$u_i^g(t) = \sum_{j=1, j\neq i}^{l} k_{ij} z_j(t) \tag{5}$$

where $k_{ij} \in R^{m_i \times m_j}$ and $u_i(t) \in R^{m_i}$.

A two-level hierarchical control is obtained by assigning proper interconnections and feedback loops to the large scale system. At the lower level we have l subsystems, S_i, $i = 1, 2, \ldots, l$, which are described in state space by the equations:

$$S_i : \dot{z}_i(t) = f_i(z_i, t) + h_i(u_i, t) + f_{i0}(z_0), \quad i = 1, 2, \ldots, l \tag{6}$$

where

$$h_i : R^{m_i} \times J \rightarrow R^{n_i}$$

$$f_{i0} : R^{n_0} \times J \rightarrow R^{n_i}$$

and $z_0 \in R_{n0}$ is the state vector of the coordinator system in the upper level. The upper level is occupied by the coordinator system S_0 described by the state space model:

$$S_0 : \dot{z}_0(t) = f_0(z_0, t) + \sum_{i=1}^{l} f_{0i}(z_i, t) + h_0(u_0, t) \tag{7}$$

where $u_0(t)$ is the control law of the coordinator and

$$h_0 : R^{m_0} \times J \rightarrow R^{n_0}$$

The structural implications of the two-level model on fault-tolerant control become evident when we consider a linearized version of the system dynamics. Here, we use a structural state model which consists of N independent linear subsystems $S_1, S_2, \ldots, S_N$ interconnected with a control common linear dynamic subsystem S_0. The mathematical model of the time- invariant large scale system, in a decomposed form, is

$$S_i : \dot{z}(t) = A_i z_i(t) + B_i u_i(t) + A_{i0} z_0(t) \tag{8}$$

$$\text{with} \qquad z_i(t_0) = z_{i0}, \quad z_0(t_0) = z_{0_0}$$

$$S_0 : \dot{z}_0(t) = A_0 z_0(t) + B_0 u_0(t) + \sum_{i=1}^{N} A_{0i} z_i(t) \tag{9}$$

where $z_i(t) \in R^{n_i}, z_0(t) \in R^{n_0}$ and $u_i(t) \in R^{r_i}, u_0(t) \in R^{r_0}$ are the state and control vectors for subsystems S_i and S_0, respectively.

The standard overall description of (8),(9) is

$$\dot{z}(t) = Az(t) + Bu(t), \qquad z(t_0) = z_0 \tag{10}$$

$$A = \begin{bmatrix} A_1 & 0 & \dots & A_{10} \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & A_l & A_{l0} \\ A_{01} & \cdots & A_{0l} & A_0 \end{bmatrix} \in R^{n \times n}, \qquad B = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & B_l & 0 \\ 0 & & 0 & B_0 \end{bmatrix} \in R^{n \times r}$$

Since the dynamics matrix (10) consists of Block elements arranged in an Arrow Structure, the system (10) is called a Block Arrow Structure(BAS) decentralized large scale system.

By appropriately defining a quadratic cost functional, a time-invariant BAS decentralized feedback controller may be designed which has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

We are currently investigating critical properties of LSS, from the fault- tolerant control point of view, that include restructurability and reconfigurability. Specifically, we have identified the following key properties for restructurable/reconfigurable systems:

- Flexibility: based upon the strength of subsystem interconnections (interlevel or intralevel).
- Stabilizability: in terms of structural controllability and structural observability concepts.

Our investigations thus far indicate that the modeling approach pursued is representative of a large class of engineered systems, that, because of critical performance requirements, require some degree of fault tolerance in the control design.

2 Fault Detection and Identification

A major effort has been undertaken, during this initial phase of the project, to develop an integrated methodology to fault detection and identification that capitalizes upon a combination of conventional techniques and artificial intelligence. New and innovative concepts are introduced in the FDI approach that maximize symptomatic evidence, account for sensor and system uncertainties and combine failure data even when the latter are of a conflicting nature.

FDI techniques are considered for sensor, actuator and component failures. Both single and multiple faults are treated. The research objective is to develop a systematic and thorough FDI procedure that is maximally sensitive to failures while avoiding false alarms. The approach is systematic because it relies on a modular architecture to (a) trigger efficiently the FDI routines, (b) validate sensor data, (c) combine failure evidence from such diverce sources as analytic redundancy, dectection/estimation theory and limit checking, (d) utilize expert system tools and Dempster-Shafer evidential theory to manage uncertainty and assess the symptomatic evidence, i.e. detect and identify faulty components, and (e) finally, provide trending information as well as the best value of critical variables and parameters for monitoring and control purposes.

This research team has previously developed FDI procedures based upon parity space by utilizing analytic redundancy. We will highlight below only some recent developments in detection/estimation and evidential theory. Symptoms derived from this technique will augment the failure evidence available from the parity space procedure to assure a robust FDI approach.

2.1 New Directions in FDI

Decision strategies for FDI based on a Bayesian approach were reviewed. Bayes' estimation theory using minimum mean squared estimation and joint detection/estimation(JDE) procedures as well as test methods such as the M-hypothesis, generalized likelihood and probability ratio tests were critically assessed as to their effectiveness in FDI and their computational cost and complexity. Trade-offs were considered and robustness issues were addressed. Such Bayesian methods as the multiple model approach were examined in detail using a general linear discrete-time multiple dynamic model. Here, failure modes are modeled as switching parameters and it is assumed that the number of failure models is finite and compact. The ith model of the linear process is represented by

$$x_i(t+1) = A_i(q,\theta_i)x_i(t) + B_i(q,\theta_i)u(t) + w_i(t) \tag{11}$$

$$y(t) = H_i(q,\theta_i)x_i(t) + v_i(t) \tag{12}$$

where $u(t), y(t)$ are the actuator input and the measured output, respectively; $x_i(t)$ is the state of the ith model; A_i, B_i, and H_i are the associated system matrices; θ_i is the parameter vector of the process which may be time-varying; q is the shift operator defined by $q^{-1}x(t+1) = x(t)$; $w_i(t) \propto N[0, Q_i]$ is the white Gaussian process noise; $v_i(t) \propto N[0, R_i]$ is the white Gaussian measurement noise.

The minimum mean squared estimate(MMSE) can be represented by

$$\hat{x}^{MS}(t|t) = E(x(t)|Y(t)] = \arg\min_{x(t)} E(x(t) - \hat{x}(t|t))(x(t) - \hat{x}(t|t))^T$$

$$\text{where } Y(t) = [y(t), y(t-1), \ldots, y(t-n)]^T$$

The ith Kalman filter equations are easily derived from these assumptions. The computational shortcomings of both optimal and suboptimal procedures following these Bayesian methods are well documented in the techniical literature. The statistical ratio test paradigms may be circumvented by using a Markov chain with an associated transition probability matrix. Consider a joint detection/estimation algorithm based on this approach. Each state of the model may be described by an integer index $s(t)$. We define a windowed Markov chain state sequence $I_i(t)$ by

$$I_i(t) = \{s(t) = i, s(t-1), \cdots, s(t-n+1)\} \tag{13}$$

where the window size is n, $s(t) \in \{1, 2, \cdots, M\}$, and M is the total number of stochastic models. $s(t)$ determines or identifies the structure of the failure model at time t.

Suppose that, given a stochastic process $s(t)$ with a finite number of possible integers, the associated Markov chain exists such that

$$p(s(t)|s(t-1), \cdots, s(0)) = p(s(t)|s(t-1)) = \pi(s(t), s(t-1)) \tag{14}$$

$$p(t) = \Pi p(t-1) \tag{15}$$

where $p(\cdot) \in R^M$ stands for probability of the state sequence, $\Pi = \{\pi(i,j)\} \in R^{M \times M}$ for all states $\{s(0), \cdots, s(t)\}$ and for all $t \geq 0$. $\pi(i,j)$ is referred to as the transition probality from the model state j to the next state of the model i. Now, we can consider state estimation and structural detection for this partitioned systems.

Given the probability density function $f(\cdot)$ of the state $x_i(t)$, the estimate of the ith model is

$$\hat{x}(t|t) = E[x_i(t)|Y(t), I_i(t)] \tag{16}$$

and the detection/estimation approach can be summarized as follows:

- **state estimation:**

$$\hat{x}(t|t) = \sum_{i=1}^{M} \hat{x}_i(t|t)p(I_i(t)|Y(t)) \tag{17}$$

$$\text{where } p(I_i(t)|Y(t)) = \frac{f(Y(t)|I_i(t))p(I_i(t))}{\sum_{j=1}^{M} f(Y(t)|I_j(t))p(I_j(t))} \tag{18}$$

- **structural detection**

$$p(i|Y(t)) = \sum_{\forall I_i(t)} p(I_i(t)|Y(t)) \tag{19}$$

where $f(\cdot|\cdot)$ is a conditional pdf of $Y(t)$ given the sequence $I_i(t)$.

It is noted that this state estimation approach requires a bank of Kalman filters for the fixed window and the number of states must be the same in each failure model. These problems may be alleviated by employing an alternate approach which is based on multiple parametric models. The proposed approach relies on the error equation of auto-regressive moving average (ARMA) models. The ith model of the system is described by

$$A_i(q,\theta) = q^{-d}B_i(q,\theta)u(t) \tag{20}$$

where $A_i(q,\theta)$ is a monic polynomial, $B_i(q,\theta)$ is a stable polynomial, and d is the system delay. If the process is considered to be linear, then from the Kalman filter interpretation for the time-varying system, the parameter estimate $\hat{\theta}_i(t)$ can be calculated as follows:

$$\hat{\theta}_i(t) = E[\theta_i(t)|Y(t)] = \hat{\theta}_i(t-1) + K_i(t)\{y(t) - \phi_i(t)^T\hat{\theta}_i(t-1)\} \tag{21}$$

where $\hat{\theta}_i(t) = [a_1, \cdots, a_n; b_1, \cdots, b_m]^T$, $\phi_i(t) = [-y(t-1), \cdots, -y(t-n); u(t-1), \cdots, u(t-m)]^T$, and $K_i(t), P_i(t)$ are the familiar Kalman gain and covariance matrices, respectively.

Now, parameter estimation and structural detection may be accomplished by using the following computations:

- **parameter estimation:**

$$\hat{\theta}(t) = \sum_{i=1}^{M} \hat{\theta}_i(t)p(I_i(t)|Y(t)) \tag{22}$$

$$\text{where } p(I_i(t)|Y(t)) = \frac{f(Y(t)|I_i(t))p(I_i(t))}{\sum_{j=1}^{M} f(Y(t)|I_j(t))p(I_j(t))} \tag{23}$$

- **structural detection**

$$p(i|Y(t)) = \sum_{\forall I_i(t)} p(I_i(t)|Y(t)) \tag{24}$$

The generalized likelihood ratio (GLR) test is shown to be appropriate here to execute the detection and estimation strategy.

A new strategy to the detection/estimation problem is based on possibilistic methods and is implemented using expert system tools. A new failure detection scheme is obtained by utilizing Zadeh's fuzzy set theory in constructing a decision-making rule base. Two possible approaches are considered: The first one involves the design of membership functions for the a posteriori probability or likelihood ratio of each failure model followed by a determination of the fuzzy relational mapping for the rules. The second refers to the collection of all measures of the failure modes in evidence, which may be functions of standardized normal variables, to evaluate statistical ratio tests for them, and finally, to fire those rules that satisfy a particular failure mode.

For, the fuzzy failure detection scheme, some appropriate measure such as the a posteriori probability or likelihood of each failure model is used. To illustrate the point, the fuzzy rulebase consists of rules of the form:

- If 'p1 is large' and 'p2 is small', then 'prob model is 1' or
- If 'r1 is large' and 'r2 is small', then 'likely model is 1'

where p1 and p2, r1 and r2 are the a posteriori probabilities and likelihood ratios for models 1 and 2, respectively. The goal here is to find the fuzzy relational mapping or fuzzy relational matrix for failure detection. The advantages of this methodology are a significant reduction in decision errors by using linguistic variables and a considerable saving in computational time through parallel (disjunctive) rule implementation.

The coupling of the new FDI scheme to the multiple parametric model approach is achieved as follows: For each kth component of the parameter vector θ_i, the parametric space is divided into several crisp cells that represent some specific symptoms or failure evidence in the associated parameter. These cells can be chosen according to probabilistic specifications such as mean, variance, etc. Next, under real time parameter estimation conditions, these values for each model in a particular cell can be used to determine the failure mode at hand and infer structural detection through the output of the fuzzy rulebase.

Failure detection may also be accomplished using production rules. In this case, the decision depends on a description of crisp failure situations under the assumption that all possible failure modes are completely specified.

A typical rule in the production rulebase for failure detection of impulse lines is

- If 'reading is full-scale', then 'line is blocked'.

This last methodology holds a great deal of promise when all possible failure modes are considered.

Presently, simple dynamical systems are modeled and both Bayesian and possibilistic approaches are simulated with the intent of arriving at the optimum detection/estimation algorithm that satisfies the performance criteria and yet is computationally efficient.

The final thrust of our current research activity is concerned with the study of the Dampster-Schafer mathematical theory of evidence and its possible implications to the FDI problem. Dempster-Schafer theory provides an excellent tool for combining several pieces of evidence from a knowledge source. We intend to use this tool in order to combine failure symptomatic evidence resulting from the application of parity space, possibilistic detection/estimation and limit checking techniques. The power of D-S theory is in the representation of subjective(as opposed to probabilistic) uncertainty, mainly by virtue of its expression of ignorance. The groundwork for applying D-S theory to the FDI problem has been laid and we expect to report on this task in the near future.

In summary, substantial progress has been made in conceptualizing and developing new and innovative techniques to address the FDI and modeling problems of dynamical large scale systems that undergo component or sensor failures. The distinguishing charasteristic of our approach is a blend of analytical and symbolic representations. Computational tools are used to develop, simulate and demonstrate the power and effectiveness of these techniques. For the remainder of this fiscal year, we will continue to pursue this research direction so that we may produce a viable and generic package of FDI routines that can be applied to any complex physical system.

A HYBRID ANALYTICAL/INTELLIGENT
APPROACH TO FAULT-TOLERANT
CONTROL SYSTEM DESIGN
CONTRACT NO: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

Presentations and Technical Contributions

1. In September 1989, a presentation was given to the staff of NASA's MSFC at Huntsville, Alabama by Drs. Vachtsevanos and Arkin on possible applications of fault-tolerant control technology to the space station program.
2. On 11/9/89, Dr. G. Vachtsevanos was invited to present a seminar to the technical staff of Rockwell International Corp., the Science Center at Thousand Oaks, California on fault-tolerant control techniques. He held technical discussions with Rockwell personnel on issues of fuzzy logic and the application of fault-tolerant control techniques to aerospace systems.
3. In November 1989, discussions were held and a statement on fault- tolerant control was mailed to Nothrop Corp.
4. Interest on our fault-tolerant control activities has been expressed by the Federal Aviation Administration. A position paper on this subject was mailed to FAA research staff.
5. In June 1989, Dr. Vachtsevanos chaired an invited session on Intelligent Control at the 1989 Automatic Control Conference and presented a paper entitled "Fault Tolerant Control of Dynamical Large Scale Systems". A copy of the paper is attatched to this report.
6. In December 1989, Dr. Vachtsevanos will present a paper, co-authored by Dr. H. Kang, on "Model Reference Fuzzy Control" at the 28th IEEE Conference on Decision and Control, Tampa, Florida.
7. A paper entitled "Stability and Robustness of a Nonlinear Fuzzy Controller Based on the Phase Portrait Assignment Algorithm", co-authored with H. Kang, has been submitted for publication to the IEEE Trans. on Systems, Man and Cybernetics.

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Project Participants

The parincipal Investigator of this project is

Dr. George Vachtsevanos, Professor of Electrical Engineering at the Georgia Institute of Technology, Atlanta, Georgia.

Dr. Ronald Arkin, Assistant Professor at the School of Information and Computer Science, the Georgia Institute of Technology, serves as a co- investigator to the project.

Dr. Hoon Kang, a recent Ph.D. graduate at the Georgia Institute of Technology, is participating in project activities as a post-doctoral fellow.

Three graduate students in the School of Electrical Engineering at Georgia Tech are currently receiving partial support from this project.

FAULT-TOLERANT CONTROL OF DYNAMICAL LARGE SCALE SYSTEMS

George Vachtsevanos & Young-Tae Kim
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250

Manolis Christodoulou
Technical University of Crete
Chania, Greece

ABSTRACT

A fault-tolerant design procedure for large scale engineered systems is introduced. The proposed methodology is based on a hybrid approach that capitalizes upon beneficial attributes of conventional systems and control-theoretic techniques, as well as methods of artificial intelligence. The paper focuses on a fault propagation algorithm that assesses quickly the impact of a failure on other healthy components and subsystems. Qualitative reasoning techniques are employed to determine the state transitions. An example from a major subsystem of the space station is used to illustrate the algorithmic developments.

1. INTRODUCTION

Modern dynamical systems, such as aircraft and space vehicles, nuclear power plants, and many industrial and manufacturing processes, are complex systems that place a heavy burden on performance monitoring, status evaluation, and control strategies. As the technological sophistication of these systems increases, they are required to be more fault tolerant so that the system availability is kept high at the maximum possible rate.

The problem of designing fault-tolerant control systems has been addressed in the recent past in terms of two general and fundamental approaches [1-5]. The first one relies upon modern control-theoretic techniques to recast the fault tolerant system design into a classical control problem, while the second is based on decision-theoretic concepts related to the field of artificial intelligence in order to provide partial answers to the problem. A critical review of both AI-based and control-theoretic techniques for fault-tolerant control reveals that there is no unified methodology to integrate those diverse issues of system modeling, fault detection and isolation, fault propagation, system restructuring, and controller reconfiguration.

This need is addressed by introducing a problem-focused system integration philosophy that capitalizes upon a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. AI techniques allow a marked reduction of the set of hypotheses, thus permitting traditional control-theoretic approaches to be applied efficiently.

When one or more failures or abrupt parameter changes are detected, the system enters an emergency mode. The proposed fault-tolerant control system design procedure takes over in that event and entails the following tasks (Figure 1):

- Fault Detection and Identification (FDI)
- Fault Propagation (FP)
- System Restructuring
- Controller Reconfiguration.

A methodology for fault diagnosis of large scale dynamical systems is introduced which is based upon a combination of signal redundancy techniques and fuzzy logic.

Suppose that a fault or parameter deviation occurs in some subsystem of a large scale process. Then the deviation propagates to adjacent subsystems before the faulty component is isolated. The fault might propagate through an adjacent subsystem without causing any further fault, it might be absorbed in this subsystem (depending on its transition pattern), or it might cause this subsystem to fail. When a faulty behavior or a marked deviation in the system's behavior is observed, then it will be necessary to track the current behavior pattern of the system and its major subsystems and take fault-tolerant action before a permanent failure occurs and cripples the system. This paper addresses this issue by recognizing the unconventional nature of the problem. For a fault propagation model to be effective, it must be capable of predicting the impact of a failure on other system components well in advance before the actual effect is realized. Only then may the system have opportunity to restructure itself by isolating failed or potential faulty subsystems so that overall system survivability is assured.

2. FAULT PROPAGATION

A fault propagation model is defined here to be a representation of the propagation of input system variable deviations that originate at the faulty unit to other system components. A fault is initiated in a unit which by definition is unhealthy, propagates through a set of units which are generally healthy (although in some units, an additional enabling fault is a condition for further propagation), and terminates in a unit or units which are thereby rendered unhealthy.

Fault Tree Analysis (FTA) [6] has been classically used to discover the paths to failure in complex systems.

In a different approach [7], it is possible to model the subsystems of the large scale system as nonvariant stochastic automata with a transition matrix between the input and output states.

The proposed approach to modeling of fault propagation begins with a system structural diagram and decomposes it into its constituent units or components; it then represents the system as a causal network via a qualitative simulation.

Qualitative Reasoning (QR) has been recently an area of intense research activity within the artificial intelligence and cognitive sciences [8-11]. Investigators in these two fields have studied qualitative causal reasoning models of physical systems or mechanisms.

Considering the nature and complexity of large scale dynamical systems, Kuipers' approach to qualitative simulation seems to be the most appropriate. We propose to augment his original approach by including explicitly constraints that represent conservation of energy via scalar Lyapunov functions. This addition eliminates most spurious behaviors and it is hoped that will lead to predicting the single correct behavior of the faulted system.

2.1 QUALITATIVE SIMULATION

The goal of qualitative physics is to provide a theoretical framework for understanding the behavior of physical systems.

From the qualitative structure of the physical system, Qualitative Simulation (QSIM) uses a local propagation strategy to predict the qualitative behavior of a mechanism. The qualitative structure of a mechanism is described by a set of qualitative constraints. Each constraint specifies a relationship that holds between two or more functions of the mechanism. Each function has a qualitative state consisting of a qualitative value and direction of change. A qualitative value can be a landmark value such as 0 or it can be an open interval between two landmark values. At first, QSIM determines a complete initial state by propagating known qualitative values and directions of change (given the initial condition) through the given constraints before proceeding to predict possible behaviors. With a complete set at a distinguished time point, QSIM uses a strategy of proposing and filtering sets of qualitative transitions to predict all possible subsequent states in the following open time interval. A number of real valued parameters, which vary continuously over time, characterize a physical system. Each parameter can be considered as a function $f:[a,b] \to R^*$, where $R^* = [-\infty,\infty]$, the extended real number line. The qualitative behavior of f on [a,b] is the sequence of qualitative states of f:

$$QS(f,t_0), QS(f,t_0,t_1), QS(f,t_1), QS(f,t_1,t_2), \ldots, QS(f,t_{n-1},t_n), QS(f,t_n)$$

Constraints holding between parameters in the structural description of the system serve to limit the possible combinations of qualitative behavior. Constraints are expressed as predicates rather than as functions for two reasons: First, they will be used as predicates in the QSIM algorithm to test the consistency of sets of qualitative values. Second, if a constraint were to be treated as a function, it is unclear how to define precisely the function's range. At large there are two types of constraints which are used in QSIM:

- Arithmetic constraints
- Qualitative function constraints.

An example of an arithmetic constraint is:

- ADD(f,g,h): iff $f(t) + g(t) = h(t)$ for every $t \in [a,b]$.

Qualitative function constraints are defined as follows:

- M^+ is a two-place predicate on reasonable functions $f,g:[a,b] \to R^*$.

$M^+(f,g)$ is true, iff $f(t) = H(g(t))$ for all $t \in [a,b]$, where H is a function with domain $g([a,b])$ and range $f([a,b])$, differentiable with $dH(x)/dt > 0$ for all x in the interior of the domain. Similarly, M^- can be defined except that $dH(x)/dt < 0$.

We propose additional constraints related to energy functions. A scalar Lyapunov function, associated with a subsystem of the engineered large scale system, is viewed as an energy function.

Let V(x) be a candidate Lyapunov function with $\dot{V}$ its time derivative. Qualitative constraints may be imposed by asserting the following conditions:

(1) If $\dot{V} < 0$, the system tends to be stable and thus its energy decreases.

(2) If $\dot{V} = 0$, the system is marginally stable and its energy remains constant.

(3) If $\dot{V} > 0$, the system may be unstable, thus its energy may be increasing.

2.2 FROM STATE SPACE TO QUALITATIVE SPACE

A large scale engineered system is considered to be compared of a number of linearly interconnected linear or nonlinear subsystems. A control law is applied to each subsystem which consists of a local and a global feedback term. Let us focus attention first to a typical subsystem without regard to interconnections. We will define some useful concepts for linear systems only although the extension to the nonlinear case is straightforward except for the number and the nature of the qualitative constraints.

Consider the state space model of a controllable system described as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad (1)$$

where x is an n-dimensional state vector and A,B,C are matrices with appropriate dimensions.

With the controllability assumption, the system equations may be written in the controller canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u \qquad (2)$$

Or, more explicitly as:

$$\begin{aligned} x_1 &= x \\ x_2 &= \dot{x}_1 \\ &\vdots \\ x_n &= \dot{x}_{n-1} \\ \dot{x}_n &= -a_1 x_n - a_2 x_{n-1} - \cdots - a_n x_1 + u \end{aligned} \qquad (3)$$

Equations (3) define n+1 qualitative variables and their derivatives. We seek the qualitative behavior of these variables starting from some given initial values. The qualitative value of a variable is either a symbolic or numeric value and its direction of change, i.e. increasing, descreasing, or steady.

A set of constraint equations defined on the basis of constituent component models and their interconnections (the state equations), as well as functional and global (energy) relations, specify the transition from state space to qualitative space. The qualitative operations do not define a mathematical field. Only a few qualitative solutions or combinations are consistent with the equations. The set of qualitative constraints is intended to limit the behavioral modes or interpretations of the system to the correct ones.

From the above relations, an appropriate set of qualitative constraints may be derived as:

$\mathrm{DERIV}(x_1,x_2), \mathrm{DERIV}(x_2,x_3), \ldots, \mathrm{DERIV}(x_{n-1},x_n)$.

The last equation of the set is expressed qualitatively as:

$$\mathrm{DERIV}(x_n, X)$$

where X is composed of addition constaints as depicted in Figure 2a. A total of n-derivative, n-multiplicative, and n-additive constraints are required to convert from the state space model to the qualitative model, as shown in Figure 2b.

The tree of behavioral modes can be pruned by the consideration of functional and energy-type constraints. A transformation of the state model to a Schwartz matrix form provides the most appropriate setting for the scalar Lyapunov function as a qualitative constraint criterion.

Similar qualitative state transition diagrams can be derived for multivariable systems.

2.3 FAULT PROPAGATION TO OTHER MAJOR SUBSYSTEMS OF THE LSS

When a system undergoes structural perturbations (faults), the interactions between constituent subsystems are expected to have major effects on stability. In assessing the impact of propagating faults on other (healthy) subsystems, we will use the following assertion: If the subsystems are stable to a degree larger than the strength of the interconnection, then the composite system is stable.

Consider a large scale unforced system described by:

$$\dot{x} = f(x,t) \qquad (4)$$

which is assumed to consist of N-subsystems:

$$\dot{x}_i = f_i(x_i,t) + g_i(x,t) \quad , \quad i = 1,\ldots,N \qquad (5)$$

and assume that the origin e = 0 is an equilibrium state.

When the i-th subsystem is completely decoupled, then:

$$\dot{x}_i = f_i(x_i,t) \qquad (6)$$

i.e. the i-th subsystem is totally isolated from the rest of the system.

Assume a Lyapunov function $v_i(x_i,t)$ for the isolated subsystems (6). Use a weighted sum:

$$r(x,t) = \sum_{i=1}^{N} c_i v_i(x_i,t) \qquad (7)$$

as a potential Lyapunov function for the large scale system. The c_i's are positive constants.

In stability studies of large scale systems, we define a positive definite function $w_i(x_i)$ and we check for:

$$\dot{v}_i(x_i,t) \lessgtr -a_i\{w_i(x_i)\}^2 \qquad (8)$$

The constant a_i is considered as the degree of stability in view of the fact that it gives a lower bound for the decrease in $\dot{v}$ with respect to $w_i^2(x_i)$. Next we check the interconnection terms:

$$|g_i(x,t)| \lessgtr \sum_{j=1}^{N} b_{ij} w_j(x_j) \qquad (9)$$

where the nonnegative constants b_{ij} indicate the interconnection strength in the sense that they provide an upper bound with respect to $w_j^2(x_j)$. Therefore, a qualitative measure of the impact of a faulty j-th subsystem on the behavior of the i-th subsystem may be obtained by comparing the interconnection strength:

$$|g_i(x,t)| \text{ to } \dot{v}_i(x_i,t) \ .$$

In terms of the constants a_i and b_{ij}, we may form the N x N matrix $E = (e_{ij})$, where $e_{ii} = a_i - b_{ij}$, $e_{ij} = -b_{ij}$, $i \neq j$, and compute the leading principle minors of E; a check on their sign and relative magnitude will determine again the fault propagation impact. The evolution of the time derivatives of the Lyapunov functions and their relative values compared to the

interconnections strength over the first distinguished time points after the occurrence of a fault will provide the information required to eliminate spurious behaviors.

3. AN EXAMPLE

The proposed qualitative simulation approach is demonstrated, in outline form, with an example. The test system is the Thermal Control System of the space station's common module. Jointly with the Boeing Aerospace Company, a hierarchical control algorithm and fault diagnostic routines for this major large scale system of the space station have been developed [12]. The algorithms have been implemented and successfully tested on a subscale laboratory prototype constructed by Boeing. In our example, a fault condition is hypothesized and propagated through the system's major components. If certain threshold or tolerance limits are exceeded, then the affected subsystems are isolated, before the hierarchical controller is reconfigured, in order to preserve the system integrity. A simplified schematic of the Thermal Control System is shown in Figure 3.

The qualitative simulation begins with a structural description which consists of a set of constraints holding among time varying, real valued parameters. The behavioral description consists of a finite set of time points representing the qualitative distinct states of the system and values for each parameter at each time point. This description consists of the ordinal relations (i.e. >, <, and =) holding among the different values in the behavioral description. Certain values are thermal distinguished or landmark values and play a special role in the qualitative simulation. Beginning with a set of assertions about the initial state of the system, the process takes place through the propagation/prediction cycle.

Algebraic relations of the system may, therefore, be represented as qualitative constraints. Let us demonstrate the procedure by considering the example of component HX-2 (Figure 4a). The algebraic equations for HX-2 are:

$$\begin{aligned} Q_2 &= \dot{m}_2 C_v \Delta T_2 \\ \Delta T_2 &= T_{o2} - T_{i2} \\ \dot{m}_2 &= (1-\beta)\,\dot{m}_0 \\ T_{i3} &= T_{o2} - (1-\beta)\,\Delta T_2 \end{aligned} \tag{10}$$

where β denotes the bypass control valve variable (i.e. $\beta = 0$ means closed valve and $\beta = 1$ means fully opened valve). And constraints for HX-2 may be written as:

$$(C1)\,\Delta T_2 < 5°F \quad , \quad (C2)\,T_{i2} = (70 \pm 2.5)°F \tag{11}$$

We assume that C_v is constant. Qualitative constraints relations for HX-2 are as follows:

$$\begin{aligned} &(QC1)\,M^-(\beta,\dot{m}_2) \quad , \quad (QC2)\,MULT(\dot{m}_2,\Delta T_2,Q_2) \\ &(QC3)\,MULT(1-\beta,\Delta T_2,T) \quad , \quad (QC4)\,ADD(T_{i2},T,T_{i3}) \end{aligned} \tag{12}$$

A qualitative constraints model for HX-2 is shown in Figure 4b. We can begin the process with active states at the distinguished time $t = (t_0,t_1)$. We assume that the qualitative states of each variable at time t are as follows:

$$QS(\beta,t_0,t_1) = [(0,1),dec] \tag{q1}$$
$$QS(\dot{m}_2,t_0,t_1) = [0,\dot{m}_0),inc] \tag{q2}$$
$$QS(Q_2,t_0,t_1) = [(0,\overset{o}{Q}_2),inc] \tag{q3}$$
$$QS(\Delta T_2,t_0,t_1) = [(0,5),inc] \tag{q4}$$
$$QS(T,t_0,t_1) = [(a,b),inc] \tag{q5}$$
$$QS(T_{i2},t_0,t_1) = [67.5,72.5),std] \tag{q6}$$

At the distinguished time $t = (t_1,t_2)$, these states undergo the following transitions:

$$QS(\beta,t_1,t_2) = \begin{bmatrix}(0,\ 1),dec\\(0,\ 1),inc\\(0,\ 1),std\end{bmatrix} \quad \begin{matrix}(q1\text{-}1)\\(q1\text{-}2)\\(q1\text{-}3)\end{matrix}$$

$$QS(\dot{m}_2,t_1,t_2) = \begin{matrix}[(0,\dot{m}_0),dec]\\ [(0,\dot{m}_0),inc]\\ [(0,\dot{m}_0),std]\end{matrix} \quad \begin{matrix}(q2\text{-}1)\\(q2\text{-}2)\\(q2\text{-}3)\end{matrix}$$

$$QS(Q_2,t_1,t_2) = \begin{matrix}[(0,\overset{o}{Q}_2),dec]\\ [(0,\overset{o}{Q}_2),ind]\\ [(0,\overset{o}{Q}_2),std]\end{matrix} \quad \begin{matrix}(q3\text{-}1)\\(q3\text{-}2)\\(q3\text{-}3)\end{matrix}$$

$$QS(\Delta T_2,t_1,t_2) = \begin{bmatrix}(0,\ 5),dec\\(0,\ 5),inc\\(0,\ 5),std\end{bmatrix} \quad \begin{matrix}(q4\text{-}1)\\(q4\text{-}2)\\(q4\text{-}3)\end{matrix}$$

The assumed initial qualitative state at time (t_0,t_1) has three possible transitions at time (t_1,t_2). Let us further assume that a failure has occurred at t_1 and the faculty component was identified to be a valve stuck in one position. A stuck valve means that β = const(std) (i.e. (q1) can undergo a transition to only (q1-3) at time (t_1,t_2)). Let us investigate next how this fault propagates through the system by checking the qualitative constraints under possible state transitions. Here we will check only for (QC1) and (QC2) to show how the search is conducted and the effect of the failure event on other components is determined.

$$(QC1)\,M^-(\beta,\dot{m}_2) \ ,$$

from the assumption that β = const

(q1-3)(q2-1) : not consistent
(q1-3)(q2-2) : not consistent
(q1-3)(q2-3) : consistant

$$(QC2)\,MULT(\dot{m}_2,\Delta T_2,Q_2) \ ,$$

by checking (QC1) we get (q2-3) for $QS(\dot{m}_2,t_1,t_2)$.

Among nine possible transitions, we determine readily that there are three consistent ones as: (q2-3)(q4-1)(q3-1),

(q2-3)(q4-2)(q3-2), or (q2-3)(q4-3)(q3-3). Therefore, failure of the valve causes a variation of the temperature difference dependent on the absorbed heat. Similarly, we can check the other qualitative constraints (QC3,QC4) related to this particular system. Energy function related constraints will also be used to narrow the behavioral interpretations and optimize the search algorithm.

The fault propagation model thus provides the information sought as to the impact of this fault condition on other system components.

4. CONCLUSIONS

The proposed fault propagation procedure proceeds as follows:

(1) Construct the qualitative model of the physical system.

(2) Estimate the initial qualitative state of the faulty system.

(3) Using qualitative simulation, find all possible behaviors.

(4) Construct the constraints related to the total energy function of the system.

(5) Using the constraints in (4), eliminate spurious behaviors.

(6) Determine the genuine behavior caused by the failure.

The key difficulty in employing such a qualitative technique to assess the impact of fault propagation is due to the generation of behavioral modes that are not consistent with the physical behavior of the system. Functional and energy related constraints that augment arithmetic qualitative constraints seem to effectively reduce spurious behaviors, thus leading to the correct interpretation.

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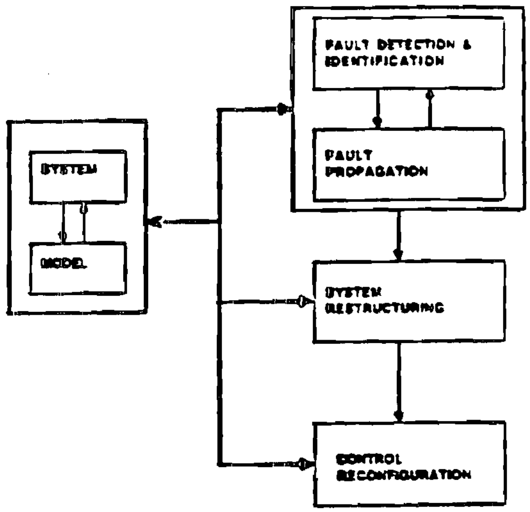

Figure 3. Block Diagram of Proposed Fault Tolerant Control Philosophy

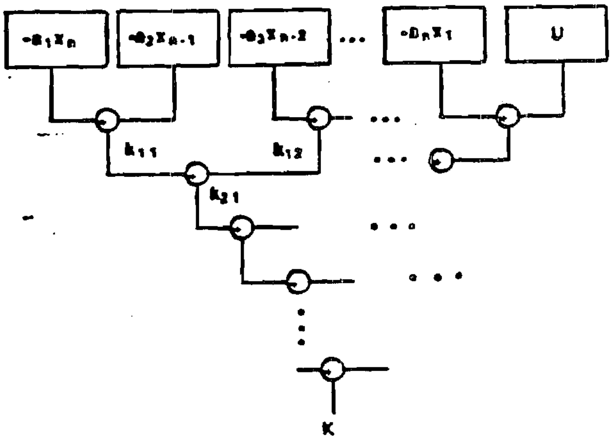

Figure 2a. Qualitative Constraints from $\dot{x}_n = -a_1x_n - a_2x_{n-1} - \dots$

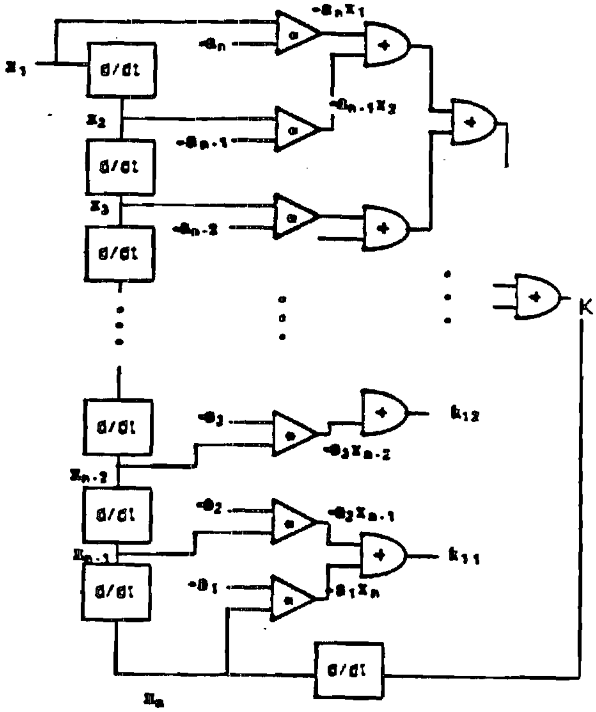

Figure 2b. Block Diagram of Qualitative Constraints Model

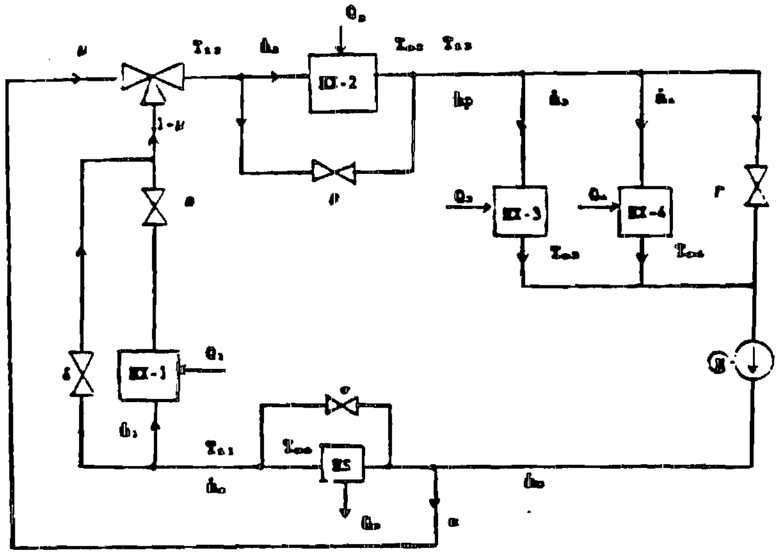

Figure 3. A Schematic of the Thermal Control System

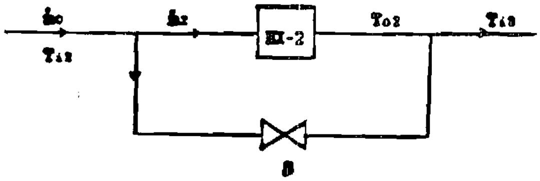

Figure 4a. Block Diagram of Heat Exchanger 2

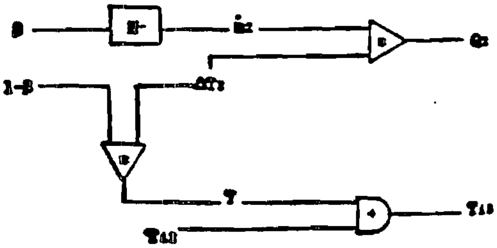

Figure 4b. Qualitative Constraints Model of HX-2

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Stability and Robustness of A Nonlinear Fuzzy Controller Based on The Phase Portrait Assignment Algorithm

by

Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering

Georgia Institute of Technology

Atlanta, Georgia 30332

(404)-894-6252 (404)-894-3834

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Abstract

This paper presents a new design technique and the associated stability analysis of fuzzy logic controllers for a class of nonlinear systems. The design approach is called 'the phase portrait assignment algorithm' (P^2A^2), and it establishes a stabilizing rulebase by utilizing fuzzy measures as introduced by Zadeh. This method can substitute for classical control paradigms when the latter suffer from dynamic, parametric, and stochastic uncertainties. The P^2A^2 technique not only provides a bridge between qualitative reasoning and asymptotic stability of nonlinear feedback control systems, but also gives an insight into the dynamical system behavior. A single-link robot arm is used to illustrated the performance and advantages of the proposed fuzzy control mechanism.

Keywords: *fuzzy-logic controller, nonlinear* feedback system, stability analysis

1 Introduction

The introduction of fuzzy sets and possibility measures by Zadeh [1,2] made it possible to cope with the approximate estimation and control problem under large-grain uncertainty. A fuzzy set is defined as a class of objects with a continuum of grades of membership using certainty or confidence factors in order to handle uncertainties or complexity in a particular domain of knowledge [3] and a fuzzy number can be described by linguistic terms such as 'large', 'medium', 'small', 'very small', and so on, whose fuzziness provides many degrees of freedom in dealing with uncertainty as a non-uniform possibility distribution [3]. Furthermore, a fuzzy confidence interval represents information that reduces (expands) the uncertainty by using lower and upper bounds as the presumption level or the α-cut increases (decreases). The characteristic functions of linguistic variables represent the associated crisp subsets by using uniform distributions of the feasible subset (possibility = 1) and the non-feasible subset (possibility = 0). Referring to the characteristic functions, membership functions are determined by using non-uniform possibility distributions.

Since Zadeh's introduction of fuzzy mapping and control [4], fuzzy concepts can be applied to the control of dynamic systems whereas stability of a fuzzy feedback control system has been an important issue. Mamdani [5] and Tong[6] suggested means for the application of fuzzy systems to control engineering. Kickert et al.[7] approximated a nonlinear fuzzy-logic controller with a multi-level relay in a simple case which was analyzed by the describing function method since an oscillation was anticipated. Tong [8] showed some results of asymptotic properties of fuzzy feedback systems. Mamdani et al.[9] introduced a self-organizing control system of a fuzzy rule modifier by using a performance table. The concept of L_2-stability was examined in [10] by using Nyquist criteria of a compensated system and a fuzzy-logic controller. Kania et al.[11] introduced the concept of energistic stability for fuzzy dynamic systems. Chen et al.[12] applied Hsu's cell-to-cell mapping theory [13] to the global behavior of the cell state space implicitly by grouping the cells. Finally, Kang et al.[14] emphasized evidential aspects of the nonlinear system dynamics and the transformation from crisp relations to fuzzy relations by using the vector field of the phase plane approach.

The objective of this paper is to provide a general method for constructing rule sets for fuzzy feedback controllers so that global asymptotic stability may be guaranteed with respect to a class of nonlinear systems. This is based upon 'the principle of increasing precision with decreasing intelligence' [15] where the basic idea is to relate evidence of the asymptotic system behavior to crisp subsets by using vector fields, explicitly, and then, to fuzzify each crisp subset (non-fuzzy cell). Periodic behavior of the closed-loop system may be avoided by applying the proposed algorithm. To preview the organization of the paper, Section 2 deals with the design procedure and stability analysis of the proposed scheme whereas convergence and robustness are also considered. A single-link robot arm is used to show the performance of the fuzzy regulator in Section 3 of this paper. Finally, Section 4 contains conclusions and a discussion

of the contributions.

2 Nonlinear Fuzzy Control

A design procedure for a fuzzy expert control mechanism, 'the phase portrait assignment algorithm' (P^2A^2), is proposed for a closed-loop feedback system consisting of a multi-input nonlinear process and a fuzzy rulebase controller as shown in Figure 1. The compositional rule of inference and the modified mean-of-maxima defuzzifier are used for deciding the control feedback input.

2.1 Nonlinear Fuzzy-Logic Feedback Control Systems

Let a nonlinear system be described by a vector differential equation of the form [16]

$$\dot{x}(t) = f(x(t), u(t), t) \qquad t \geq t_0 \tag{1}$$

where $x(t) \in \mathcal{R}^n$ represents the state vector with $x_0 = x(t_0)$; $u(t) \in \mathcal{R}^m$ is the control input vector; and $f(\cdot,\cdot,\cdot)$ denotes a nonlinear function. In brief,

$$\begin{aligned} x(\cdot) &: \mathcal{R}_+ \longrightarrow \mathcal{R}^n \\ u(\cdot) &: \mathcal{R}_+ \longrightarrow \mathcal{R}^m \\ f(\cdot,\cdot,\cdot) &: \mathcal{R}^n \times \mathcal{R}^m \times \mathcal{R}_+ \longrightarrow \mathcal{R}^n \end{aligned} \tag{2}$$

where $\mathcal{R}_+$ is a subset of $\mathcal{R}^1$ such that $\mathcal{R}_+ = [t_0, \infty)$. A general method of constructing a fuzzy rule set is necessary so that the rulebase updates the control input $u(t)$ to ensure global asymptotic stability with respect to a given nonlinear system. The rulebase can be represented as follows:

- rule 1: if (x_1 is $L^1_{x_1}$) and $\cdots$ and (x_n is $L^1_{x_n}$) then (u_1 is $L^1_{u_1}$) and $\cdots$ and (u_m is $L^1_{u_m}$).

$$\vdots \qquad\qquad \vdots \tag{3}$$

- rule s: if (x_1 is $L^s_{x_1}$) and $\cdots$ and (x_n is $L^s_{x_n}$) then (u_1 is $L^s_{u_1}$) and $\cdots$ and (u_m is $L^s_{u_m}$).

where x_j ($j = 1..n$) and u_k ($k = 1..m$) are fuzzy representations for $x_j(t)$ and $u_k(t)$, the elements of $x(t)$ and $u(t)$, respectively; $L^i_{x_j}$ and $L^i_{u_k}$ are linguistic variables such as 'positive large', 'negative small', and so on. The number of rules 's' depends on the number of possible linguistic variables 'ℓ', and $s = \ell^n$ when all possible combinations are considered but this number is finally determined by the P^2A^2. Reduction in the number of rules is possible only when the control $u(t)$ has no influence on the system behavior for some rules [17].

Consider the constraints of the control inputs and the states.

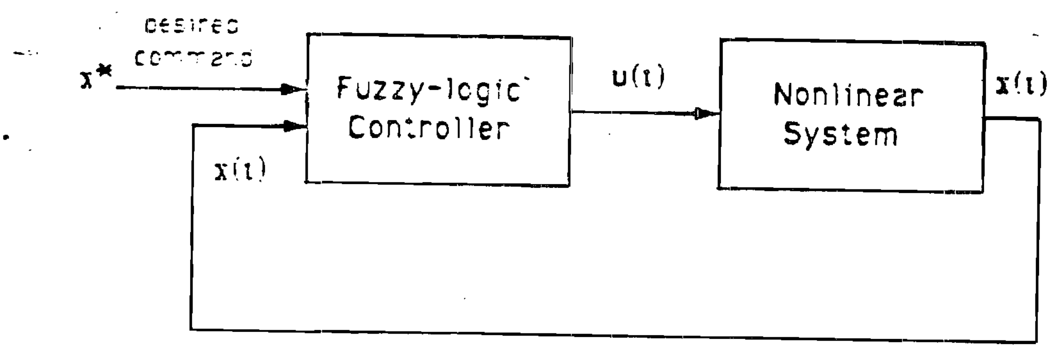


Figure 1: Block diagram of a fuzzy-logic feedback system

Assumption 2.1 (admissible controls) : *The control input $u(t)$ is bounded within some range*

$$u_{k\min} < u_k(t) < u_{k\max} \quad \text{for } k = 1..m \tag{4}$$

implying that the maximum and minimum values, $u_{k\max}$ and $u_{k\min}$, of the actuator are bounded. If the set of admissible controls is defined by

$$\mathcal{U}(t) = \{u_k(t) : t \in [t_0, t_1], k = 1..m\} \in \mathcal{R}^m \tag{5}$$

then the above constraints may be described by

$$\mathcal{U}_c(t) = \{u(t) : \|u(t) - \bar{u}\|_p < M_u, t \in [t_0, t_1]\} \tag{6}$$

for some fixed vector $\bar{u} = \{\bar{u}_k\}$ where M_u is a finite positive real number; $\bar{u}_k$ may be $\frac{1}{2}(u_{k\min} + u_{k\max})$; and $\|\cdot\|_p$ represents p-norm.

Assumption 2.2 (the domain-of-interest in the state space) : *The domain-of-interest (DOI) in the phase plane is considered to provide global asymptotic stability for a local positive definite function. The state $x(t)$ is bounded as follows:*

$$x_{j\min} < x_j(t) < x_{j\max} \quad \text{for } j = 1..n \tag{7}$$

where $x_{j\max}, x_{j\min}$ are real numbers. The set of allowable state trajectories is denoted by

$$\mathcal{X}_s(t) = \{x_j(t) : t \in [t_0, t_1], j = 1..n\} \in \mathcal{R}^n \tag{8}$$

and the DOI may be described by

$$\mathcal{X}_c(t) = \{\|x(t) - \bar{x}\|_p < M_x, t \in [t_0, t_1]\} \tag{9}$$

for some fixed vector $\bar{x} = \{\bar{x}_j\}$, *where* M_x *is a finite positive real number and* $\bar{x}_j$ *may be* $\frac{1}{2}(x_{j\,\min} + x_{j\,\max})$.

Note that $\|x(t)\|_\infty$ is related to the Cartesian coordinates while $\|x(t)\|_2$ is related to polar or spherical coordinates.

Definition 2.1 (extended reachability) : *Let* S_u *and* S_x *be subsets of* $\mathcal{U}_c(t)$ *and* $\mathcal{X}_c(t)$, *respectively, such that*

$$S_u = \{u(t) : \|u(t) - u^*\|_p < \delta_u\} \tag{10}$$

$$S_x = \{x(t) : \|x(t) - x^*\|_p < \delta_x\} \tag{11}$$

for fixed vectors $u^* \in \mathcal{R}^m$, $x^* \in \mathcal{R}^n$, *and for small positive real numbers* δ_u, δ_x. *Extended reachability is stated as follows:*

For the nonlinear system (1) with the initial state $x_0 = x(t_0)$, *there exists* S_u *such that* S_x *is reachable (accessible) from* x_0 *at* $t = t_0$ *with respect to* S_u *for some finite time* $t = T$.

Remark : Roughly speaking, extended reachability implies that $u(t)$, within a δ_u-neighborhood of u^*, leads $\mathcal{X}_s(t)$ to the δ_x-neighborhood of x^* in a finite time. In short, if $u(t) \in S_u$ for $t \in [t_0, T]$,

$$\|x_0 + \int_{t_0}^{T} f(x(\tau), u(\tau), \tau)d\tau - x^*\|_p < \delta_x. \tag{12}$$

Extended reachability relaxes the existing reachability condition by accommodating the fuzzified subsets of $x(t)$ and $u(t)$.

Some definitions relating to fuzzy sets and fuzzy numbers are appropriate for a better understanding of the theoretical developments in this paper (see [18]):

Definition 2.2 *An ordinary subset, A, of a referential set, X, is defined by its 'characteristic function'* $\mu_A(x) \in \{0, 1\}, \quad \forall x \in X$.

A feasible set $F(X)$ is a crisp set described by a characteristic function. A linguistic description in X such as 'large' resides in a feasible subset $F(X) \subset X$ such that $\mu_{F(X)}(x) = 1$ if $x \in F(X)$ and $\mu_{F(X)}(x) = 0$ otherwise.

Definition 2.3 *A fuzzy subset, A, from the universe of discourse, X, is defined by its characteristic function* $\mu_A(x) \in [0, 1]$, $\forall x \in X$, *which is called the 'membership function'.*

Now consider the following mappings of the fuzzy controller in (3):

$$\mathcal{X} = \{(x, \mu(x))\}; \quad \mu : X \longrightarrow [0,1]; \quad x \in X$$

$$\mathcal{U} = \{(u, \mu(u))\}; \quad \mu : U \longrightarrow [0,1]; \quad u \in U$$

$$R_F : \mathcal{X} \longrightarrow \mathcal{U} \tag{13}$$

where $\mathcal{X}, \mathcal{U}$ are families of fuzzy sets defined on X, U. respectively. and R_F is a fuzzy relation which can be interpreted as a mapping with its domain in $\mathcal{X}$ and a space of values constrained in $\mathcal{U}$. Let $A \in \mathcal{X}$ and $B \in \mathcal{U}$, then a formal fuzziness system is defined as

$$A \circ R_F = B \tag{14}$$

where '$\circ$' denotes the operator for the compositional rule of inference. A membership function may represent a non-uniform possibility distribution for the associated linguistic variable described by the characteristic function, and moreover, a fuzzy set is an invariant and consonant set as the presumption level increases.

Consider a controller structure of the fuzzy relational system represented by

$$u(t) = r(x(t)) \qquad t \geq t_0 \tag{15}$$

where $r(\cdot) : \mathcal{R}^n \longrightarrow \mathcal{R}^m$ is a memoryless nonlinear mapping described by the compositional rule of inference (3). Now the fuzzy feedback system can be represented by

$$\dot{x}(t) = f(x(t), r(x(t)), t) \triangleq g(x(t), t). \tag{16}$$

Our goal is to find a fuzzy controller $r(\cdot)$ that stabilizes the nonlinear feedback system $\dot{x}(t) = g(x(t), t)$. and to provide a generic procedure of constructing the stabilizing rule set by using fuzzy logic. If we denote the fuzzy assignments (fuzzy sets) by

$$A_{ij} = x_j \text{ is } L^i_{x_j} = L^i_{x_j}(x_j) \qquad (i = 1..s; j = 1..n) \tag{17}$$

$$B_{ik} = u_k \text{ is } L^i_{u_k} = L^i_{u_k}(u_k) \qquad (i = 1..s; k = 1..m) \tag{18}$$

then the fuzzy relation R_F of the parallel implications in (3) can be obtained as [3]

$$R_F = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s) = \bigvee_{i=1}^{s} (A_i \Rightarrow B_i) \tag{19}$$

where $A_i = \bigwedge_{j=1}^{n} A_{ij}$ and $B_i = \bigwedge_{k=1}^{m} B_{ik}$ with $\bigwedge$ and $\bigvee$ denoting the minimum and maximum operators. respectively.

Definition 2.4 *A 'fuzzy implication' is a mapping $S_i : A_i \Longrightarrow B_i$ and the membership function for S_i is $\mu_{S_i}(u,x) = \mu_{A_i}(x) \bigwedge \mu_{B_i}(u)$ where $x \in X$ and $u \in U$.*

If we consider a fuzzy relational matrix, the quantization of the support in X determines the size of the matrix, and the element of R_F is

$$R_F(p,q) = \mu_{A_i}(x_p) \bigwedge \mu_{B_i}(u_q) \tag{20}$$

where p,q are integers; x_p, u_q are the quantized representatives in X, U, respectively.

Definition 2.5 *The compositional rule of inference can be stated as follows [7]: let the i-th fuzzy implication be $\Sigma_i : A_i \Longrightarrow B_i$ whose membership function is*

$$\mu_{\Sigma_i}(y,x) = \mu_{A_i}(x) \wedge \mu_{B_i}(y) \tag{21}$$

then, for any implication Σ_i^* such that $\Sigma_i^* : A_i^* \Longrightarrow B_i^*$, the membership function for B_i^* is defined by

$$\mu_{B_i^*}(y) = \bigvee_x \{\mu_{A_i^*}(x) \wedge \mu_{\Sigma_i}(y,x)\} \tag{22}$$

and R_F may be represented by

$$R_F = \bigcup_{i=1}^{s} \Sigma_i. \tag{23}$$

The design of membership functions is an important issue closely linked to the possibility distribution of linguistic variables. Fuzzy membership functions should satisfy the properties of convexity and normality [18]. One approach to the design of membership functions is to make use of the S, Π, and Z curves as proposed by Zadeh [2]. The parameters of these functions should be carefully determined according to the scope of each linguistic variable [17]. We define the S, Π, and Z membership functions as follows:

$$S(x;a,b,c) = \begin{cases} 0 & x < a \\ 2(\frac{x-a}{c-a})^2 & a < x < b \\ 1 - 2(\frac{x-c}{a-c})^2 & b < x < c \\ 1 & x > c \end{cases} \tag{24}$$

$$\Pi(x;b,c) = \begin{cases} S(x; c-b, c-\frac{b}{2}, c) & x < c \\ S(x; c, c+\frac{b}{2}, c+b) & x > c \end{cases} \tag{25}$$

$$Z(x;a,b,c) = 1 - S(x;a,b,c) \tag{26}$$

For example, A_{11} = 'x_1 is positive large' can be described by

$$\mu_{A_{11}}(x_1(t)) = S(x_1(t);a,b,c). \tag{27}$$

The parameters of the membership functions are a, b, and c. The transition points, defined as the value of the universe of discourse for which the grade of membership is equal to $\frac{1}{2}$, are $x = b$ for S, Z: $x = c \pm \frac{b}{2}$ for Π. The maximum grade of 1 occurs at $x = c$ for all these functions and the minimum value of 0 at $x = a$ for S, Z: $x = c \pm b$ for Π. These design parameters are chosen to include the state trajectories of $x(t)$ for all x_0 in the DOI [19]. Typical S, Π, Z curves are shown in Figure 2. Other kinds of membership functions are triangle, trapezoid, arctangent, normal distribution functions, and so forth.

A number of mathematical properties describe the relationship between membership functions and the

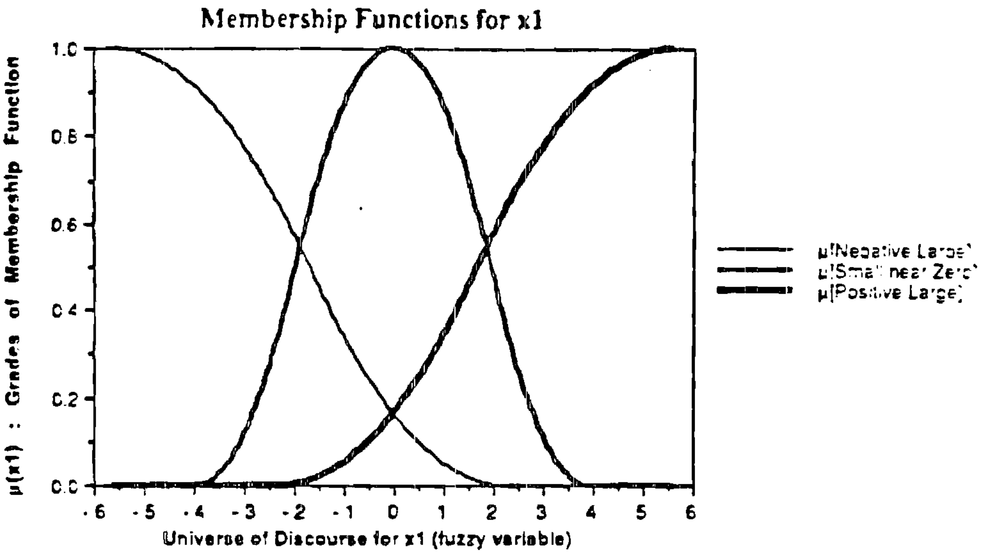


Figure 2: S, Π, Z membership functions

inferred control action. Kania and his co-workers [20] proposed an α-stability property, an α-decision stability property, an α-β strong decision stability property, a good mapping property, a γ-good mapping property, and the associated conditions. Some definitions and mathematical properties of the formal fuzziness system are shown as follows [20,21]:

Definition 2.6 *The formal fuzziness system (14) has the α-stability (α-S) property with respect to some family $\mathcal{A}$ and some feasible subset $F(U) \in U$ if $\mu_B(u) \leq \alpha \quad \forall u \in U - F(U)$ for every fuzzy set $A \in \mathcal{A}$ of $A \circ R = B$ where the operator $\circ$ is defined in (14).*

Theorem 2.1 *Consider the system where $R = A \Rightarrow B$ is given. Then $\bigvee_{i=1}^{n} \mu_A(x_i) \leq \alpha$ or $\mu_B(u_j) \leq \alpha$ $\forall u_j \in U - F(U)$ if and only if the system (14) has the α-S property with respect to a family $\mathcal{A}$.*

Definition 2.7 *A system* (14) *has the α-decision stability (α-DS) property with respect to some family $\mathcal{A}$ of fuzzy sets defined on X if the system has the α-S property with respect to $\mathcal{A}$ and* $\max_{U-F(U)} \mu_{A\circ R}(u) < \max_{F(U)} \mu_{A\circ R}(u)$.

The α-DS property assures that the grade of membership in the feasible subset $F(U)$ is greater than that in the nonfeasible subset $U - F(U)$. The next definition is a stronger version of the α-DS property [20]:

Definition 2.8 *A system* (14) *has the α-β strong decision stability (α-β SDS) property with respect to some family $\mathcal{A}$ if and only if the system* (14) *has the α-DS property with respect to $\mathcal{A}$ and* $\bigvee_{u\in U} \mu_{A\circ R}(u) \geq \beta > \alpha$.

A membership function should follow the α-DS ($\alpha - \beta$ SDS) property for the feasible subset of a linguistic variable, and for every rule, the γ-good mapping property should hold.

Definition 2.9 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given. Then R has the good mapping (GM) property if and only if $A_i \circ R = B_i$ for every $i = 1, ..., s$.*

Definition 2.10 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given where $A_i \in \mathcal{X}$ and $B_i \in \mathcal{U}$ for $i = 1, \cdots, s$. Then a fuzzy relation R has the γ-good mapping (γ-GM) property in the weaker sense if and only if $A_i \circ R = \tilde{B}_i$ for every $i = 1, ..., s$ where $B_i \subseteq \tilde{B}_i$, and $\mu_{\tilde{B}_i} = \mu_{B_i}$ for $\mu_{B_i} \geq \gamma, 0 \leq \gamma \leq 1$.*

Remark: Note that, when $\gamma = 0$, the γ-GM property reduces to the GM property as defined above [21]. It is recommended that from the γ-GM property, at the boundary of each crisp subset, overlapping of the associated adjacent membership functions takes place where the grade of membership is $\gamma = 0.5$ [22]. This overlapping concept is important in the sense that we can minimize the decision errors by overlapping adjacent membership functions where $\mu_A(x_i) = 0.5$.

2.2 Design: The Phase Portrait Assignment Algorithm (P^2A^2)

Let us consider the P^2A^2 for a nonlinear fuzzy controller described by (3) or (15). The following steps describe how to establish the fuzzy rulebase that guarantees aymptotic stability of the closed-loop system consisting of (1) and (15). The key point here is that the procedure entails three basic steps: first, the classification of *integral* manifolds according to the evidence in the phase plane, second, the selection of nonfuzzy cells from the crisp relations, and finally, the transformation of each crisp relation into a fuzzy relation.

- (1) Find $u^* \in S_u$ with which $x(t)$ converges to or passes through S_x in a finite time T, such that $x(T) \in S_x$. There may be several u^*'s for some x_0's that satisfy the extended reachability condition.

These trajectories are called 'the primary-attracting manifolds' (the PAM's). This concept is closely related to the switching curves [23,24].

- (ii) Divide an admissible control element $u_k(t)$ into m_k regions resulting in $\bar{u}_{kr}$'s ($r = 1..m_k$) in such a way that some u_k^*'s $\in \{\bar{u}_{kr}\}$ where $\bar{u}_{kr}$ represents the mean of $u_k(t)$ in the r-th region. A linguistic variable $L_{u_k}^i$ is defined and is assigned to *each* $\bar{u}_{kr}$. The maximally assignable controls are mm_k constant $\bar{u}_{kr}$'s.

- (iii) Plot the state trajectories $C_t(x_0, \bar{u}_{kr}) \in \mathcal{X}_s(t)$ for every x_0 in the DOI and for each $\bar{u}_{kr}$. The PAM's are denoted by $C_t^*(x_0, u_k^*)$. Define 'the primary-attracting cells' (the PAC's) R^*'s $\in \mathcal{R}^n$ that cover the PAM's $C_t^*(x_0, u_k^*)$ in the DOI and assign u_k^* to R^*. The size of R^* partially determines the number of fuzzy rules as the size of the DOI is fixed.

- (iv) Plot the state trajectories $C_t(x_0, \bar{u}_{kr})$ for every x_0 and for some $\bar{u}_{kr}$'s ($\neq u_k^*$). If such $\bar{u}_{kr}$'s lead to or intersect with $C_t^*(x_0, u_k^*)$ either directly or indirectly (via another $C_t(x_0, \bar{u}_{kr})$) in a finite time, then these $C_t(x_0, \bar{u}_{kr})$'s are defined as 'the secondary-attracting manifolds' (the SAM's) $C_t^\dagger(x_0, u_k^\dagger)$ while these $\bar{u}_{kr}$'s are denoted by $u_k^\dagger$'s.

- (v) Define 'the secondary-attracting cells' (the SAC's) $R^\dagger \in \mathcal{R}^n$ that include $C_t^\dagger(x_0, u_k^\dagger)$'s by assigning $u_k^\dagger$ to *each* $R^\dagger$. $R^\dagger$'s are chosen in a such way that $C_t^\dagger(x_0, u_k^\dagger)$'s may have the shortest path, if possible, to $C_t^*(x_0, u_k^*)$ or to nearby $C_t^\dagger(x_0, u_k^\dagger)$ that eventually leads to $C_t^*(x_0, u_k^*)$ and so that $u_k^\dagger$'s may not change abruptly between $R^\dagger$'s. Also, $C_t^\dagger(x_0, u_k^\dagger)$'s may not form a closed-cycle.
  <u>Remark</u> : Steps (iv) and (v) may be difficult since there are many choices for assigning the SAC's. But there should be only one u_k^* or $u_k^\dagger$ for each R^* or $R^\dagger$, respectively.

- (vi) Repeat steps (iv) and (v) until the whole DOI is filled with R^*'s and $R^\dagger$'s. The rest of the manifolds $C_t(x_0, \bar{u}_{kr})$ not satisfying $C_t^*(x_0, u_k^*)$ or $C_t^\dagger(x_0, u_k^\dagger)$ are defined as 'the diverging manifolds' (the DVM's) $C_t^\triangle(x_0, \bar{u}_{kr})$ and these x_0's and $\bar{u}_{kr}$'s should be avoided. Similarly, 'the diverging cells' (the DVC's) that include $C_t^\triangle(x_0, \bar{u}_{kr})$'s are defined as $R^\triangle$'s.

- (vii) The final *attracting* cells, R^*'s (PAC's) and $R^\dagger$'s (SAC's), constitute the <u>cells</u> of the P^2A^2. The total number of cells is equal to the total number of rules. On the boundary of the DOI, all the manifolds $C_t^\dagger(x_0, u_k^\dagger)$ and $C_t^*(x_0, u_k^*)$ should be directed toward the inner regions.
  <u>Remark</u> : $C_t(x_0, \bar{u}_{kr})$ can be obtained analytically by finding the n-dimensional vector fields for the nonlinear system as described above; heuristically by exploiting the expertise of experts; or experimentally for unknown complex systems. Figure 3 shows an example of the attracting cells, R^*'s and $R^\dagger$'s, and the DVC's $R^\triangle$'s for a simple second-order system.

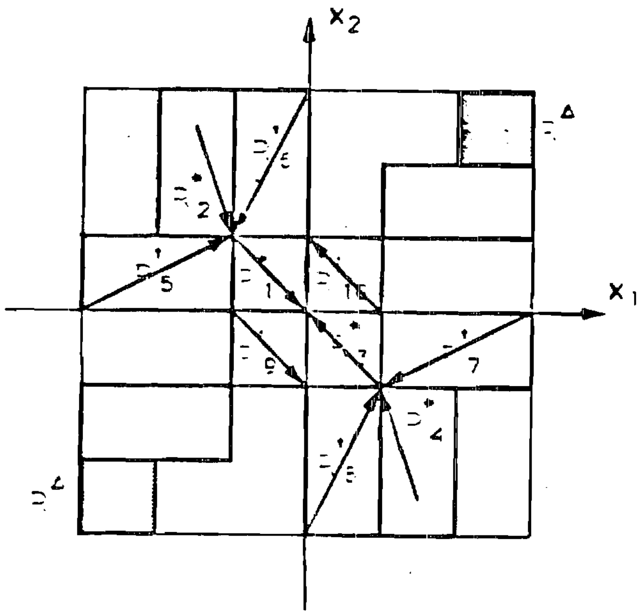


Figure 3: Typical cells and the vector fields of the P^2A^2

- (viii) If there are s cells in the DOI, the i-th cell is defined by the pair $c\{R[i], \bar{u}[i]\}$ where $\bar{u}[i] \in \mathcal{R}^m$ is a control vector of the chosen m constant elements among $\{u_k^*, u_k^\dagger; k = 1..m\}$ for $i = 1..s$, and for each cell, a linguistic characterization is performed by assigning n $L_{x_j}^i$'s (for $j = 1..n$) to each $R[i] \in \{R^*, R^\dagger\}$ in the DOI. Every fuzzy assignment $\{A_{ij}, B_{ik}\}$ in the i-th rule is determined according to the above steps, since each admissible control $\bar{u}_{kr}$ has its own linguistic description $L_{u_k}^i$ for $k = 1..m$ for each $R[i]$. Each cell $R[i]$ in the DOI is assigned to its particular linguistic characterizations $\{L_{x_j}^i : j = 1..n\}$ by using the connectivity of the i-th cell $c\{R[i], \bar{u}[i]\}$. The membership functions are described by $\mu_{A_{ij}}(x_j(t))$ for A_{ij} = 'x_j is $L_{x_j}^i$' and $\mu_{B_{ik}}(u_k(t))$ for B_{ik} = 'u_k is $L_{u_k}^i$'.

- (ix) The control input $u_k(t)$ in (15) is determined via defuzzification by

$$u_k(t) = \frac{\sum_{i=1}^{s} d_i(x(t))\bar{u}_k[i]}{\sum_{i=1}^{s} d_i(x(t))} \qquad \text{for } k = 1..m \tag{28}$$

where $\bar{u}_k[i]$ is the k-th element of $\bar{u}[i]$ and $d_i(x(t))$ is the degree of fulfilment for the i-th rule defined by

$$d_i(x(t)) = \wedge_{j=1}^{n} \mu_{A_{ij}}(x_j(t)) \qquad \text{for } i = 1..s \tag{29}$$

$$d_i(\cdot) : \mathcal{R}^n \longrightarrow [0,1] \tag{30}$$

and $\Sigma_{i=1}^{s} d_i(x(t)) \neq 0$ is required for the existence of a solution of the defuzzification scheme. It is noted that

$$r_k(\cdot) = \frac{\Sigma_{i=1}^{s} \bar{u}_k[i] d_i(\cdot)}{\Sigma_{i=1}^{s} d_i(\cdot)} \tag{31}$$

where $r_k(\cdot)$ represents the k-the element of $r(\cdot)$ in (15).

<u>Remark</u> : The number of cells in the DOI is the same as the number of rules in the fuzzy rule set. When the fuzzy characterization functions are mapped into the fuzzy membership functions, α-DS property [20] should hold for the feasible subsets. Moreover, for each linguistic variable, overlapping of the adjacent membership functions is necessary to ensure that $\Sigma_{i=1}^{s} d_i(x(t)) \neq 0$ for any $x(t)$ in the DOI (the γ-GM property) [17]. Note that the DOI is decomposed into two kinds of cells, PAC's and SAC's. The crisp relational mapping R_C may be represented using the crisp cells as

$$R_C = \bigcup_{i=1}^{s} c\{R[i], \bar{u}[i]\} \tag{32}$$

where $R_C : \mathcal{R}^n \longrightarrow \mathcal{R}^m$. If we introduce the fuzziness transformation $\mathcal{F}$, the fuzzy relational mapping (19) may be defined by

$$R_F = \mathcal{F}[R_C] = \mathcal{F}[\bigcup_{i=1}^{s} c\{R[i], \bar{u}[i]\}]. \tag{33}$$

Lemma 2.1 *If the γ-GM property and the α-DS property (or the α-β SDS property) hold, then there exists a $\delta_u[i] > 0$ for all $x(t) \in R[i]$ such that*

$$|u_k(t) - \bar{u}_k[i]| < \delta_{u_k}[i] \qquad \text{for} \quad k = 1..m \tag{34}$$

or

$$||u(t) - \bar{u}[i]|| < \delta_u[i] \tag{35}$$

where $\bar{u}[i] = \{\bar{u}_k[i] : k = 1..m\}$ and $\delta_u[i] = \max_k\{\delta_{u_k}[i]\}$.

<u>Proof</u> : Let overlapping (the γ-GM property) occur at $\gamma = \alpha$ for the design, then there exists only one $d_i(x(t))$ such that $d_p(x(t)) > \alpha$ if $i = p$, that is, the p-th rule is the most contributing fuzzy rule at time t.

$$d_p(x(t)) = \bigwedge_{j=1}^{n} \mu_{A_{pj}}(x_j(t)) = \alpha_p > \alpha \qquad \text{if} \quad i = p \tag{36}$$

$$d_i(x(t)) = \bigwedge_{j=1}^{n} \mu_{A_{ij}}(x_j(t)) = \alpha_i \ll \alpha \qquad \text{if} \quad i \neq p \tag{37}$$

and from the α-DS property

$$\Sigma_{\substack{i \neq p \\ i=1}}^{s} \alpha_i \ll \alpha_p \tag{38}$$

is satisfied by approximation subject to the minimum operators. From (28).

$$u_k(t) = \frac{\alpha_p \bar{u}_k[p] + \Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}$$

$$= \frac{\alpha_p}{\Sigma_{i=1}^{s} \alpha_i} \bar{u}_k[p] + \frac{\Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}. \tag{39}$$

Since $\alpha_p \cong \Sigma_{i=1}^{s} \alpha_i$ from (38),

$$u_k(t) = \bar{u}_k[p] + \Delta_k[p] \tag{40}$$

where $\Delta_k[p] \triangleq \frac{\Sigma_{i \neq p} \alpha_i \bar{u}_k[i]}{\Sigma_{i=1}^{s} \alpha_i}$. Therefore,

$$|u_k(t) - \bar{u}_k[p]| < \delta_{u_k}[p] \tag{41}$$

where $\delta_{u_k}[p] = |\Delta_k[p]|$ for $k = 1..m$.

<u>Remark</u> : If the granularity of the fuzzy rulebase is dense enough, (38) is strongly confirmed, and therefore, $\delta_{u_k}[p]$ in (41) is reduced. Moreover, reduction in the fuzzy rules is possible as long as the system behavior is not affected by deleting some rules.

Theorem 2.2 (global convergence) *Let the extended reachability condition hold for a nonlinear system described by (1) and let the P^2A^2 be applied to the control input $u(t)$. Then, for any x_0 in the DOI of the state space,*

$$\|x(t) - x^*\|_p < \epsilon \tag{42}$$

is satisfied for $t \geq T$ where $\epsilon > 0$ and $x(t)$ is globally convergent to an ϵ-neighborhood of x^*.

<u>Proof</u> : Let the index 'i' of the most contributing rule be omitted for simplicity. The proof is divided into two parts.

case (i) $x_0 \in R^*$: $x(t)$ is found by

$$x(t) = x_0 + \int_{t_0}^{t} f(x(\tau), u(\tau), \tau) d\tau. \tag{43}$$

In the worst case, $u(\tau) = u^* + \Delta^* \in S_u$, and from Lemma 2.1 of the P^2A^2, $x(t)$ continues to remain in R^*'s reaching S_x in a finite time $t_x = t_1 - t_0$. From (12),

$$\|x(t_1) - x^*\|_p = \|x_0 + \int_{t_0}^{t_1} f(x(\tau), u^* + \Delta^*, \tau) d\tau - x^*\|_p < \delta_x \tag{44}$$

at $t = t_1 > t_0$.
case (ii) $x_0 \in R^\dagger$: There exists $t = t_2 > t_0$ such that $x(t)$ is transferred from $R^\dagger$ to R^* by applying the worst-case control $u(t) = u^\dagger + \Delta^\dagger$ from Lemma 2.1 of the P^2A^2 such that

$$x(t_2) = x_0 + \int_{t_0}^{t_2} f(x(\tau), u^\dagger + \Delta^\dagger, \tau) d\tau \in R^* \tag{45}$$

and then, similarly from (i), we can use $u(t) = u^* + \Delta^* \in S_u$, resulting in $x(t) \in S_x$ at time $T = t_2 + t_x$. Therefore,

$$\|x(t) - x^*\|_p < \epsilon \tag{46}$$

in a finite time $t \geq T$ where $\epsilon = \delta_x > 0$.

2.3 Analysis: Stability of The Nonlinear Fuzzy-logic Controller

The fuzzy-logic control system is analyzed by using Lyapunov's direct method [25] in this section. Since the P^2A^2 forces the vector field of the dynamic equation (1) to be directed toward S_x by using the fuzzy-logic rulebase control $u(t)$, the following proposition is considered for stability analysis of nonlinear fuzzy control systems considering the vector fields.

Proposition 2.1 *Let the extended reachability condition hold and let the P^2A^2 be applied to (1). If $x(t) \in R^*$, then for some x^*,*

$$\dot{x}(t) = \Psi(t)(x(t) - x^*) + \xi_x \tag{47}$$

is satisfied $\forall t \in \mathcal{R}_+$ where the eigenvalues of $\Psi(t)$ are negative, i.e., $\lambda_i[\Psi(t)] < 0$ for i=1..n, $\Psi(t) = \Psi(x(t), u(t), t) \in \mathcal{R}^{n\times n}$; and $\xi_x \in \mathcal{R}^n$ is a point in the σ-neighborhood of $\dot{x}(t)$ when $x(t) = x^*$ such that

$$\|\dot{x}(t) - \xi_x\|_p|_{x(t)=x^*} < \sigma. \tag{48}$$

<u>Proof</u> : Since $x(t) \in R^*$, the vector field of $\dot{x}(t) = f(x(t), u^*, t)$ is directed toward $x^* \in S_x$ by applying the fuzzy control input $u(t) = u^* + \Delta^* \in S_u$ from the P^2A^2. If $x_i(t), \xi_{xi}, x_i^*$ are the elements of $x(t), \xi_x, x^*$, respectively, then the following is satisfied.

$$\frac{\dot{x}_i(t) - \xi_{xi}}{x_i(t) - x_i^*} < 0 \qquad \forall t \in [t_0, t_1] \tag{49}$$

for $i = 1..n$. Thus, for some function of time $\Psi_i(t) < 0, \forall t \in \mathcal{R}_+$.

$$\dot{x}_i(t) = \Psi_i(t)(x_i(t) - x_i^*) + \xi_{xi} \tag{50}$$

where $\Psi_i(t)$ may be a diagonal element of $\Psi(t)$. Therefore, in general,

$$\dot{x}(t) = \Psi(t)(x(t) - x^*) + \xi_x \tag{51}$$

where $\lambda_i[\Psi(t)] < 0$ for $i = 1..n$ and $\forall t \in [t_0, t_1]$. t_1 can be extended to infinity by considering piecewise time-scale analysis in the different R^*'s.

Remark : It is noted that (49) is related to a Lipschitz condition of (1) when $u(t) = u^*$ in such a way that the uniqueness of the solution to (1) is guaranteed if $\Psi_i(t)$ is finite $\forall t \in \mathcal{R}_+$ [16].

Theorem 2.3 *Let a local positive definite function $v(t)$ in R^* be defined by*

$$v(t) = (x(t) - x^*)^T \mathcal{P}(x(t) - x^*) \tag{52}$$

where $\mathcal{P}$ is a symmetric positive definite matrix. If the P^2A^2 is applied to (1) with the extended reachability condition, x^* is asymptotically stable with a ball attractor S_z for all $x_0 \in R^*$, $\forall t \in [t_0, \infty)$.

Proof : Considering $\dot{v}(t)$.

$$\dot{v}(t) = \dot{x}(t)^T \mathcal{P}(x(t) - x^*) + (x(t) - x^*)^T \mathcal{P}\dot{x}(t) \tag{53}$$

and if we define $z(t) = x(t) - x^*$ then from (47) in Proposition 2.1.

$$\dot{v}(t) = (\Psi(t)z(t) + \xi_x)^T \mathcal{P}z(t) + z(t)^T \mathcal{P}(\Psi(t)z(t) + \xi_x)$$

$$= -z(t)^T \mathcal{Q}(t)z(t) + 2\xi_x^T \mathcal{P}z(t) < 0 \tag{54}$$

is required for asymptotic stability where $\mathcal{Q}(t) \in \mathcal{R}^{n\times n}$ is a positive definite matrix defined by

$$\mathcal{Q}(t) = -(\Psi(t)^T \mathcal{P} + \mathcal{P}\Psi(t)). \tag{55}$$

Therefore, for asymptotic stability.

$$z(t)^T \mathcal{Q}(t)z(t) > 2\xi_x^T \mathcal{P}z(t) \tag{56}$$

should be satisfied and this is implied by

$$||z(t)||_p > \frac{2\lambda_{max}[\mathcal{P}]||\xi_x||_p}{\inf_t \lambda_{min}[\mathcal{Q}(t)]} \tag{57}$$

where $\lambda_{max}[A], \lambda_{min}[A]$ represent the maximum and minimum eigenvalues of a matrix A. If $S_z \in \mathcal{R}^n$ is defined by

$$S_z = \{z(t) : ||z(t)||_p < \delta_x\} \tag{58}$$

where $\delta_z = \frac{2\lambda_{max}[\mathcal{P}]||\xi_z||_p}{\inf_t \lambda_{min}[\mathcal{Q}(t)]} > 0$. then S_z is a ball attractor outside of which $\dot{v}(t) < 0$ holds with asymptotic stability. and it is easy to realize $S_z = S_z$.

Remark : Note that $\lambda_{min}[\mathcal{Q}(t)]$ may change but $\inf_t \lambda_{min}[\mathcal{Q}(t)] < 0$ is satisfied by Proposition 2.1. If $||\xi_z||_p$ is small enough compared with $\inf_t \lambda_{min}[\mathcal{Q}(t)]$. the attractor (58) reduces to a point.

Corollary 2.1 *Let the extended reachability condition and the P^2A^2 be satisfied for a multi-input nonlinear system in (1). Then x^* is globally asymptotically stable with a ball attractor S_x for all x_0 in the DOI and $\forall t \in [t_0, \infty)$.*

Proof : (i) If $x(t) \in R^\dagger$, there exists a control in the δ_u-neighborhood of $u^\dagger$ that transfers $x(t)$ from $R^\dagger$ to R^* in a finite time by the P^2A^2.
(ii) If $x(t) \in R^*$, there exists a control in the δ_u-neighborhood of $u^* \in S_u$ that forces $x(t)$ to converge to S_x by applying Theorem 2.3 and x^* is asymptotically stable.
From (i) and (ii), for all x_0 in the DOI, global asymptotic stability is guaranteed.
Remark : (i) is referred to as 'transferring capability' since $x(t) \in R^\dagger$ is moved to $x(t) \in R^*$ in a finite time by using the P^2A^2. The corollary 2.1 can be applied to the fuzzy regulators for nonlinear systems.

Theorem 2.4 (fuzzy robustness) *Given the nonlinear system in (1), let the extended reachability condition hold and let the P^2A^2 be applied to (1). The size of the attractor in (58) can be reduced to the minimum-size attractor S_x if we can select $\xi_x^* = \min_x[\xi_x]$.*

Proof : The proof is obvious from Theorem 2.3 since $S_x(S_x)$ in (58) can be minimized if there exist a sequence of u^* so that the minimum Euclidean distance can be obtained by applying the P^2A^2 such that $\xi_x = \xi_x^*$ in (47).
Remark (fuzzy robustness) : As the proposed fuzzy controller deals with fuzzified cells in the DOI, the controller is robust against disturbances as long as dynamic and parametric uncertainties of nonlinear systems, or some external noise, are bounded within some allowable regions. It is asserted that any state $x(t)$ outside the ϵ-neighborhood of x^* is attracted to S_x in a finite time, and therefore, the bounded region can allow the corresponding magnitude of any disturbance with the same performance.

3 Example: Simulation of A Single-Link Robot Arm

As an illustrative example, consider a single-link robot arm shown in Figure 4 with a mass of 1.0 [Kg] at the end of the link of 1.0 [m] length. The dynamics can be derived by using the Lagrange-Euler equation of motion [26].

$$u(t) = m\ell^2\ddot{\theta}(t) - mg\ell \sin\theta(t) \tag{59}$$

and since $m = 1[Kg]$, $\ell = 1[m]$, and $g = 9.8[m/sec^2]$ if we define $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$, the following vector equations can be obtained

$$\dot{x}_1(t) = f_1(x(t), u(t), t) = x_2(t)$$

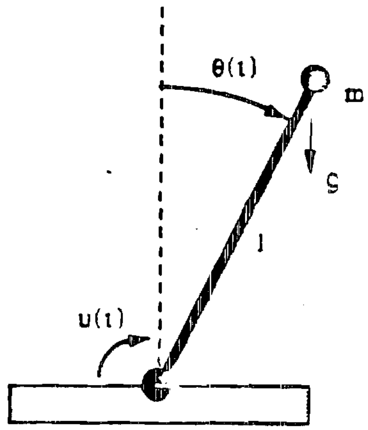


Figure 4: Single-link robot arm

$$\dot{x}_2(t) = f_2(x(t), u(t), t) = 9.8 \sin x_1(t) + u(t). \tag{60}$$

The direction of the vector field of (60) is found by

$$\varphi(t) = \tan_2^{-1}(f_1, f_2) \tag{61}$$

where $\tan_2^{-1}(\cdot,\cdot)$ can access any direction in the (x_1, x_2) coordinates. Making use of the vector field $\varphi(t)$ in the P^2A^2, the fuzzy rulebase for the control input $u(t)$ in (3) and (15) can be constructed to regulate $x_1(t)$ and $x_2(t)$ to zero ($x^* = 0$) as follows:

- R_1: if (x_1 is PL) and (x_2 is PL) then (u is NL)
- R_2: if (x_1 is PL) and (x_2 is SZ) then (u is NS)
- R_3: if (x_1 is PL) and (x_2 is NL) then (u is SZ)
- R_4: if (x_1 is SZ) and (x_2 is PL) then (u is NS)
- R_5: if (x_1 is SZ) and (x_2 is SZ) then (u is SZ)
- R_6: if (x_1 is SZ) and (x_2 is NL) then (u is PS)
- R_7: if (x_1 is NL) and (x_2 is PL) then (u is SZ)

- R_8: if (x_1 is NL) and (x_2 is SZ) then (u is PS)
- R_9: if (x_1 is NL) and (x_2 is NL) then (u is PL)

where we use abbreviations for various linguistic variables: 'PL' for 'positive large', 'PS' for 'positive small', 'SZ' for 'small near zero', 'NS' for 'negative small', and 'NL' for 'negative large'. Membership functions for the fuzzy variables x_1, x_2, and u, are the S, Π, and Z curves. The above 9 rules guarantee global asymptotic stability in the sense of Lyapunov.

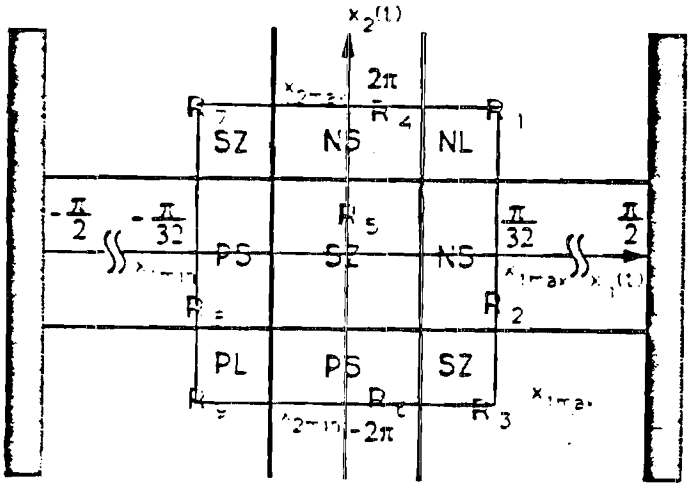
In the simulations, the initial conditions are chosen to be $x_1(0) = 30$ [deg] and $x_2(0) = 0$. The maximum value for the control input $u(t)$ is $\pm 300[Nm](M_u)$ while the boundary values for $x(t)$ are $\pm\frac{\pi}{32}$ for $x_1(t)$; $\pm 2\pi$ for $x_2(t)$. In the case of 9 rules, Figure 5 (a) and (b) show the non-fuzzy cells and the phase portrait of the nonlinear system (60) for different $\bar{u}[i]$'s of the fuzzy rulebase established by the P^2A^2 utilizing the vector field where the PAC's and the SAC's are represented by R^*'s and $R^\dagger$'s, respectively. The results of 9 rules and those of 25 rules of the fuzzy-logic controller are compared in Figures 6, 7, and 8. Figure 9 shows actual state trajectories of 9 rules and 25 rules. As mentioned earlier, more rules usually imply higher convergence rates since the granularity of the DOI reduces the fuzziness in the phase plane. Moreover, an impact force of $100[N]$ is applied to the end of the link during $0.01[sec]$ starting at $t = 4.0[sec]$ to consider some uncertainty at the joint.

Although the simulations exclude the actuator dynamics, if the actuator modes are fast enough compared to the robot dynamics, the 9 rules can be applied without further modifications. If not, the number of states should be increased by adding the actuator dynamics to the fuzzy rulebase. For more information on joint flexibility in connection to actuators, see [27].

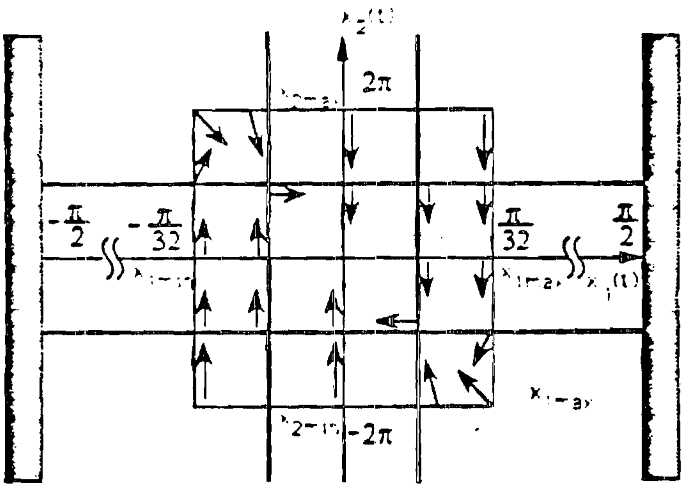
4 Conclusions and Open Problems

The research work reported in this paper supports a connection between artificial intelligence and analytic methods by linking the 'if-then' rules of the control input to a systematic design procedure called the P^2A^2 for nonlinear systems. Advantages of fuzzy-logic controllers include robustness and flexibility in modifying the particular domain by providing qualitative reasoning for a specific control decision. Also, we can reduce the decision errors by transforming crisp sets into fuzzy sets.

Global asymptotic stability of nonlinear fuzzy-logic control is considered by using Lyapunov's direct method. Some difficult problems associated with fuzzy control, such as ambiguity in completeness of the fuzzy rulebase,updating and calibrating the rulebase controllers, and exhaustive use of the linguistic rules, can be solved by applying the P^2A^2. We can easily change, add, or delete some rules according to the organization level.



(a)



(b)

Figure 5: The cells (a) and the phase portrait (b) of $x_1(t)$ and $x_2(t)$ from the P^2A^2

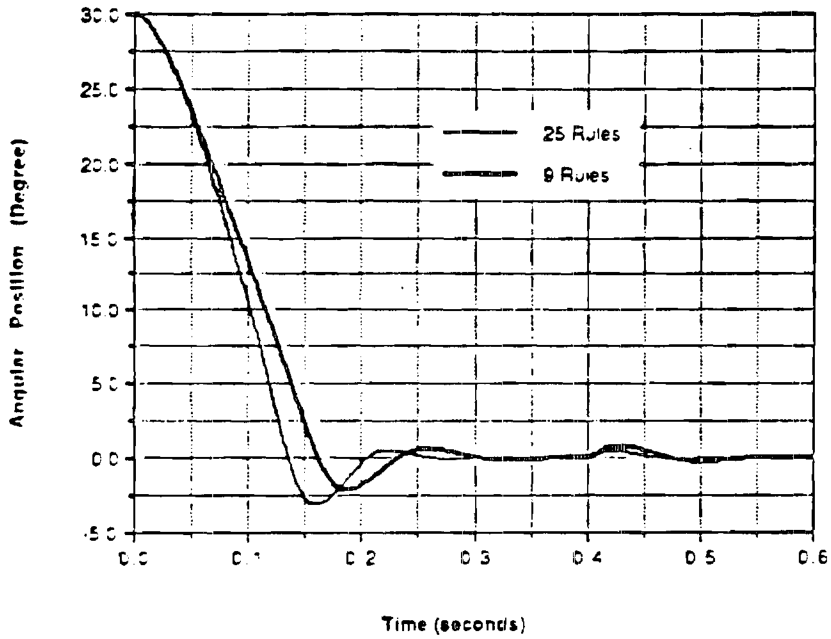


Figure 6: Angular position of the single-link robot arm

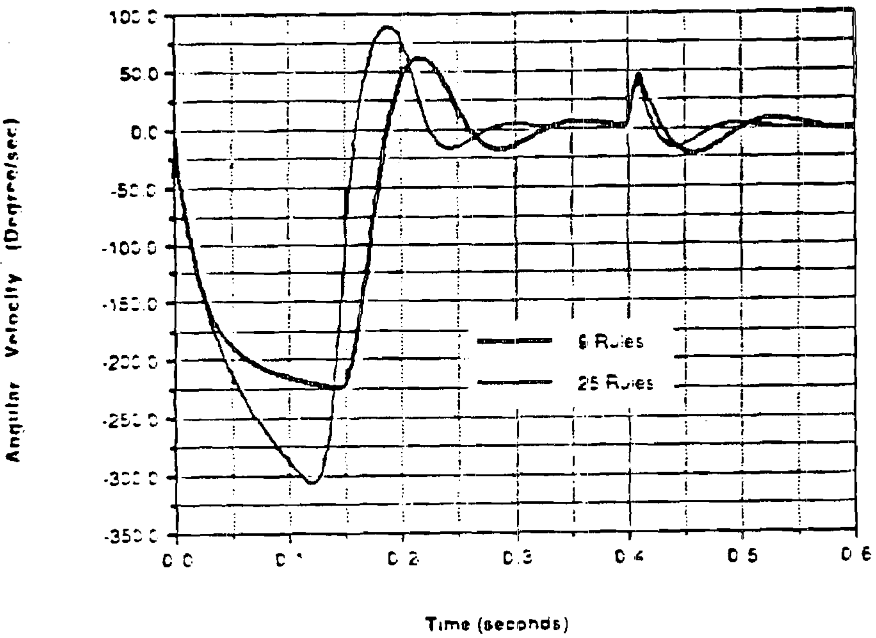


Figure 7: Angular velocity of the single-link robot arm

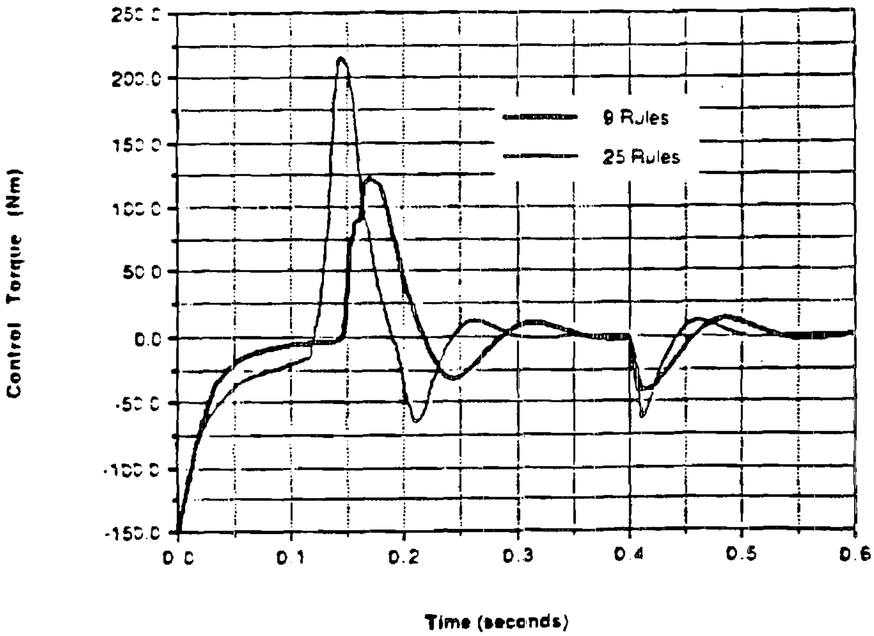


Figure 8: Fuzzy control torque input applied to the single-link robot arm

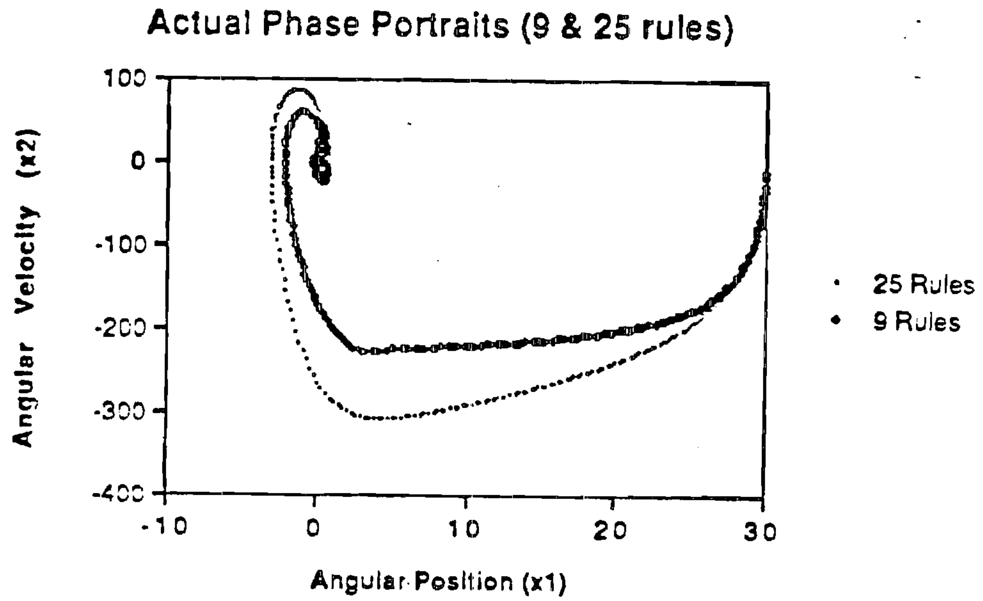


Figure 9: Actual phase portrait of the single-link robot arm

Simulation results show that fast convergence, asymptotic stability, and robustness with respect to disturbances are achieved. Although P^2A^2 may requires a large amount of fuzzy information for a system of higher dimension, it is easy to expand the P^2A^2 concept by using the n-dimensional vector fields. Fuzzy systems need more samples for the decision making process compared with analytical methods, however it handles unreliable information by using linguistic variables and fuzzy relations. In the simulations, an infinite-valued logic is applied to fuzzy sets, but, in the practical sense, simplified fuzzy relational matrices can be used if we use multi-valued logics of the quantized domain. A fuzzy controller can be implemented via a parallel computer architecture as shown in Figure 10 since the rules are disjunctive.

Some important aspects of the fuzzy rulebased controllers are as follows: First, the fuzzy dynamic controller $\dot{u}(t) = r_u(x(t), u(t), t)$ is a substitute for the fuzzy regulator $u = r(x(t))$. However, limit cycles are experienced probably caused by the double integrator in the closed-loop system. Second, there is a proportional relationship between the size of the fuzzy rules and the convergence rates. Finally, robustness properties of the fuzzy controller can be enhanced by adding convergent learning algorithms that, in other words, are the static rules that change the fuzzy rulebase itself which is dynamic.

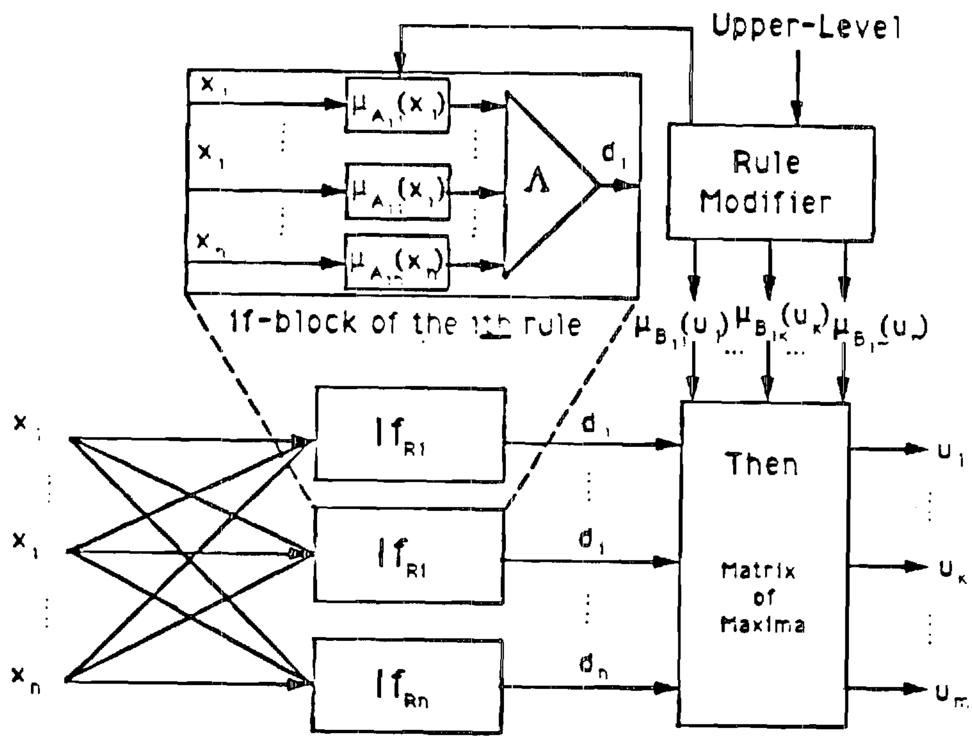


Figure 10: The parallel computer architecture of the fuzzy rulebased systems

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List of Figures

1	Block diagram of a fuzzy-logic feedback system	3
2	S, Π, Z membership functions	7
3	Typical cells and the vector fields of the P^2A^2	10
4	Single-link robot arm	16
5	The cells (a) and the phase portrait (b) of $x_1(t)$ and $x_2(t)$ from the P^2A^2	18
6	Angular position of the single-link robot arm	19
7	Angular velocity of the single-link robot arm	20
8	Fuzzy control torque input applied to the single-link robot arm	20
9	Actual phase portrait of the single-link robot arm	21
10	The parallel computer architecture of the fuzzy rulebased systems	22

Model Reference Fuzzy Control

by

Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

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Hoon Kang and George J. Vachtsevanos

School of Electrical Engineering, Georgia Institute of Technology

Atlanta, Georgia 30332

Abstract

This paper is concerned with a fuzzy logic controller for linear systems. A fuzzy rulebase is employed to stabilize the closed-loop system consisting of a linguistic controller and a process. By utilizing a reference model, impulse-like control inputs can be avoided. A generic guideline for building a control rulebase is developed and membership functions for linguistic variables are chosen to be S, Π, and Z functions. The rules guarantee asymptotic stability. Simulation results show robustness and convergence of the proposed symbolic controller.

1 Introduction

Since the introduction of fuzzy sets and possibility measures by Zadeh [1,2] to cope with the approximate estimation of uncertainty, many learning and adaptation schemes have been developed for various applications. A fuzzy set is defined as a class of objects with a continuum of grades of membership using certainty or confidence factors in order to handle uncertainties or complexity in a particular domain of knowledge [3].

Compared with a probabilistic representation of systems caused by random phenomena, a fuzzy system is an algebraic relation derived from possibility measure theory and it can be represented either by the input-output property or by a transition of states in which case it is called a fuzzy dynamic system. When only imprecise or indefinite information about a process's dynamic behavior is available, then fuzzy sets and linguistic variables may be used to build better models for a process. Several definitions and properties of fuzzy sets and fuzzy numbers are included in Appendix.

A fuzzy number can be described by linguistic terms ('large', 'medium', 'small', 'very small', etc.) whose fuzziness provides many degrees of freedom in dealing with uncertainty by using non-uniform possibility distributions [3]. For any presumption level α $(0 < \alpha < 1)$, the confidence interval represents information that reduces the uncertainty by using lower and upper bounds. From an objective point of view, it may be a confidence interval between two measured data as defined in statistics; from a subjective viewpoint, it may be interpreted as the available expertise about the process. As α increases, the confidence interval decreases in most cases [4].

It is possible to apply fuzzy sets to a classical tracking control problem. Performance criteria of energy functions or least-squares error approaches are suitable for control purposes and rulebases should be constructed to obey an energy-dissipating property. The objective of this paper is to construct a rulebase having the properties of asymptotic stability and robustness under uncertainties such as bounded disturbances, parametric disturbances, and unmodeled dynamics. By providing sufficient conditions for the stabilizing rules, a prototype guideline for the control update is proposed. Simple SISO linear plants are used as examples.

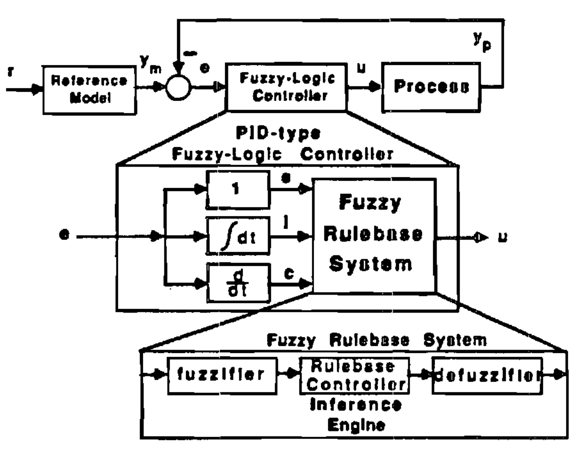

Figure 2.1: Block diagram of the MRFC structure

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2 Model Reference Fuzzy Control

In this section, an expert control system is proposed which consists of a fuzzy rule-based controller and a linear process. A model reference fuzzy control (MRFC) concept is introduced in order to guarantee the model-following property of a fuzzy controller. The mean of maxima procedure is applied to the inference scheme that estimates the control feedback.

2.1 Structure of MRFC

A rule set represents a control objective in fuzzy relations. From measured data such as the speed of a dc motor, the joint angle or velocity of a robot arm, aircraft elevator deflection, and so on, a rulebased controller interprets such data via a fuzzy decision maker and provides control signals which satisfy the performance criterion.

A reference model is implemented in parallel with the process so that the tracking error between the model and the process prevents impulse-like signals in the inferred control value. Figure 2.1 shows the block diagram of the proposed MRFC. The control input $u[k] = u(kt_s)$ can be represented by

$$u[k] = u[k-1] + \delta u[k] \tag{2.1}$$

where t_s is a sampling period and $\delta u(t) = \delta u[k]$ is the update for the control input $u[k]$ from the rulebase. The tracking error $e(t) = e[k]$ is defined by

$$e(t) = y_m(t) - y_p(t) \tag{2.2}$$

where $y_m(t)$ and $y_p(t)$ are the model output and the process output, respectively. The error $e(t)$ is fuzzified in the linguistic controller. The reference model is chosen to be stable and it may be a 1st-order SISO linear system.

Proportional-integral-derivative (PID) and proportional-derivative (PD) control concepts are used as fuzzy inputs to the symbolic controller. The derivative and integral of the tracking error are also fuzzified in the rulebased controller and are defined as follows:

$$c(t) = e[k] - e[k-1] \cong t_s\dot{e}(t) \tag{2.3}$$

$$i(t) = \int_0^t e(\tau)d\tau \tag{2.4}$$

where $c(t)$ and $i(t)$ are the change in error and the integral error, respectively. $\delta u(t), e(t), c(t)$, and $i(t)$ are fuzzified next by specifying the universe of discourse and by assigning appropriate membership functions for each variable as described in the following section.

The rulebased controller may be interpreted as

$$\mathcal{F} : e \Longrightarrow \delta u \quad \text{or} \quad \delta u = F(e) \tag{2.5}$$

where $\mathcal{F}$ and $F(\cdot)$ represent a nonlinear mapping and the corresponding nonlinear function, respectively, that describes a fuzzy relationship including a fuzzifier and a defuzzifier. Let the associated universe of discourse be $\Delta U, E, C, I$ for $\delta u, e, c, i$, respectively, then $\mathcal{F} : E \times C \times I \Rightarrow \Delta U$ for PID control and $\mathcal{F} : E \times C \Rightarrow \Delta U$ for PD control. If the Jacobian of $F(\cdot)$ is defined by

$$J_F = \frac{\partial F}{\partial e} \tag{2.6}$$

then $\dot{u} = \frac{du}{dt}$ may be written as

$$\dot{u} = J_F\dot{e}. \tag{2.7}$$

From (2.1) and (2.3), $\delta u = t_s\dot{u}$ and $c = t_s\dot{e}$, and, therefore, we obtain

$$\delta u = J_F c. \tag{2.8}$$

The Jacobian system for the process is defined by

$$\delta y_p = J_P \delta u. \tag{2.9}$$

Definition 2.1 *The model-following property is stated as follows:*

$$\delta y_m > 0 \Longrightarrow \delta y_p > 0 \quad \text{or} \quad \delta y_m < 0 \Longrightarrow \delta y_p < 0 \quad \forall t > 0, \tag{2.10}$$

or equivalently,

$$\delta y_p \delta y_m > 0 \quad \forall t > 0. \tag{2.11}$$

Consider the following lemma that describes the Jacobian system (2.8).

Lemma 2.1 *Consider a nonlinear system from y_m to y_p represented by*

$$\mathcal{N} : y_m \Rightarrow y_p \tag{2.12}$$

where $y_m, y_p \in \mathcal{R}^1$ and the associated Jacobian system $J_H : \delta y_m \Rightarrow \delta y_p$ defined by

$$\delta y_p = J_H \delta y_m. \tag{2.13}$$

The model-following property holds if and only if $0 < |J_H| < \infty$, and $J_H > 0 \ \forall t > 0$ where

$$J_H = J_P[I + J_F J_P]^{-1} J_F \tag{2.14}$$

where J_P is the Jacobian system for the process.

Proof: sufficiency — Since $\delta y_p = J_H \delta y_m$, and from the model-following property,

$$\delta y_p \delta y_m = J_H \delta y_m^2 > 0. \tag{2.15}$$

Therefore, $J_H > 0$.
necessity — Let $J_H > 0$ and J_H is finite then it is obvious that $\delta y_p \delta y_m > 0$, $\forall t > 0$.
Remark: In fact, using the model-following property, we can say that y_p has the same trend of increasing or decreasing y_m.

A sufficient condition for the model-following property can be found by the following theorem.

Theorem 2.1 *Let $J_P > 0$ in (2.9) or (2.14), then the model-following property holds if there exists $\alpha > 0$ such that $J_H > 0 \ \forall t > 0$, i.e.,*

$$J_F = \alpha J_P^T \quad \forall t > 0 \tag{2.16}$$

Proof: As $J_F = \alpha J_P^T$, substituting it into (2.14), $J_H > 0$, $\forall t > 0$.
Remark: The above theorem says that the Jacobian system for the fuzzy rulebase, J_F should be positive for the model-following property of the system $\mathcal{N}$ if J_P is positive.

2.2 Design of Membership Functions

The design of membership functions is an important step in the development of the fuzzy controller and is closely linked to the possibility distribution of linguistic variables. Fuzzy membership functions should satisfy the properties of convexity and normality in usual cases.

Membership functions for $e(t), \dot{e}(t)$, and $\delta u(t)$ make use of S, Z, and Π curves as proposed by Zadeh [2]. The parameters of these functions should be carefully determined according to the scope of each linguistic variable and they are closely related to the mean and the variance of a probability distribution. It is noted that fuzzy sets do not accumulate every available evidence since the operators involved are minimum or maximum procedures. We define S, Π, and Z membership functions for the MRFC as follows:

$$S(x; \alpha, \beta, \gamma) = \begin{cases} 0 & x < \alpha \\ 2(\frac{x-\alpha}{\gamma-\alpha})^2 & \alpha < x < \beta \\ 1 - 2(\frac{x-\gamma}{\alpha-\gamma})^2 & \beta < x < \gamma \\ 1 & x > \gamma \end{cases} \tag{2.17}$$

$$\Pi(x : \beta, \gamma) = \begin{cases} S(x; \gamma-\beta, \gamma-\frac{\beta}{2}, \gamma) & x < \gamma \\ S(x; \gamma, \gamma+\frac{\beta}{2}, \gamma+\beta) & x > \gamma \end{cases} \tag{2.18}$$

$$Z(x; \alpha, \beta, \gamma) = 1 - S(x; \alpha, \beta, \gamma) \tag{2.19}$$

The parameters of the membership functions are α, β, and γ. The transition points defined as the value of the universe of discourse for which the grade of membership is equal to $\frac{1}{2}$ are $x = \beta$ for S, Z; $x = \gamma \pm \frac{\beta}{2}$ for Π. The maximum grade of 1 occurs at $x = \gamma$ for all these functions and the minimum value of 0 at $x = \alpha$ for S, Z; $x = \gamma \pm \beta$ for Π.

2.3 Stability of The Closed-Loop Fuzzy System

Stability of the closed-loop fuzzy feedback system is an important issue. Kickert et al.[5] showed that a nonlinear fuzzy-logic controller for a specific simple case is identical to a multi-level relay which can be analyzed via a describing function technique and they anticipated the autonomous oscillation of the system. Ray et al.[6] connected a fuzzy-logic controller to the concept of L_2-stability by using a compensated system with Nyquist criteria. Kiszka et al.[7] introduced the concept of energistic stability for a class of fuzzy dynamic systems.

There are few mathematical properties describing the relationship between membership functions and the inferred control actions. Kania and his co-workers [8] proposed an α-stability property, α-decision stability property, $\alpha - \beta$ strong decision stability property, a good mapping property, a γ-good mapping property, and associated conditions for the above properties as defined in Appendix.

In this paper, we propose a symbolic controller that guarantees global asymptotic stability. Consider the fuzzy relations R_F given by the statements: let the size of a rulebase be s and the number of fuzzified variables m.

- if A_1 then B_1
  ⋮
- if A_i then B_i
  ⋮
- if A_s then B_s

where $A_i = \bigwedge_{j=1}^{m} A_{ij}$. R_F may be described by

$$R_F = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s). \tag{2.20}$$

For example, A_{11} ='e is positive large', A_{12} ='$\dot{e}$ is negative small', and B_1 ='δu is negative small'. The membership functions for each case are described by $\mu_{A_{11}}(e), \mu_{A_{12}}(\dot{e})$, and $\mu_{B_1}(\delta u)$, respectively. The above fuzzy relations imply the well-known compositional rule of inference [3] (the inference scheme of modus ponens).

In order to guarantee stability of the closed-loop system, the strategic rules should be constructed so as to meet the passivity or the energy dissipating property for a given performance index. The following is a set of guidelines for building a control rule set for stabilizing fuzzy relations:

1. Choose a characteristic function from a referential subset (choose a feasible subset) and select a membership function that follows the α-decision stability property for that feasible subset of a linguistic variable.

2. Navigate the whole set of possible linguistic rules, applying the chosen membership functions to each linguistic variable.

3. For every rule, the γ-good mapping property should hold ($\gamma \neq 0$). That is to say, if the premise part of the i-th rule, A_i, consists of several sub-premises or fuzzy subsets, A_{ij}, such as $A_i = \bigwedge_{j=1}^{m} A_{ij}$, then, for any j and for some presumption level, $\alpha_1 (0 < \alpha_1 < \alpha)$, there should exist at least one A_{kj} such that $A_{ij} \wedge A_{kj} \neq \emptyset$ for $i \neq k$. This is a necessary condition for the existence of the solution of the modified mean of maxima scheme.

4. In the closed-loop feedback system, if the above conditions hold and the strategic rules are built so that every fuzzy variable has the energy dissipating property, then the fuzzy feedback system is stable and converges. A rule satisfying stability may be deleted since $\delta u = 0$.

The modified mean of maxima procedure is one approach to the defuzzification of fuzzy control variables. As long as the above guidelines are satisfied, the change in control input $\delta u(t)$ can be found by

$$\delta u(t) = \frac{\sum_{i=1}^{s} \delta u_i^* f_i(t)}{\sum_{i=1}^{s} f_i(t)} \tag{2.21}$$

where $f_i(t)$ is the degree of fulfilment defined by

$$f_i(t) = \bigwedge_{j=1}^{m} \mu_{A_{ij}}(x_j(t)) \tag{2.22}$$

with $x_j \in X_j$ (for example, $x_1 = e$, $x_2 = \dot{e}$), and δu_i^* is defined by

$$\delta u_i^* = \{\bar{v} | \mu_{B_i}(v) = \max_v [\mu_{B_i}(v)]; v, \bar{v} \in \Delta U\} \tag{2.23}$$

where ΔU is the universe of discourse of δu. From (2.8), the relationship between the Jacobian J_F and the defuzzifier is described by

$$\delta u = J_F c = \frac{\sum_{i=1}^{s} \delta u_i^* f_i}{\sum_{i=1}^{s} f_i}. \tag{2.24}$$

The error becomes

$$e = y_m - \int_0^t h_p(t-\tau) \int_0^{\tau} \frac{\sum_{i=1}^{s} \delta u_i^* f_i(\xi)}{\sum_{i=1}^{s} f_i(\xi)} d\xi d\tau \tag{2.25}$$

where $h_p(\cdot)$ represents the impulse function of the process. Let the impulse response can be separated as

$$h_p(t-\tau) = h_1(t)h_2(\tau) \tag{2.26}$$

Then the change in error $c = t_s\dot{e}$ is

$$c = t_s[J_m\dot{r} - \dot{h}_1\int_0^t h_2(\tau)u(\tau)d\tau - h_p u]. \tag{2.27}$$

It is required that the update must satisfy $ec < 0$ in some region for asymptotic stability.

2.4 A Rulebase for Asymptotic Stability

Let $E(x,t)$ be a candidate energy function then Lyapunov's direct approach can be stated as follows: If $E(x,t)$ is bounded and its derivative $\dot{E}$ exists, the following conditions should hold for asymptotic stability [9]:

$$E(x,t) > 0 \quad \forall x \neq 0 \tag{2.28}$$

$$\dot{E}(x,t) < 0. \tag{2.29}$$

The Lyapunov function candidate is defined by (2.30),

$$E(e,t) = e^T P e > 0 \tag{2.30}$$

$$\dot{E}(e,t) = \frac{1}{t_s}(c^T P e + e^T P c) \tag{2.31}$$

where P is a symmetric positive definite matrix. From (2.31), it is obvious that the signs of e and c should be opposite in order to satisfy $\dot{E}(e,t) < 0$. One possible c may be described by $c = De$ where D is a negative definite matrix.

Assumption 2.1 *Assume that the sign of the dc gain in the transfer function of a linear system is known (i.e., positive).*

The above assumption is minimal in the sense that the degree of the transfer function, relative degree, and exact system delay are not required as in the cases of analytical methods such as linear quadratic regulator (LQR), model reference adaptive control (MRAC), etc.

The following are 9 rules which guarantee global asymptotic convergence in compliance with Lyapunov stability. Here, we use abbreviations for various linguistic descriptions: 'PL' for 'positive large', 'PS' for 'positive small', 'SZ' for 'small near zero', 'NS' for 'negative small', and 'NL' for 'negative large'. Membership functions for e, c, and δu using S, Z, and Π curves are shown in Figure 2.2. A priori information about the process is necessary such as the trends in the impulse response, the approximate time delay, etc. The following is a PD rule set for the fuzzy rulebased controller.

1. if (e is PL) and (c is PL) then (δu is PL)
2. if (e is PL) and (c is SZ) then (δu is PS)
3. if (e is PL) and (c is NL) then (δu is SZ)
4. if (e is SZ) and (c is PL) then (δu is PS)
5. if (e is SZ) and (c is SZ) then (δu is SZ)
6. if (e is SZ) and (c is NL) then (δu is NS)
7. if (e is NL) and (c is PL) then (δu is SZ)
8. if (e is NL) and (c is SZ) then (δu is NS)
9. if (e is NL) and (c is NL) then (δu is NL)

The above rule set represents a PD control scheme for asymptotic stability. Rules 3,5,7 obey Lyapunov stability conditions; Rules 1,4,6,9 satisfy the model-following property of the Jacobian system J_P; and the component of $\delta u(t)$ contributed by the remaining rules 2,8 is chosen to reduce tracking errors in such a way that the process output $y_p(t)$ follows the reference model output $y_m(t)$ which is closely related to the above assumption. In the case of PID control, $i(t)$ is added to the condition part making the size of the rulebase larger. By adding i to the rule set, it is possible to increase the rate of convergence.

Theorem 2.2 *Let the sign of the dc gain in the transfer function and the Jacobian J_P of a process be known (dc gain>0, $J_P > 0$) and let the suggested guidelines be satisfied for the chosen membership functions of the linguistic variables. Let the PD rule set satisfies the following relations:*

- *case 1: if $ec < 0$ then δu is zero. (Lyapunov stability)*
- *case 2: if $ec \geq 0$ ($e \neq 0$) then $\delta u = J_P c$ with $J_P > 0$. (the model-following property)*
- *case 3: if $ec = 0$ ($e \neq 0$) then δu has the same sign as e. (known dc gain of the process)*

Then asymptotic stability is achieved and the convergence of the tracking error is guaranteed.

__Proof__: (case1) $ec < 0$ — The vector field of (e,c) directs the state trajectory toward the origin. (case 2) $ec \geq 0$ — From the model-following property, if $\delta y_m \to 0$ then $\delta y_p \to 0$ since $J_H > 0$. Thus, the system error converges to some nominal value. (case 3) In this case, $c = 0$ and the error biases. Therefore, c needs to be adjusted according to the dc gain of the process.

It is noted that, if the sign of J_P is positive, then $J_P > 0$ is a sufficient condition for the model-following property. Figure 2.3 shows the state trajectory and the regions by the 9 rules in a typical case.

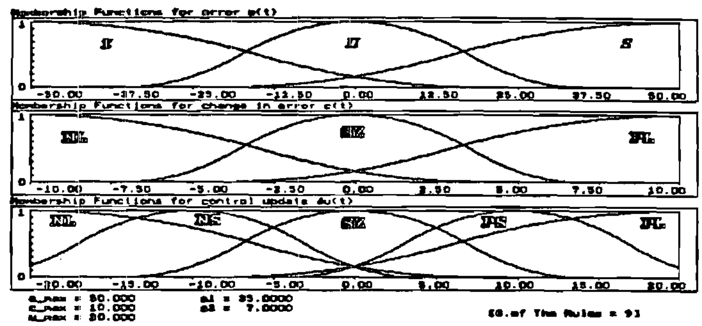

Figure 2.2: The membership functions of e, c, and δu

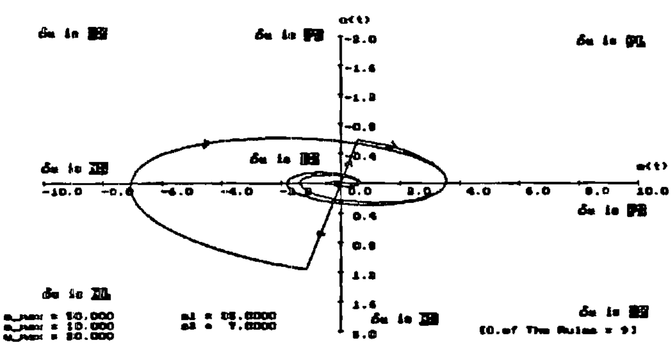

Figure 2.3: The regions and the state trajectory by the 9 rules

3 Simulation Results

A 2nd order single-input single-output (SISO) plant transfer function $H_p(s)$ and the unmodeled version of its transfer function $H_{up}(s)$ are chosen to show robustness of the proposed fuzzy controller,

$$H_p(s) = \frac{2}{s+2} \tag{3.1}$$

$$H_{up}(s) = \frac{40}{(s+2)(s+15)} \tag{3.2}$$

where $H_{up}(s)$ contains unmodeled dynamics of $H_p(s)$ multiplied by $\frac{20}{s+15}$. A reference model $H_m(s)$ is represented by

$$H_m(s) = \frac{3}{s+3}. \tag{3.3}$$

A reference input signal $r(t)$ is of a staircase type: $r(t) = 10$ at $t = 1$ sec, $r(t) = 20$ at $t = 5$ sec, and then $r(t) = 0$ at $t = 9$ sec.

In the fuzzy rulebased controller, the parameters of the membership functions are the maximum values in the region of interest such as $e_{max}, c_{max}, i_{max}$, and u_{max}; mean values of linguistic variables; the sizes of the feasible sets are related to s_e, s_c, and s_i. (s_e is the size of the feasible region between $\mu_{A_i}(x) = 1$ and $\mu_{A_i}(x) = 0.5$ in S, Π, Z functions)

- **case1: unmodeled dynamics**
  Using the above $H_{up}(s)$ as a process and $H_m(s)$ as a reference model, the fuzzy rulebased controller is applied to a tracking problem and the size of the rulebase varies from 9 to 12 to 25 or finally 27 rules. As the number of rules increases, the convergence rate improves as shown in Figure 3.1. The integral term $i(t)$ is included when the number of rules is 27. Subsequent cases are results of 27 rules with unmodeled dynamics. If the sign of the dc gain is negative, the rules should be changed in order to meet asymptotic stability of the closed-loop system.

- **case2: bounded disturbances**
  A uniform additive noise is considered to be corrupting the process output $y_p(t)$ and it is bounded between -0.5 and 0.5 as shown in Figure 3.2. Simulation results show robustness for bounded output disturbances.

- **case3: parameter variations**
  The poles of the process are changed from $t = 3$ sec to $t = 7$ sec and the transfer function $H'_{up}(s)$ in that case is described by

  $$H'_{up}(s) = \frac{60}{(s+4)(s+20)}.$$

  A change in the error dynamics and no significant changes in the process output are observed in Figure 3.3.

- **case4: unstable system**
  The process transfer function becomes unstable and is given by

  $$H_{up}(s) = \frac{4}{s-2}.$$

  Note that the control input $u(t)$ is flipped over in Figure 3.4. Since the steady-state values in the closed-loop system exist, it is possible for the fuzzy controller to find a control input satisfying the stability property.

4 Conclusions and Discussion

Simulation results indicate that the proposed fuzzy control scheme exhibits robustness and stability properties with respect to parametric disturbances, bounded noise, unstable systems, and unmodeled dynamics. The design parameters of the fuzzy controller are less sensitive than those of MRACs or LQRs.

Self-organizing techniques are required for the autonomous addition and deletion of rules and changes in parameters according to some strategic performance indices. As shown in the simulation results, corrective or learning schemes refine the structure of rulebased controllers. In the case of fuzzy relational matrices, some techniques are suggested in [10,11]. Usually, the size of the rulebase increases after a number of iterations of the learning process.

In order to apply the fuzzy linguistic system, expert knowledge is needed for a special domain application. Use of performance criteria in designing a rulebased controller is recommended and it is suggested to describe the plant dynamics as fully and completely as possible.

Some advantages and disadvantages of the rulebased control approach include [12]: Advantages are robustness compared with analytic methods, flexibility in modification of the domain, and improvement in the man-machine interface by providing reasoning behind a particular control decision. Disadvantages are ambiguity in completeness of the rulebase, difficulties in updating and calibrating the rulebased controllers, exhaustive use of the linguistic rules, and superfluous memory requirements.

In general, fuzzy systems need more samples for the decision making process compared with analytical methods but handle unreliable information by using linguistic variables and fuzzy relations. A fuzzy controller can be implemented by a parallel computer architecture since the rules are disjunctive.

5 Appendix: Mathematical Properties and Notations of Fuzzy Sets

Some definitions relating to fuzzy sets and fuzzy numbers are as follows [4]:

Definition 5.1 *An ordinary subset, A, of a referential set, $\mathcal{E}$, is defined by its 'characteristic function'* $\mu_A(x) \in \{0,1\}$, $\forall x \in \mathcal{E}$.

A feasible set $F(Y)$ may be described by a characteristic function. A linguistic description in Y such as 'large' resides in a feasible subset $F(Y) \in Y$.

Definition 5.2 *A fuzzy subset, A, from the referential set, $\mathcal{E}$ is defined by its characteristic function $\mu_A(x) \in [0,1]$, $\forall x \in \mathcal{E}$, which is called the 'membership function'.*

A membership function may represent a non-uniform possibility distribution for the associated linguistic variable described by the characteristic function. The next two definitions are necessary and sufficient conditions for good membership functions in general. A fuzzy set is an invariant set as the presumption level increases.

Definition 5.3 *A fuzzy subset A on $\mathcal{X}$ is 'convex' if and only if $\forall x_1, x_2 \in \mathcal{R}, \forall \lambda \in [0,1]$:*

$$\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \mu_A(x_1) \wedge \mu_A(x_2).$$

Definition 5.4 *A fuzzy subset A on $\mathcal{X}$ is 'normal' if and only if $\forall x \in \mathcal{R} : \max_x \mu_A(x) = 1$.*

The formulas of some fuzzy number arithmetic operators are defined as follows [3,13]:

Definition 5.5 *A 'fuzzy implication' is a mapping $S : A_i \Longrightarrow B_i$ and the membership function for S is $\mu_S(y,x) = \min[\mu_{A_i}(x), \mu_{B_i}(y)]$ where $x \in X$ and $y \in Y$.*

Definition 5.6 *The 'compositional rule of inference' is as follows: for a given fuzzy implication $S : A_i \Longrightarrow B_i$, the fuzzy set B'_i inferred by a given fuzzy set A'_i has the membership function defined by $\mu_{B'_i}(y) = \max_x \min[\mu_{A'_i}(x), \mu_S(y,x)]$.*

Given the following mappings:

$$\mathcal{X} = \{(x, \mu(x))\};\ \ \mu : X \longrightarrow [0,1];\ \ x \in X$$

$$\mathcal{Y} = \{(y, \mu(y))\};\ \ \mu : Y \longrightarrow [0,1];\ \ y \in Y$$

$$R : \mathcal{X} \longrightarrow \mathcal{Y}$$

where $\mathcal{X}$ is a family of fuzzy sets defined on X, $\mathcal{Y}$ is a family of fuzzy sets defined on Y, and R is a fuzzy relation which can be interpreted as a mapping with its domain in $\mathcal{X}$ and a space of values constrained in $\mathcal{Y}$. Let $U \in \mathcal{X}$ and $V \in \mathcal{Y}$ then a formal fuzziness system is defined as

$$\mathbb{U} \circ \mathbb{R} = \mathbb{V}. \tag{5.1}$$

Some definitions and mathematical properties of the formal fuzziness system are shown as follows [8,14]:

Definition 5.7 *The formal fuzziness system (5.1) has α-stability property with respect to some family $\mathcal{A}$ and some feasible subset $F(Y) \in Y$ if $\mu_V(y) \leq \alpha$ $\forall y \in Y - F(Y)$ for every fuzzy set $U \in \mathcal{A}$ of $U \circ R = V(\ast)$ where $\circ$ is the operator for the compositional rule of inference.*

Theorem 5.1 *Consider the system where $R = A \Rightarrow B$ is given. Then $\bigvee_{i=1}^{n} \mu_A(x_i) \leq \alpha$ or $\mu_B(y_j) \leq \alpha$ for all $y_j \in Y - F(Y)$ iff the system (5.1) has the α-stability property with respect to a family $\mathcal{A}$.*

Definition 5.8 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given. Then R has the good mapping (GM) property iff $A_i \circ R = B_i$ for every $i = 1, \ldots, s$.*

Theorem 5.2 *Let $F(Y) \in Y$ be the feasible set. Then the system (5.1) with the relation defined above has the α-stability property with respect to a family $\mathcal{X}$ of all fuzzy sets defined on X if $\bigvee_{j=1}^{s} \mu_{B_j}(y) \leq \alpha$, $\forall y \in Y - F(Y)$.*

Theorem 5.3 *Let $A_1, \cdots, A_s$ be fuzzy sets defined on X, $\Sigma_{i=1}^{s} \mu_{A_i}(x) = 1$ for any $x \in X$, and let $B_1, \cdots, B_s$ be fuzzy sets defined on Y. Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$. Then $\mu_{A_k \circ R}(y_j) = \mu_{B_k}(y_j)$ if for some $k \in \{1, \cdots, s\}$ and some $j \in \{1, \cdots, m\}$ we have $\mu_{B_k}(y_j) \leq \max_X \mu_{A_k}(x)$ and $\mu_{B_k}(y_j) \geq \mu_{B_l}(y_j), l \neq k$.*

Definition 5.9 *A system (5.1) has the α-decision stability property with respect to some family $\mathcal{A}$ of fuzzy sets defined on X if the system has α-stability with respect to $\mathcal{A}$ and $\max_{Y-F(Y)} \mu_{U \circ R}(y) < \max_{F(Y)} \mu_{U \circ R}(y)$.*

Definition 5.10 *A system (5.1) has the $\alpha - \beta$ strong decision stability property with respect to some family $\mathcal{A}$ iff the system (5.1) has the α-decision stability property with respect to $\mathcal{A}$ and $\bigvee_{y \in Y} \mu_{U \circ R}(y) \geq \beta > \alpha$.*

Definition 5.11 *Let $R = (A_1 \Rightarrow B_1) \vee \cdots \vee (A_s \Rightarrow B_s)$ be given where $A_i \in \mathcal{X}$ and $B_i \in \mathcal{Y}$ for $i = 1, \cdots, s$. Then a fuzzy relation R has the γ-good mapping (γ-GM) property in the weaker sense iff $A_i \circ R = \bar{B}_i$ for every $i = 1, \ldots, s$ where $B_i \subseteq \bar{B}_i$, and $\mu_{\bar{B}_i} = \mu_{B_i}$ for $\mu_{B_i} \geq \gamma, 0 \leq \gamma \leq 1$.*

Note that, when $\gamma = 0$, the γ-GM property reduces to the GM property as defined above [14].

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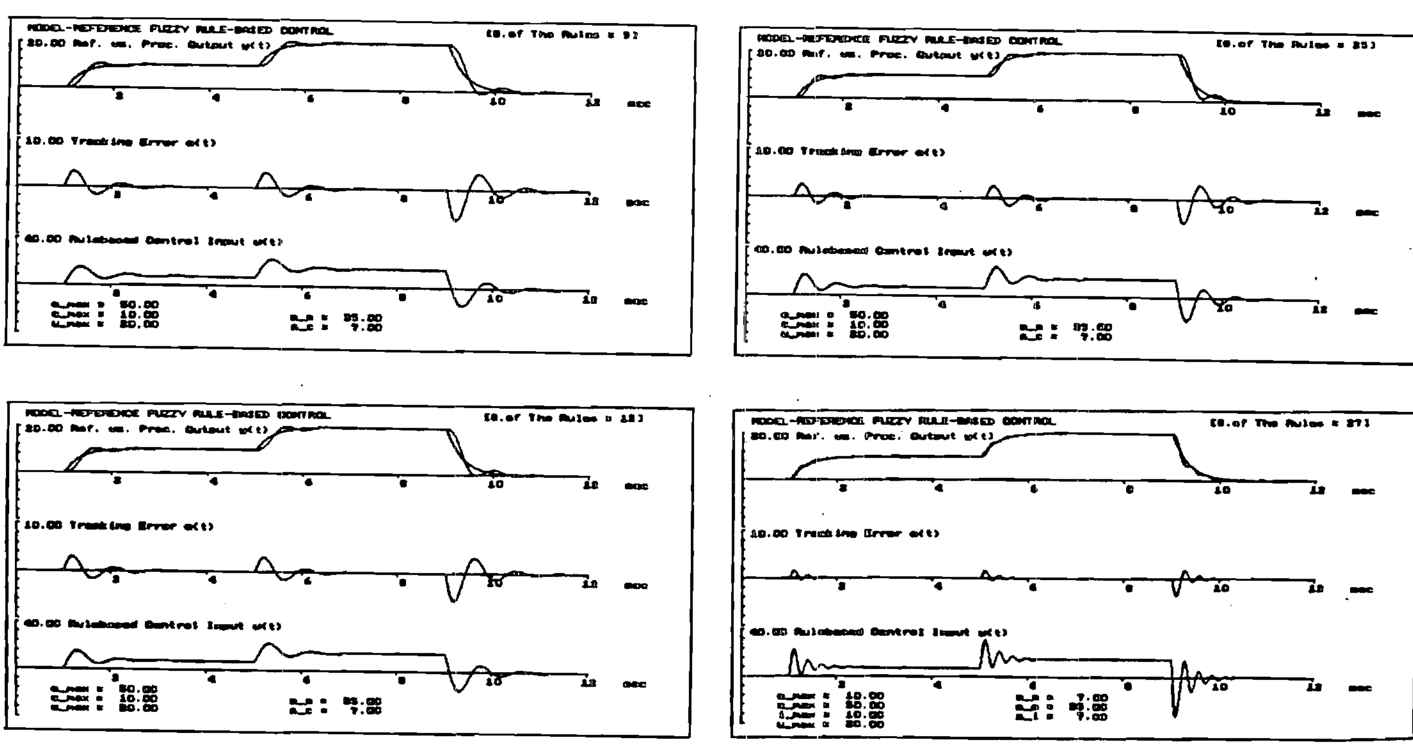

Figure 3.1: simulation of unmodeled dynamics when the number of rules changes

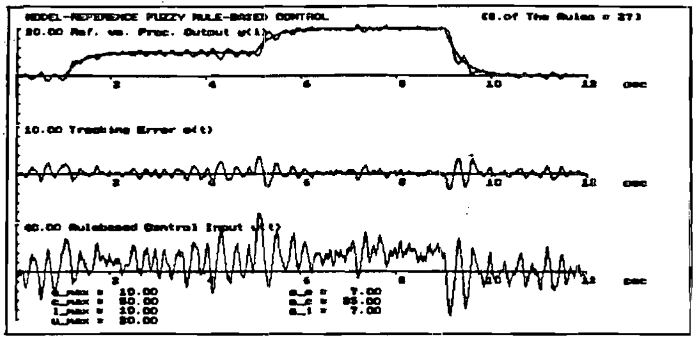

Figure 3.2: simulation of a bounded disturbance

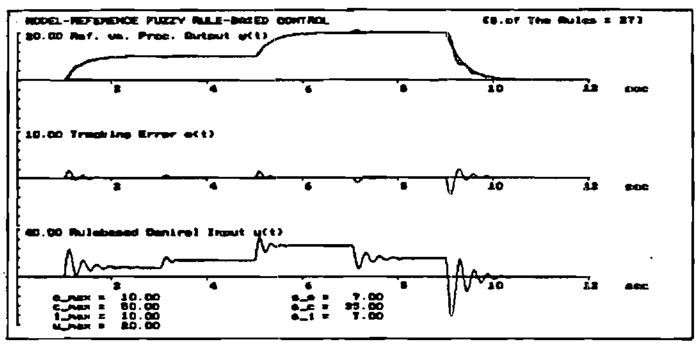

Figure 3.3: simulation of a parametric disturbance

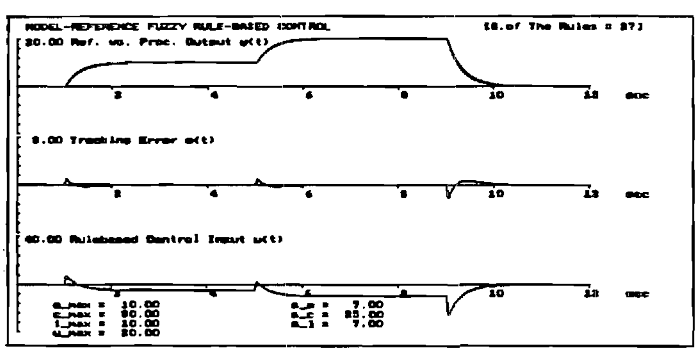

Figure 3.4: simulation of an unstable system

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Annual Letter Report for FY 90

A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design

(Contract No: N00014-89-J-3113)

Sponsor: **Office of Naval Research**
Applied Research and Technology Directorate

Principal Investigator: **Dr. George Vachtsevanos**
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Tel: (404)894-6252

September 15, 1990

A HYBRID ANAYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN

CONTRACT NO: N00014-89-J-3113

PI: Dr. George Vachtsevanos
Georgia Institute of Technology

In September 1989, the Office of Naval Research awarded contract No. N00014-89- J-3113 to the Georgia Institute of Technology to develop fault-tolerant control strategies for large scale dynamical systems. Specifically, the technically issues under investigation are: Development of a structure-based modeling methodology of large scale systems which processed features of structure flexibility, i.e. a component or subsystem may be deleted in a simple way that does not substantially disturb the global control strategy; development of robust failure detection and fault identifiaction algorithms for single and multiple faults that are maximally sensitive to true failure conditions but insensitive to noise thus reducing the possibility of false alarms; a technique to restructure the system dynamics by isolating the faulty components; a method that will lead to a structural control law which reconfigures the original system controller in order to meet the primary performance objective of guarantee stability while the system is operating in a degraded mode; finally, these algorithmic developments are to be demonstrated on an actual physical system to be designated jointly by ONR and Georgia Tech.

The technical approach pursued to meet these objectives is outlined as follows: The underlying factor of the fault-tolerant methodology capitalized upon structural features of the large scale system and employes a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. In the modeling area, the topological description of many complex dynamical processes may be cast in the form of structurally interconnected subsystems. The large scale system is composed of a number of linearly interconnected subsystems. In every subsystem, a control law is applied which consists of a local and a global feedback term, thus resulting in a two-level hierarchical control strategy. We examine first large scale systems which are described by linear state equations and exhibit this two-level hierarchical structure. The analysis will be extended eventually to nonlinear systems of a similar structure. A methodology for fault diagnosis is introduced which is based upon a combination of signal redunancy and detection/estimation procedures. An expert system, consisting of a multivalued rule base and an appropriate inferencing mechanism, assesses the aggregate of fault symptoms and determines the sensitivity of a failure condition. An innovative approach to multiple failure detection uses qualitative simulation techniques to decide on the impact of a component failure on other (healthy) system components. The proposed fault detection and identification scheme incorporates such additional functionalities as fault trending and the estimation of the "best" value of critical variables and parameters under failure states. Assume that the presence of a failure has been detected by the FDI algorithm and means for isolating the faulty or potentially faulty components are available, then the system restructuring function is undertaken by a supervisory controller which modifies the original (healthy state) description of the system by changing the appropriate entries of the local subsystem and interconnection matrices.

Finally, we propose the development of a two-level structural dynamic hierarchical approach to address the control reconfiguration problem of a faulted large scale system. The approach uses a structural state model, the Block Arrow Structure, which consists of N independent linear subsystems interconnected with a control common linear dynamic system. The objective is to find a time-invariant decentralized feedback controller which minimizes a quadratic cost functional and has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

A HYBRID ANAYTICAL/INTELLIGENT
APPROACH TO FAULT-TOLERANT
CONTROL SYSTEM DESIGN
CONTRACT NO: N00014-89-J-3113
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Georgia Institute of Technology

Research Accomplishments

During this reporting period 9/1/89 - 9/1/90, the research team focused attention on the issues of modeling of dynamical Large Scale Systems(LSS), the mechanisms of generation and propagation of faults, advanced fault detection and identification algorithms, and the application of these techniques to such critical processes as aerodynamic instabilities in axial flow compressors. A major accomplishment of this research effort related to the development of efficient "failure" or "impending failure" detection methods for compressor surge and stall instabilities which, as demonstrated via computer simulations, account for system uncertainties and incomplete data (ignorance). This hybrid analytical/intelligent approach to failure identification exhibits maximum sensitivity to failure modes while minimizing false alarms. It integrates analytical residuals generation aspects with decision processes which manage efficiently uncertainty and ignorance using fuzzy logic and Dempster-Shafer theory. It is projected to be the crucial component in a hierarchical adaptive/active control scheme for advanced jet engines.

1 LSS modeling

In the modeling area, a review of available techniques was completed and such schemes as aggregation and perturbation analysis, hierarchical and decentralized approaches and the descriptive variable method were assessed as to their suitability in fault tolerant control. The basic objective here is to develop mathematical modeling tools for large scale systems that possess attributes of modularity and structural flexibility so that a failed component or subsystem may be easily isolated without seriously affecting the global behavior of the system. More specifically, the failed system to be restructured and the control actions reconfigured so that stability(survivability) is assured for the duration of the emergency.

We are developing a two-level structural dynamic hierarchical approach to address the control reconfiguration problem. We seek a structural control law and, by effectively combining information with control action, to move closer to create a structural intelligent control concept. We outline below the innovative features of this approach.

Suppose that the large scale system consists of l interconnected nonlinear subsystems of the form

$$\dot{x}_i = f_i(x_i, t) + g_i(x_1, \ldots, x_l, t) \qquad i = 1, 2, \ldots, l \tag{1}$$

where

$x_i \in R^{n_i}$ is the state vector of the ith subsystem

$f_i : R^{n_1} \times J \rightarrow R^{n_1}$,

$$g_i : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_l} \times J \rightarrow R^{n_i}$$

Each subsystem, when isolated, is given by

$$x_i = f_i(x_i, t) \tag{2}$$

The function $g_i(x_1, \ldots, x_l, t)$ represents the interconnections of the ith subsystem with the remaining subsystems.

In every subsystem of the form (1), a control law is applied which consists of a local and a global feedback term. Specifically, for the ith subsystem (1), the control input is

$$u_i(t) = u_i^l(t) + u_i^g(t) \tag{3}$$

If the controllers are linear, then they have the form:

$$u_i^l(t) = k_{ii} x_i(t) \tag{4}$$

$$u_i^g(t) = \sum_{j=1, j \neq i}^{l} k_{ij} x_j(t) \tag{5}$$

where $k_{ij} \in R^{m_i \times m_j}$ and $u_i(t) \in R^{m_i}$.

A two-level hierarchical control is obtained by assigning proper interconnections and feedback loops to the large scale system. At the lower level we have l subsystems, S_i, $i = 1, 2, \ldots, l$, which are described in state space by the equations:

$$S_i : \dot{x}_i(t) = f_i(x_i, t) + h_i(u_i, t) + f_{i0}(x_0), \quad i = 1, 2, \ldots, l \tag{6}$$

where

$$h_i : R^{m_i} \times J \rightarrow R^{n_i}$$

$$f_{i0} : R^{n_0} \times J \rightarrow R^{n_i}$$

and $x_0 \in R_{n0}$ is the state vector of the coordinator system in the upper level. The upper level is occupied by the coordinator system S_0 described by the state space model:

$$S_0 : \dot{x}_0(t) = f_0(x_0, t) + \sum_{i=1}^{l} f_{0i}(x_i, t) + h_0(u_0, t) \tag{7}$$

where $u_0(t)$ is the control law of the coordinator and

$$h_0 : R^{m_0} \times J \rightarrow R^{n_0}$$

The structural implications of the two-level model on fault-tolerant control become evident when we consider a linearized version of the system dynamics. Here, we use a structural state model which consists of l independent linear subsystems $S_1, S_2, \dots, S_l$ interconnected with a control common linear dynamic subsystem S_0. The mathematical model of the time- invariant large scale system, in a decomposed form, is

$$S_i : \dot{x}(t) = A_i x_i(t) + B_i u_i(t) + A_{i0} x_0(t) \tag{8}$$

$$\text{with} \qquad x_i(t_0) = x_{i0}, \quad x_0(t_0) = x_{0_0}$$

$$S_0 : \dot{x}_0(t) = A_0 x_0(t) + B_0 u_0(t) + \sum_{i=1}^{N} A_{0i} x_i(t) \tag{9}$$

where $x_i(t) \in R^{n_i}, x_0(t) \in R^{n_0}$ and $u_i(t) \in R^{r_i}, u_0(t) \in R^{r_0}$ are the state and control vectors for subsystems S_i and S_0, respectively.

The standard overall description of (8),(9) is

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(t_0) = x_0 \tag{10}$$

$$A = \begin{bmatrix} A_1 & 0 & \dots & A_{10} \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & A_l & A_{l0} \\ A_{01} & \cdots & A_{0l} & A_0 \end{bmatrix} \in R^{n \times n}, \qquad B = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & B_l & 0 \\ 0 & & 0 & B_0 \end{bmatrix} \in R^{n \times r}$$

Since the dynamics matrix (10) consists of Block elements arranged in an Arrow Structure, the system (10) is called a Block Arrow Structure(BAS) decentralized large scale system.

By appropriately defining a quadratic cost functional, a time-invariant BAS decentralized feedback controller may be designed which has the features of (1)utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.

For a class of large scale systems which do not conform to the BAS structure, we propose to exploit the property of Block Triagular Structure (BTS).
For convenience, consider a linear LSS described by

$$\dot{X}(t) = AX(t) + BU(t) \tag{11}$$

Assuming that the LSS consists of l interconnected subsystems as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j=1, i \neq j}^{j=l} A_{ij} x_j(t) \tag{12}$$

and the control input is

$$u_i(t) = u_i^l(t) + u_i^g(t) \tag{13}$$

From (12) and (13), the ith subsystem is rewritten as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i^l(t) + \sum_{i \neq j, j=1}^{j=l} [A_{ij} x_j(t) + B_i u_i^g(t)] \tag{14}$$

The LSS and its ith subsystem are described in closed form as

$$\dot{X}(t) = A^c X(t) \tag{15}$$

$$\dot{x}_i(t) = A_i^c x_i(t) + \sum_{j=1, i \neq j}^{j=l} A_{ij}^c x_j(t) \tag{16}$$

where

$$A^c = \begin{bmatrix} A_1^c & A_{12}^c & & \cdots & A_{1l}^c \\ A_{21}^c & A_2^c & & & \\ \vdots & & \ddots & & \\ & & & & \\ A_{l1}^c & & & & A_l^c \end{bmatrix} \tag{17}$$

It is difficult to check the stability of the LSS (A^c) since, in general, the dimension of A^c is relatively large. We are investigating an alternative approach to verify the system stability via the stability of isolated subsystems and interconnections. The LSS of a BTS structure is stable, if all free subsystems are stable. In order to take advantage of this property of BTS, we propose the addition of a global controller to change the system structure to BTS. With the additional global controller, A^c is transformed to $\bar{A}^c$ where

$$\bar{A}^c = \begin{bmatrix} A_1^c & A_{12}^c & & \cdots & A_{1l}^c \\ 0 & & & & \\ \vdots & & \ddots & & \\ & & & & \\ 0 & & & 0 & A_l^c \end{bmatrix} \tag{18}$$

is stable if A_i^c is stabilized via local controllers, $u_i^l(t)$. The block triangular structured LSS has the features of (1)utilizing the interconnections, (2) structure flexibility.

We also propose to incorporate hierarchically an intelligent control module for a fast control input decision making. We utilize a blackboard architecture for the intelligent control module. It is intended to modify the control objectives of the local subsystems based on the information of system status and the desired (if possible) mission plan.

We are currently investigating critical properties of LSS, from the fault-tolerant control point of view, that include restructurability and reconfigurability. Specifically, we have identified the following key properties for the restructurable/reconfigurable systems:

- Flexibility: based upon the strength of subsystem interconnections (interlevel or intralevel).
- Stabilizability: in terms of structural controllability and structural observability concepts.

Our investigations thus far indicate that the modeling approach pursued is representative of a large class of engineered systems that, because of critical performance requirements, require some degree of fault tolerance in the control design.

2 Fault Detection and Identification

A major effort has been undertaken, during this first year of the project, to develop an integrated methodology for fault detection and identification that capitalizes upon a combination of conventional techniques and artificial intelligence. New and innovative concepts are introduced in the FDI approach that maximize symptomatic evidence, account for sensor and system uncertainties and combine failure data even when the latter are of a conflicting or incomplete nature.

FDI techniques are considered for sensor, actuator and component failures. Both single and multiple faults are treated. The research objective is to develop a systematic and thorough FDI procedure that is maximally sensitive to failures while avoiding false alarms. The approach is systematic because it relies on a modular architecture to (a) trigger efficiently the FDI routines, (b) validate sensor data, (c) combine failure evidence from such diverse sources as analytic redundancy, dectection/estimation theory and limit checking, (d) utilize expert system tools and Dempster-Shafer evidential theory to manage uncertainty and assess the symptomatic evidence, i.e. detect and identify faulty components, and (e) finally, provide trending information as well as the best value of critical variables and parameters for monitoring and control purposes.

This research team has previously developed FDI procedures based upon parity space and utilizing analytic redundancy. We will highlight below only recent developments in detection/estimation and evidential theory. Symptoms derived from this technique will augment the failure evidence available from parity space procedure to assure a robust FDI.

2.1 Failure Model via Recursive Parameter Estimation

A failure mode is defined as a one-to-one mapping relationship with a failure symptom such as 'the impulse line is leaking' or 'the motor inertia is very large'. In fact, the failure mode is directly related to the system parameters and can be additively incorporated in a failure model. Therefore, we can easily interpret a failure mode as a physical representation of such a failure while a failure signature is defined as a time history of each failure mode. The actual system may be described by a state space representation in terms of discrete nonlinear vector equations of the form,

$$x(t+1) = f(x(t), \theta(t), u(t)) \qquad z(t) = h(x(t), \theta(t)) \tag{19}$$

where $u(t) \in U$ and $z(t) \in Z$ are the scalar system input and output, respectively; $\theta(t) \in \Theta$ is the time-varying system parameter vector; $x(t) \in X$ denotes the state vector; and f, h represent the nonlinear vector fields for the state and output vectors, respectively. A nonlinear model can be obtained for a class of nonlinear systems where the corresponding equations are linear in the parameter vector $\theta(t) \in R^n$ such as

$$z(t) = \phi(t, \bar{\theta})^T \theta(t) \tag{20}$$

where $\bar{\theta}$ is the nominal solution for the parameters during the time window $J_T = [t_0, t_0 + T]$ and $\phi(t, \bar{\theta}) \in R^{p \times n}$ is the nonlinear regressor associated with $z(t-i)$ for $i = 1 \ldots r_1$ and $u(t-d-j)$ for $j = 0 \ldots r_2 - 1$ (d is an integer designating the system delay and $n = r_1 + r_2$). In the linear case, it is easy to derive the above relation from a auto-regressive moving-average (ARMA) model. The failure model can be represented by

$$\theta(t+1) = \theta(t) + \omega(t) + \xi(t) \tag{21}$$

$$z(t) = \phi(t)^T \theta(t) + v(t) + \eta(t) \tag{22}$$

where $\omega(t) \in R^n$ and $v(t) \in R^p$ are white gaussian parametric and measurement disturbances, respectively; $\phi(t) = \phi(t, \bar{\theta}) \in R^n$ is the regression vector; $\xi(t) \in R^n$ is the system + actuator failure signature while $\eta(t) \in R^p$ represents the sensor failure signature; For the initial conditions, it is required that $\theta_0 = E[\theta(0)]$ and $P_0 = Cov[\theta(0) - \bar{\theta}]$. Let the parameter residue be $\tilde{\theta}(t) \in \tilde{\Theta}$.

$$\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta} \tag{23}$$

where $\hat{\theta}(t)$ is the estimate of the parameter $\theta(t)$. The parameter residues are quantized in a finite number of bins, q_i, with respect to the i-th element of $\tilde{\theta}(t)$ in order to generate a histogram of the parameter residues by counting the frequency of each bin $\bar{\theta}_k$, $f^i = \{f^i_k\} \in Q_i$ for $k = 1 \ldots q_i$ and for $i = 1 \ldots n$, and $\bar{\theta}_k$ is the bin symbol with total q_i bins. Here, we call the vector $f^i \in R^{q_i}$ the failure signature histogram (FSH) of the i-th system parameter. The time-window function W_T is introduced such that

$$W_T : \tilde{\Theta} \Longrightarrow Q. \tag{24}$$

Note that selection of the design constants is important to the performance of the hybrid FDI scheme. In the normal case, the failure signatures are

$$\xi(t) = 0, \eta(t) = 0.$$

Normal Mode Operation: When $\xi(t) = 0$ and $\eta(t) = 0$, the recursive formula is obtained from the Kalman filter interpretation of equation (21) -(22).

Failure Mode Operation: Let us assume that either one of $\xi(t)$ or one of $\eta(t)$ is nonzero and let $\xi(t) \neq 0$ for simplicity. The steady state assumption assures that

$$\hat{\theta}(t) \Longrightarrow \bar{\theta} \quad as \quad t \Longrightarrow \infty$$

but if the failure occurs $t = t_f < T$,

$$\hat{\theta}(t) \Longrightarrow \bar{\theta} + \sum_{\tau = t_f}^{T} \xi(\tau) \quad as \quad t \Longrightarrow T.$$

Hybrid FDI

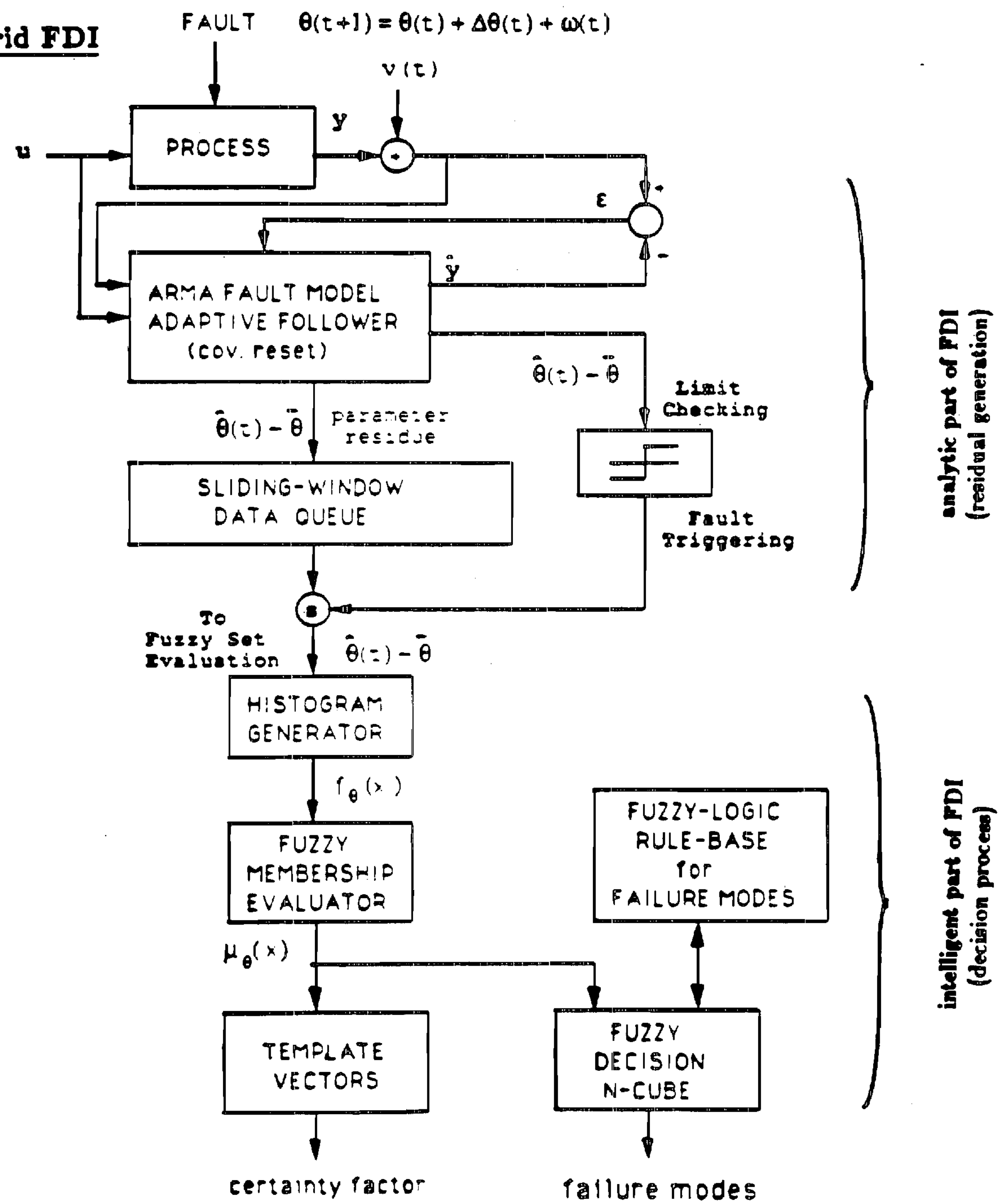


Figure 1: Block Diagram of Proposed FDI Scheme

Therefore, the parameter residue $\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta}$ plays an important role in representing the unique failure signature by the certainty equivalence principle. This failure signature can be obtained by the windowing function W_T that stores the residue $\tilde{\theta}(t)$ in J_T into the histogram f^{θ}. Note that $\xi(t)$ of the impulse function shows a biased failure signature in the parameter while $\xi(t)$ of the step function corresponds to a parameter drift. The block diagram of the FDI scheme is shown in Figure 1.

2.2 Failure Identification Using Fuzzy Decision Hypercubes

We present a new failure identification method using the concept of a rulebase implemented by a multi-dimensional decision making system called 'Fuzzy Decision Hyper-Cube' (FDHC). Now, let us consider the FSH for only one parameter and let the number of the associated failure mode be l. Given a FSH f^{θ}, there exists a transformation π from the probabilities to the corresponding fuzzy sets μ^{θ} which is based on possibility theory. A numerical evaluation is given via the formula

$$\pi(\{\theta_k\}) = \sum_{j=1}^{q} min\{f_k^{\theta}, f_j^{\theta}\} \tag{25}$$

where f_k^{θ} is sorted according to the frequency of each bin θ_k with the condition $\sum_{k=1}^{q} f_k^{\theta} = 1$. The possibility transformation π provides the maximum probability of the parameter residue by a mapping

$$\pi : Q \Longrightarrow I = [0,1]^m \tag{26}$$

where we define $\mu_k^{\theta} = \pi(\{\theta_k\})$ and m is the number of quantization. The fuzzy vector μ^{θ} is stored as a fuzzy input vector to FDHC. Thus, each failure mode can be represented uniquely by the failure signature fuzzy set (FSF). We also define a priori failure mode fuzzy sets (FMF) which correspond to each crisp failure mode. The procedure requires two transformations

$$\tilde{\Theta} \xrightarrow{W_T} Q[FSH] \xrightarrow{\pi} I[FSF] \tag{27}$$

The FDHC is constructed along each rule on the basis of a priori information about FSF and FMF. In general, the j-th rule is described by a fuzzy implication whose premise consists of conjunctive fuzzy sets.

Matching Process of FDHC: The input-output relationship of FDHC can be written as

$$B = \Sigma \circ A \tag{28}$$

where '$\circ$' denotes the max-min operation and $A \subset I$ is the FSF acquired from the FSH f^{θ}. In brief, the output fuzzy vector for each parameter failure can be expressed by

$$\mu_y^* = \bigvee_z \mu_{y,z}^{\Sigma} \bigwedge \mu_z^{\theta} \tag{29}$$

where $\mu^\theta = \{\mu_z^\theta\}$ is the stored fuzzy set from the actual failure signature which will be applied to the input of the FDHC, and μ_y^* is the output of the FDHC. Therefore, the resultant FMF output y^* excited by FSF vector μ^θ is defuzzified via

$$y^* = \arg \max_y \mu_y^*. \tag{30}$$

Now, consider the FDHC in detail.

2.2.1 Fuzzy Decision Hypercube: An Approximate Learning Tool

A fuzzy variable u in the universe of discourse U represents a quantity with uncertainty. A fuzzy set can be described by

$$A = \{(u, \mu_F(u)) | u \in U\} \tag{31}$$

where μ_F is a membership function defined by

$$\mu_F : U \longrightarrow [0,1]^m \tag{32}$$

where m is the number of quantization. A linguistic variable is a quintuple denoted by

$$V = \{u, L(u), U, Syn, Sem\} \tag{33}$$

where $L(u)$ is the set of the linguistic terms used, and Syn and Sem are the syntactic and the semantic rules, respectively. The degree of possibility are modeled in terms of membership functions for each linguistic rule that represents the relative degree of extent.

An FDHC consists of independent fuzzy condition sets and a crisp action set. Let the number of fuzzy inputs are m then the dimension of a FDHC is $(m+1)$. Each input and output subsets contained in a FDHC can be regarded as a fuzzy symbolic rule. A fuzzified symbolic rule is a fuzzy implication or fuzzy Modus Ponens using the 'if-then' proposition. It is noted that the premise part is 'fuzzy' while the decision part is 'crisp'. In other words, the inputs u's are fuzzy and the output v is crisp in a FDHC. The i-th rule is

rule i: if (u_{i1} is F_{i1}) and $\cdots$ and (u_{im} is F_{im}) then v_i is G_i.

where $L(u_i) = \{F_{i1}, \cdots, F_{im}\}$, F_{ij} is a linguistic term, $A_i = A_{i1} \cap A_{i2} \cap \cdots \cap A_{im}$, $A_{ij} = u_{ij}$ is F_{ij}, and G_i is a crisp decision. Let us denote

$$x_{ij} = \mu_{F_{ij}}(u_j), \qquad y_i = \mu_{G_i}(v)$$

$$x_i = \mu_{F_i(u)} \tag{34}$$

where $x_{ij} \in [0,1]^{m_j}$, $y_i \in [0,1]^p$, and $x_i \in [0,1]^{q_1} \times \cdots \times [0,1]^{q_m}$. Therefore, x_{ij} is a fuzzy vector, y_i is a crisp vector, and x_i is a m-dimensional hypercube. The n rules given by the heuristics or the expertise are stored in a FDHC M_n by the following recursive procedure:

$$\begin{aligned} M_n &= \bigoplus_{i=1}^{n} \{x_i \bigotimes y_i\} \\ &= M_{n-1} \bigoplus \{x_n \bigotimes y_n\} \end{aligned} \tag{35}$$

where $M_n \in [0,1]^{q_1} \times \cdots \times [0,1]^{q_m} \times [0,1]^p$; $\oplus$, $\otimes$ are the maximum and the orthogonal minimum operators, respectively; and the membership function for fuzzy inputs are further decomposed into the m dimensional premise-hypercube such as

$$x_i = \bigotimes_{j=1}^{m} x_{ij}. \tag{36}$$

If we define $x_{ij}[k]$ as the k-th element of the fuzzy vector x_{ij}, the above equation can be described in detail,

$$x_i[k_1, \cdots, k_m] = x_{i1}[k_1] \otimes \cdots \otimes x_{im}[k_m] \tag{37}$$

where $k_1 = 1..q_1, \cdots, k_m = 1..q_m$. During the approximate learning process, selection of the design parameters in fuzzy membership functions requires the following procedure:

(i) Model Setup: simulation model, complex model, experimental model, or heuristic model
(ii) Rigorous Simulations: simulations, expertise, experiments, or heuristics
(iii) Numerical Assignments via Anticipated Results: quantization size and number, multivalued (discretized) levels, and sampling interval

Although we need a meta-level rulebase for learning the changing environment, some guide lines can be substituted for a portion of the dynamic learning algorithm.

2.2.2 Fuzzy Decision Hypercube: An Approximate Reasoning Tool

In this section, a FDHC is described in terms of the roles in approximate reasoning or association. We start with a histogram in which conditional information is submerged. This histogram contains vague and ambiguous signatures about the problem we must solve. Thus, it is important that we choose the appropriate design parameters for membership functions in each rules. Let the vector histogram of the i-th fuzzy set be f_{0i} and its k-th quantization bin be $f_{0i}[k]$, then

$$f_{0i} = \{f_{0i}[1], \cdots, f_{0i}[q_m]\}. \tag{38}$$

The fuzzy sets are evaluated based on the equation given by Dubois and Prade and Yager,

$$x_{0i}[j] = \sum_{k=1}^{q_i} \{ f_{0i}[j] \otimes f_{0i}[k] \} \tag{39}$$

and the fuzzy reasoning mechanism is

$$y = M_n \circ x \tag{40}$$

where x is defined as

$$x = x[k_1, \cdots, k_m] = \otimes_{j=1}^{m} x_{0j}[k_j] \tag{41}$$

and $\circ$ is the max-min operator. Therefore, a crisp decision is made according to

$$v^* = \arg\max_v \mu_G(v) = \arg\max_v y \tag{42}$$

and v^* is the resultant mode.

2.3 Degree Of Certainty via Dempster-Shafer Theory

At the last stage of the FDI algorithm, the decision is made on the basis of a reliability index called "Degree of Certainty" which takes values in the interval [-1,1]. First, a likelihood measure is defined by

$$L_i = \sum_j min(T_{ij}, D_j)$$

where T_{ij} is the jth value of template vector T_i and D_j is the count of the jth interval. The L_i's are normalized since they are subsequently interpreted as possibility distributions. The next step is to reorder the L_i's in decreasing order so that a basic assignment m may be computed on the basis of the Dempster-Shafer theory. The basic assignment can be interpreted as a measure of belief that is committed exactly by its focal element. Let π_i's be the normalized and reordered L_i's such that π_1=1 and $\pi_i \geq \pi_{i+1}$, then $m(A_i) = \pi_i - \pi_{i+1}$, where A_i has the first ith largest mode. For example, if $L_1 = 0.4, L_2 = 1, L_3 = 0.6$ then $\pi_1 = 1, A_1 = x_2, \pi_2 = 0.6, A_2 = x_2x_3, \pi_3 = 0.4, A_3 = x_2x_3x_1$. Thus, $m(x_2) = 0.4, m(x_2x_3) = 0.2, m(x_2x_3x_1) = 0.4$. If we have more than one estimate as a clue for identifying a failure mode, the same number of sets of basic assignments are available. Two sets of basic assignments are combined into one by using Dempster's rule of combination. Finally, we obtain one set of basic assignments after combining all sets. Here we use the concept of 'Degree of Certainty' which is defined as $m(X) - Bel(\bar{X})$ where $Bel(X) = \sum_{Y \in X} m(Y)$. $Bel(\bar{X})$ is the measure of disbelief on X, whereas $m(X)$ is that of belief on X. It is intuitively plausible to choose $m(X) - Bel(\bar{X})$ as an index of certainty because the equality of belief and disbelief produces zero certainty. Therefore, $m(X) - Bel(\bar{X})$ is a proper choice for the 'Degree of Certainty'.

2.4 Simulation Example

A simple linear SISO system is used to illustrate the FDI procedure. Especially, we emphasize a hard failure such as abruptly changing parameters in the time constant or the gain of a d.c. motor. The transfer function $H(s)$ from the armature voltage to the d.c. motor speed can be represented by

$$H(s) = \frac{\kappa}{s+\alpha} \tag{43}$$

where κ, α denote the d.c. gain and the inverse time constant of the open-loop system of the d.c. motor speed control, respectively. Discretizing the above system, we can simply transform (10) into

$$z(t) = \phi(t)^T \theta(t) \tag{44}$$

where $\phi(t)^T = [-y(t-1)u(t-1)], \theta(t) = [\alpha \;\; \kappa], t_s = 5[msec]$ is the sampling period, and the system parameters are

$$a = 1 - \alpha * t_s, \qquad k = \kappa * t_s. \tag{45}$$

These parameters are subject to hard failures such as

$$\alpha \Longrightarrow \alpha + \Delta\alpha \tag{46}$$

$$\kappa \Longrightarrow \kappa + \Delta\kappa \tag{47}$$

The normal parameter values are chosen to be

$$\bar{\alpha} = 0.5, \qquad \bar{\kappa} = 10.0 \tag{48}$$

and the parameter estimate $\hat{\theta}(t)^T = [\hat{a}(t)\hat{k}(t)]$ where the actual parameters are

$$\hat{\alpha}(t) = \frac{1 - \hat{a}(t)}{t_s} \qquad \hat{\kappa}(t) = \frac{\hat{k}(t)}{t_s} \tag{49}$$

The recursive estimation algorithm for the parameter vector is chosen to be the covariance resetting when the residual $\tilde{z}(t)$ exceeds the specified boundary value δ.

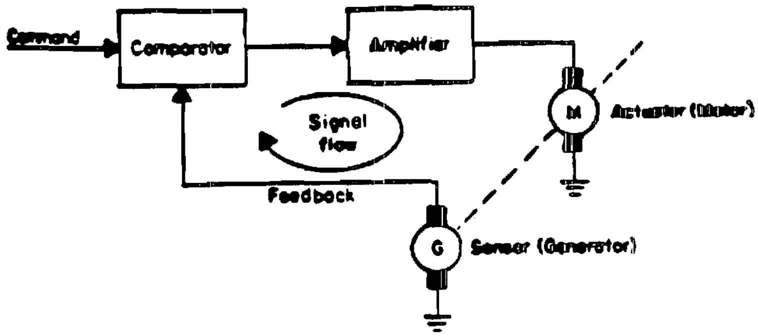
$$\hat{\theta}(t+1) = \hat{\theta}(t) + K(t+1)\{z(t+1) - \phi(t+1)^T\hat{\theta}(t)\} \tag{50}$$

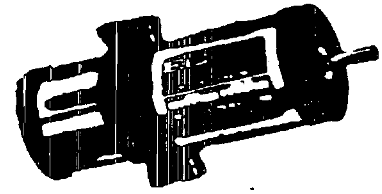
$$K(t+1) = P(t)\phi(t+1)\{\phi(t)^T P(t)\phi(t+1) + 1\}^{-1} \tag{51}$$

$$P(t+1) = P(t) - K(t+1)\phi(t)^T P(t) \tag{52}$$

$$P(t) \Longleftarrow P_0 \;\; if \;\; \|\tilde{z}(t)\| = \|z(t) - \phi(t)^T\hat{\theta}(t-1)\| > \delta \tag{53}$$

In the simulation, the parameters are disturbed by white gaussian noise both in the normal and in the failure cases with means 0.008 $\sim$ 0.017 and standard deviations 0.326 $\sim$ 0.347. Also,





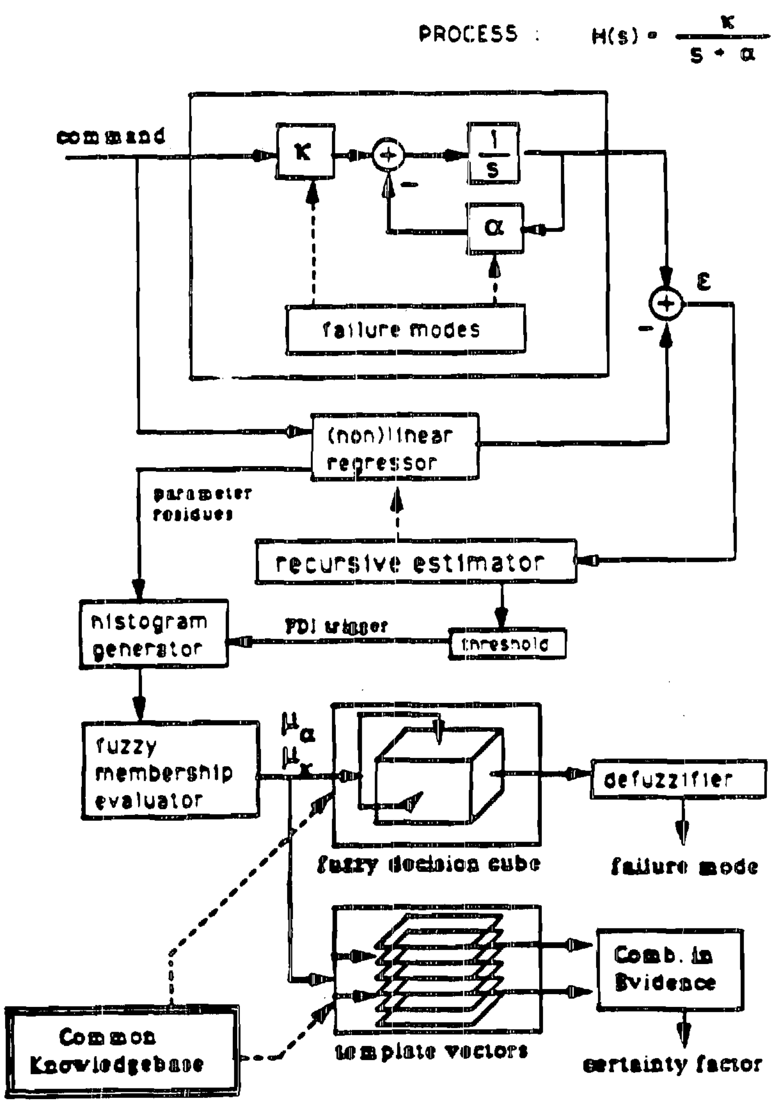
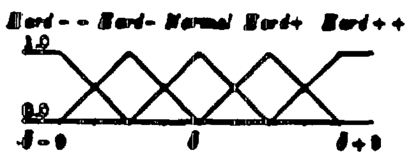


Figure 2: Hybrid Analytical/Intelligent FDI for DC Motor Speed Control

Fuzzy Matrices

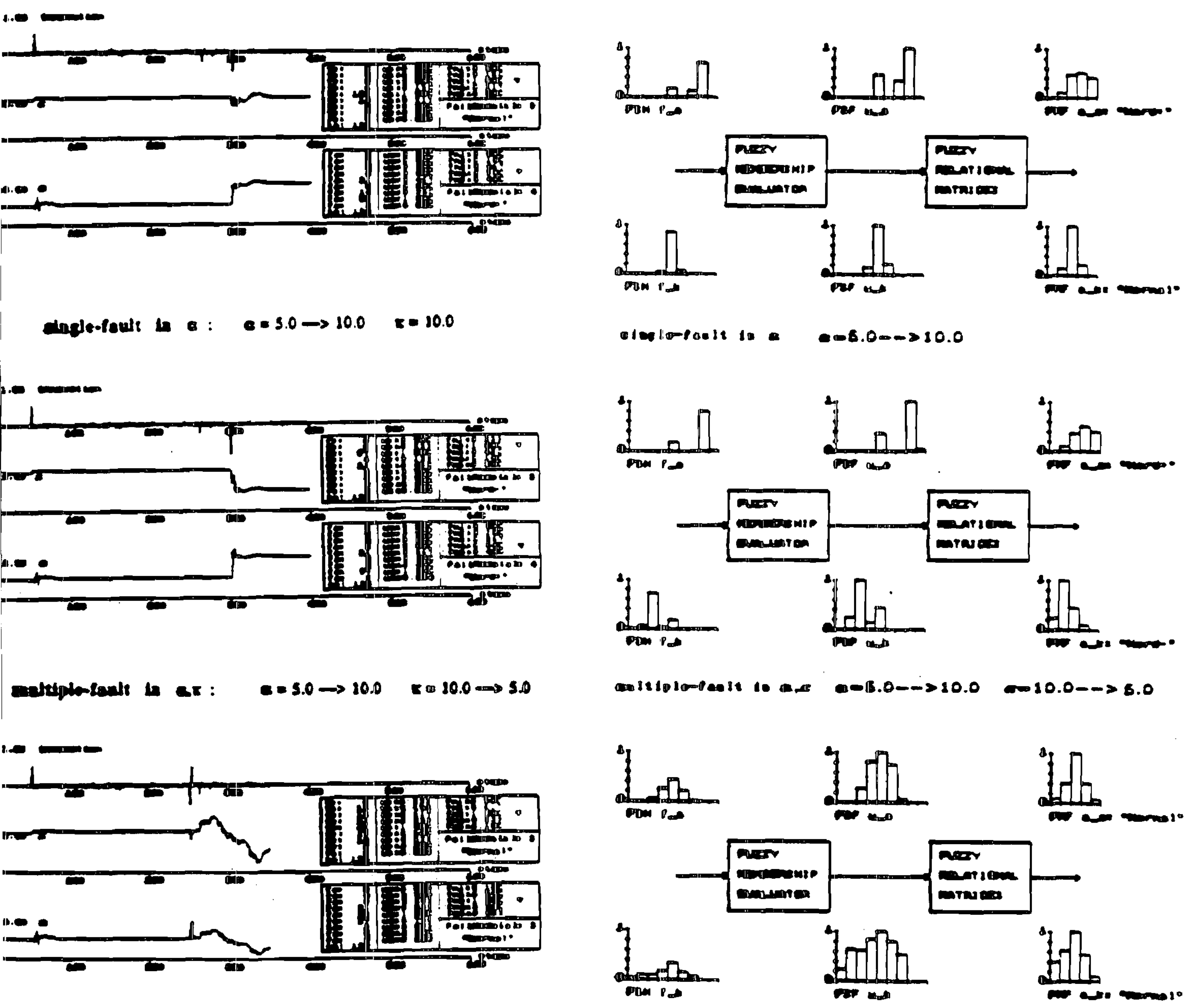
for α, κ

$$R = \begin{bmatrix} 1.0 & 0.4 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.1 & 0.5 & 1.0 & 0.4 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.1 & 0.5 & 1.0 & 0.5 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.1 & 0.4 & 1.0 & 0.5 & 0.1 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1 & 0.4 & 1.0 \end{bmatrix}$$



Fuzzy membership functions for each FRM

failure signature detection **failure mode identification**



false alarm Figure 3: Simulation Results **false alarm**

another white gaussian noise is added to the output with a mean of 1.142×10^{-4} and a standard deviation of 3.804×10^{-3}. Figures 2 and 3 show a schematic representation of the motor speed control system, the functional blocks of the FDI algorithm, and some typical simulation results for single and multiple failures as well as for a false alarm. The algorithm performs efficiently and robustly with minimum sensitivity to false alarms and maximum sensitivity to failures.

2.5 An Application of FDI to the Axial Flow Compressor Problem

The application of the hybrid analytical/intelligent FDI approach to a simple motor speed control problem led to encouraging results, as reported in the previous section. It was felt, therefore, that an attempt should be made to demonstrate the viability and robustness of the FDI algorithmic developments by applying the technique to a major system of a turbojet engine. The surge and rotating stall post instability behavior of axial flow compressors has received considerable attention over the past years and constitutes a major thrust of the ONR program. Aerodynamic instabilities of the compressor can severely limit its operating envelope. When the compressor is near its maximum achievable pressure rise, a moderate disturbance can result in instability, or even loss of the nominal operating point. The compression system can assume these two types of dynamic behavior: a large amplitude, low frequency oscillation, known as surge, in which the compressor experiences a series of rapid flow reversals and recovery; the second, known as rotating stall, is characterized by very inefficient engine operation at constant mass flow and pressure rise. Recovery from a stall condition usually necessitates engine shutdown and restart. We pose the following question: Is it possible to identify an instability condition (and, preferably, an impending instability) by monitoring key compressor dynamic variables? If the answer to this question is affirmative, then, can we use these data in the form of "failure signatures" to detect and identify specific instability conditions by assigning to them the role of "failure modes"?

The technical approach we pursued to address this problem includes the tasks: (a) nonlinear modeling of axial flow compressor dynamics, (b) identification of "failure modes" as instability conditions, (c) definition of measurable "failure signatures", and (d) application of the hybrid FDI algorithm, assessment and evaluation of simulation results.

It should be noted that this analysis procedure will be extended, during the forthcoming reporting period, to the failure detection and identification problem of a combustion engine as a complex dynamical system and control strategies will be investigated to maintain an acceptable level of engine performance.

Compressor Dynamics

We have adopted Greitzer's fourth order lumped parameter model augmented by Nett's B-forcing term and an α-forcing term introduced by us to account for the dynamics of throttle area variations. The full 6th order 1-D model (in nondimensional form) is given by

- Compressor:

$$\frac{\partial \tilde{C}}{\partial \tilde{t}} = \frac{1}{\tilde{\tau}}(\tilde{C}_{ss}(\dot{\tilde{m}}_c) - \tilde{C}) - 2\frac{\tilde{C}}{B}\frac{\partial B}{\partial \tilde{t}} \tag{54}$$

- Compressor Duct:

$$\frac{\partial \dot{\tilde{m}}_c}{\partial \tilde{t}} = B(\tilde{C} - \Delta\tilde{P}) - \frac{\dot{\tilde{m}}_c}{B} \tag{55}$$

- Throttle Duct:

$$\frac{\partial \dot{\tilde{m}}_t}{\partial \tilde{t}} = \frac{B}{G}(\Delta\tilde{P} - \tilde{F}) - \frac{\dot{\tilde{m}}_t}{B}\frac{\partial B}{\partial \tilde{t}} \tag{56}$$

- Plenum:

$$\frac{\partial \Delta\tilde{P}}{\partial \tilde{t}} = \frac{1}{B}(\dot{\tilde{m}}_c - \dot{\tilde{m}}_t) - 2\frac{\Delta\tilde{P}}{B}\frac{\partial B}{\partial \tilde{t}} \tag{57}$$

- Rotor Speed:

$$\frac{\partial B}{\partial \tilde{t}} = \frac{1}{\tau_B}(B_\infty - B) \tag{58}$$

- Throttle Area Dynamics:

$$\frac{\partial \alpha}{\partial \tilde{t}} = \frac{1}{\tau_\alpha}(\alpha_\infty - \alpha) \tag{59}$$

The algebraic equations describe the nonlinear compressor performance and throttle characteristics. They are approximated by:

$$\tilde{C}_{ss} = -22.35\dot{\tilde{m}}_c^3 + 22.86\dot{\tilde{m}}_c^2 - 5.157\dot{\tilde{m}}_c + 1.04 \tag{60}$$

$$\tilde{F} = (\frac{\dot{\tilde{m}}_t}{\alpha})^2 \tag{61}$$

In addition to the nondimensional compressor pressure rise, mass flow rates and plenum pressure, some key parameters in the compressor dynamic description include:

$$B(t) \stackrel{\text{def}}{=} \frac{U(t)}{2\omega L_c} \qquad \text{the so-called B-parameter}$$

whose value is proportional to rotor speed $U(t)$, which was found to be a major determinant of post-instability compressor behavior; ω is the Helmholtz frequency and L_c is the length of the compressor duct.

$$\alpha(t) = \frac{A_T(t)}{A_c}$$

a parameter directly proportional to the controlled throttle actuator disk area, $A_T(t)$.

$$G(t) = \frac{L_T A_c}{L_c A_T(t)}$$

a geometric parameter inversely proportional to $\alpha(t)$. τ_B and τ_α are the time constants of the compressor rotor and compressor throttle, respectively.

Figures 4 and 5 depict typical dynamic compressor characteristics derived from the simultaneous solution of Equations (55)- (61). The figures show the compressor axial velocity parameter (or nondimensional mass flow) and nondimensional plenum pressure as functions of time. The results are also displayed as a compressor map type of format in which the instantaneous pressure rise is plotted against the compressor axial velocity parameter. This latter representation is analogous to the phase plane diagrams often used for illustrating nonlinear oscillations. Both limit cycle and damped oscillatory behavior are illustrated in the figures. The mathematical model described above simulates the compressor dynamic behavior and is subsequently used to emulate the actual physical process.

Failure Signatures (FS)

Three measured parameters define corresponding failure signatures for FDI purposes. They are:

FS1 $B(t)$ - through a direct measurement of rotor rotational speed (rpm). Small values of B are indicative of a stall condition whereas the compressor tends to a surge instability for large values of this parameter (normal value of $B(t) = 0.7 \sim 0.8$).

FS2 $\alpha(t)$ - through a direct measurement of the throttle area (normally $\alpha > 0.6$).

FS3 $\Delta\tilde{P}$ - through a direct measurement of the differential pressure between the compressor and atmospheric conditions. An electronic circuit basically consisting of a zero-crossing counter provides information about the frequency of the $\Delta\tilde{P}$ oscillations.

Failure Modes (FM)

The following failure modes are identified:

FM1 Normal Operation = NOP

FM2 Potentially Unstable Operation = POP

FM3 Impending Stall = INS

FM4 Rotating Stall = RST

FM5 Impending Surge = ISG

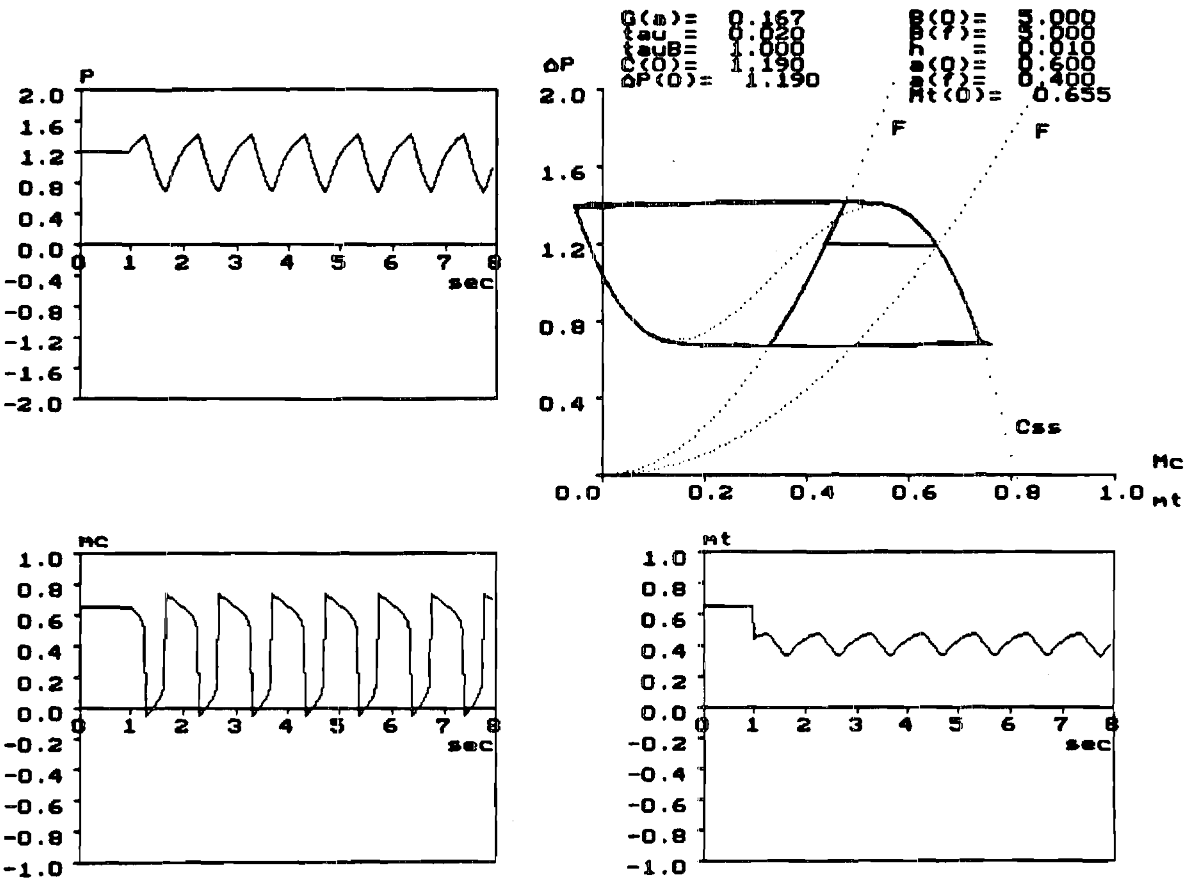


Figure 4: Dynamic compressor characteristics - limit cycle response

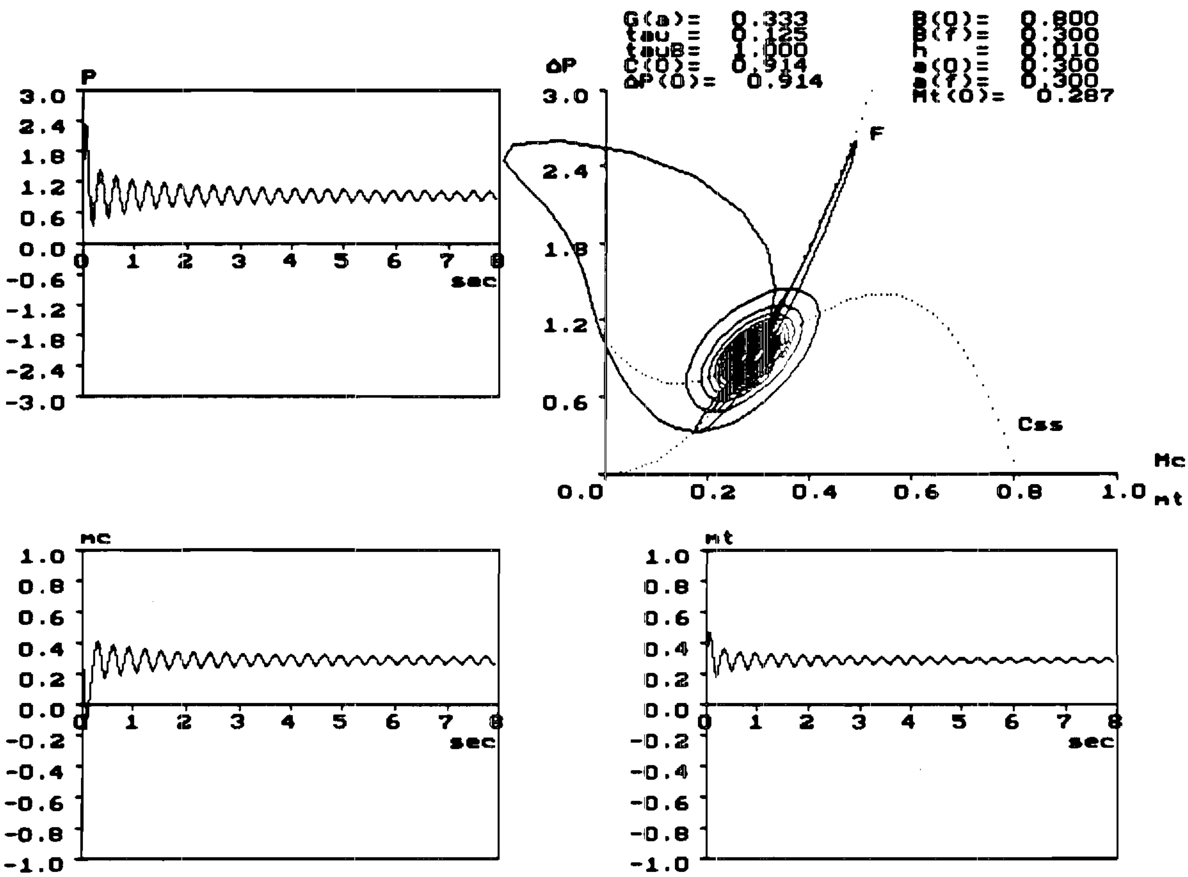


Figure 5: Dynamic compressor characteristics - damped oscillatory response

FM6 Abrupt Surge = ASG

FM7 Deep Surge = DSG

The rationale for this particular choice of failure signatures and failure modes stems from the analysis of the compressor dynamic behavior at or near stall and surge conditions. Specifically, the normal operation designation is obvious but must be capable of avoiding false alarms. Under a potentially unstable operation, the Compressor Performance Table (CPT) and the Throttle Characteristics (TC) are both within their normal range but the value of B is either too large or too small indicating that the compressor is vulnerable to entering an unstable operating region. An impending rotating stall is signified by the CPT and TC operating at their corresponding peak points; small disturbances about the operating point due to throttle behavioral uncertainty may cause the system to move towards a rotating stall condition. The compressor exhibits a rotating stall mode when the TC has fallen below the peak operating point. A very large value of B in the absence of $\Delta\tilde{P}_c$ oscillations or an uncertain TC position in combination with a large value of B will cause the compressor to exhibit eventually a surge instability; an impending surge condition is not accompanied by pressure oscillations. Large values of B in combination with a high frequency oscillatory behavior of $\Delta\tilde{P}_c$ and uncertain $\alpha(TC)$ values signal the presence of an abrupt surge. Finally, a deep surge condition is evidenced by very large values of B accompanied by slow oscillations and small values of $\alpha(TC)$.

Decision making (i.e. declaring a failure mode in the presence of failure signature evidence) is based on a possibilistic approach in order to account for large-grain uncertainty in the compressor dynamic behavior. Fuzzy rule based systems are finding applications in such areas as identification and control, expert systems and fuzzy decision tools. A fuzzy system may be expressed by a functional relation, known as fuzzy semantics, of the form:

$$V = \{x, L(x), U, \text{Syn}, \text{Sem}\}$$

where x is a fuzzy variable, $L(x)$ denotes a set of linquistic terms, U is the universe of discourse, Syn is a syntactic rule, and Sem is a semantic rule that governs this fuzzy relation. Typical sources of this static knowledgebase are heuristics, expert opinion, learning via simulation and training through experimentation.

For the compressor case, the linquistic terms for the failure signatures and failure modes are defined as:

FS1 B - Compressor Speed

$$L(B) = \{\text{Small, Medium, Large, Very Large}\} = \{S, M, L, VL\}$$

FS2 α - Throttle Nozzle Area

$$L(\alpha) = \{\text{Small, Medium, Large}\} = \{S, M, L\}$$

FS3 ΔP - Pressure Difference

$$L(\Delta P) = \{\text{No Osc., Small Osc., Very Osc.}\} = \{NO, SO, VO\}$$

$$L(FM) = \{\text{NOP, POP, IST, RST, ISG, ASG, DSG}\}$$

As exhaustive enumeration of all possible FS and FM combinations leads to a total of thirty-six rules. With prescribed membership functions derived primarily from simulation studies, the rule base is shown in Figure 6.

Each rule is of the form:

If B is (   ) and α is (   ) and ΔP is (   ), then FM is (   )

i.e. the fuzzy decision hypercube (FDHC) uses fuzzy set inputs and produces a crisp output which corresponds to the identified failure mode.

Dempster-Shafer theory provides a suitable framework for ignorance management. A likelihood distribution is derived first from the failure signature possibilistic distribution. Basic assignments are evaluated next from the likelihood values, after the latter have been reordered and normalized as described in a previous section of this report. The evidence is finally combined and the degree of certainty is computed via

$$\text{Degree of Certainty} = m(X) - Bel(\bar{X})$$

The failure identification procedure is summarized by the following steps:

step(i) Failure signature histogram

$$FS \Longrightarrow FSH$$

step(ii) Failure signature fuzzy set (FSF) via membership evaluator

$$FSH \Longrightarrow FSF$$

step(iii-1) uncertainty (vagueness) management

$$FSF \Longrightarrow FMF$$

step(iii-2) ignorance management

$$FSF \Longrightarrow DOC$$

Figures 7,8, and 9 show typical computer simulation results for an impending stall, a rotating stall, and an abrupt surge instability condition, respectively. Extensive simulation runs under

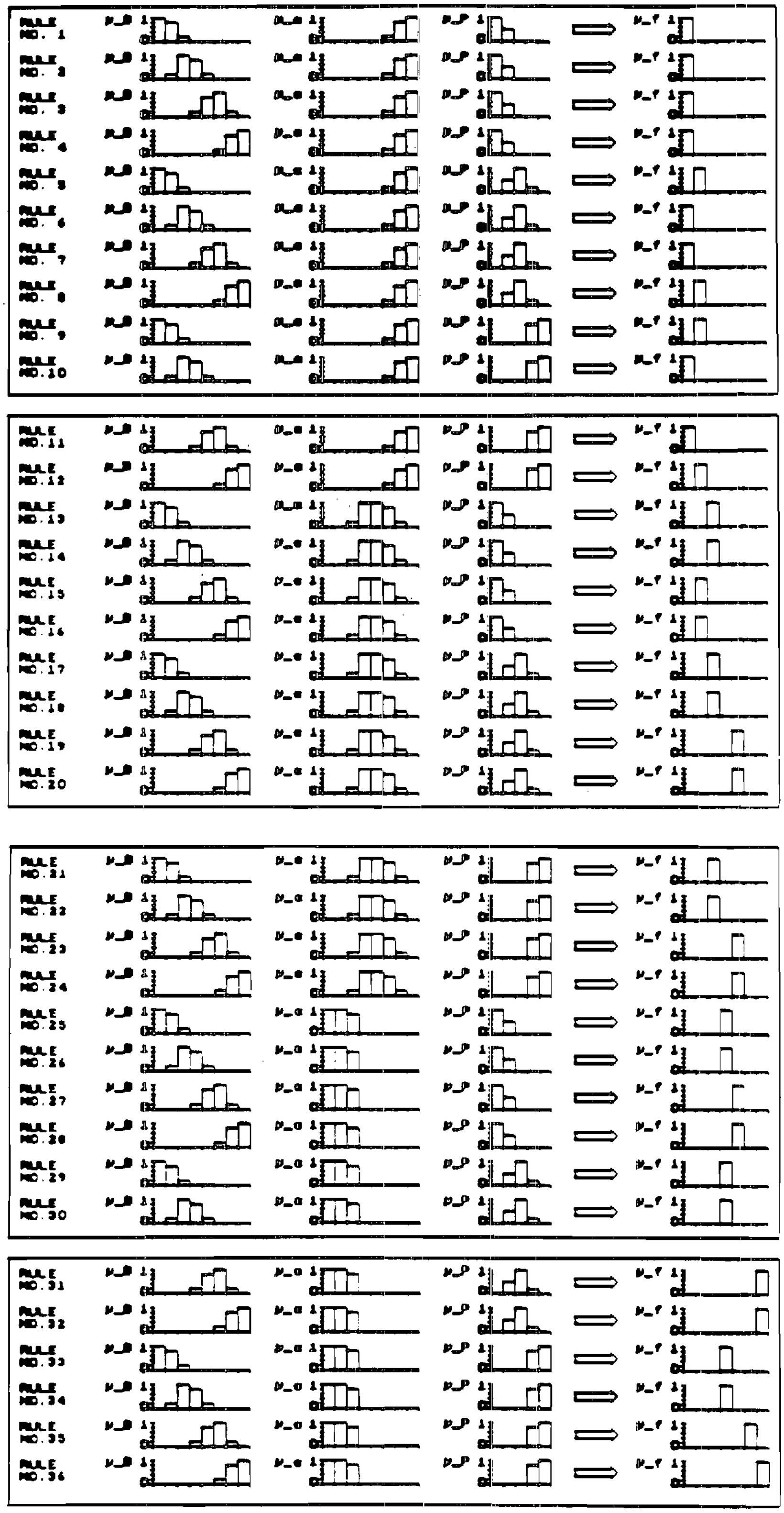


Figure 6: Membership function of failure signatures

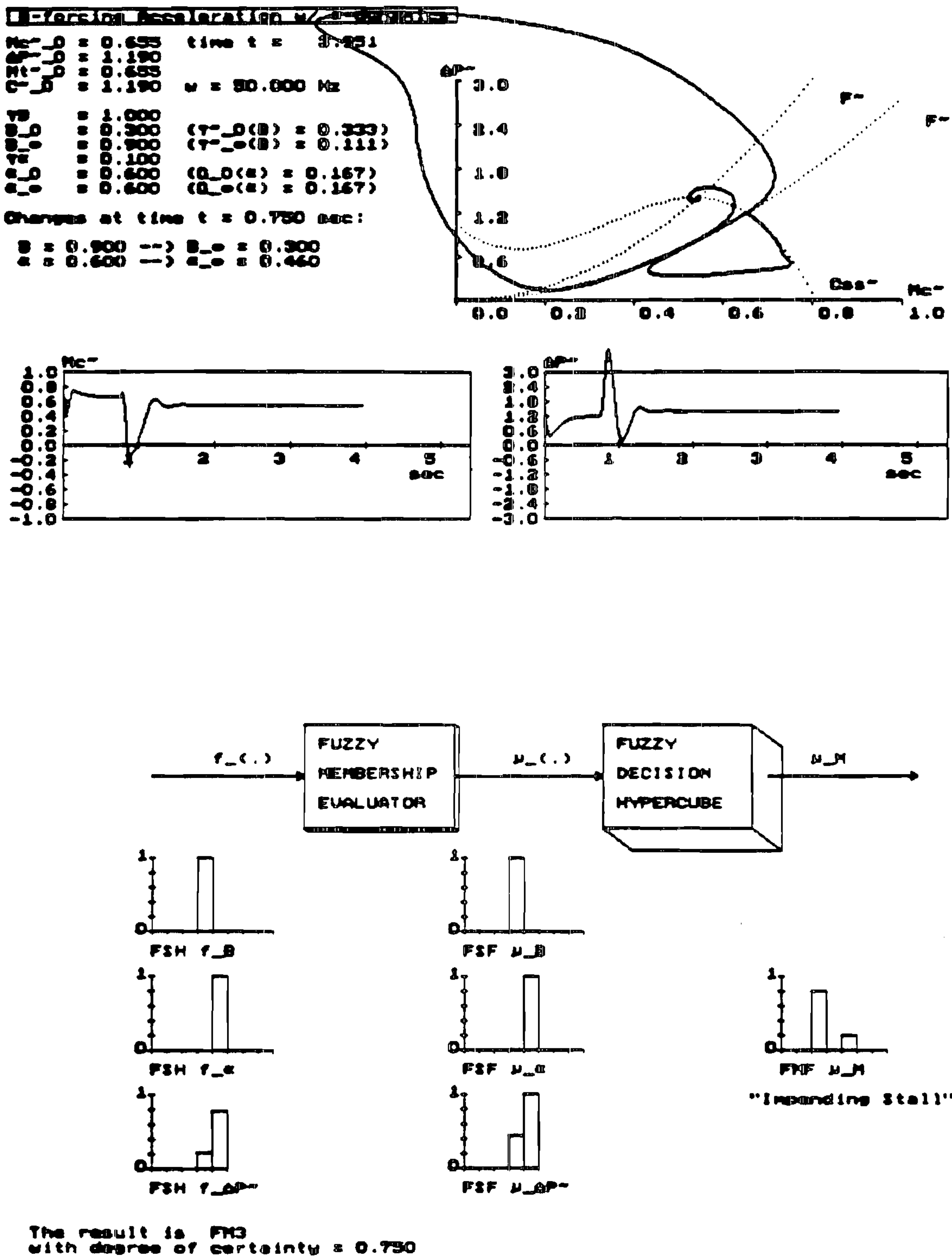


Figure 7: Computer simulation results - Impending Stall condition

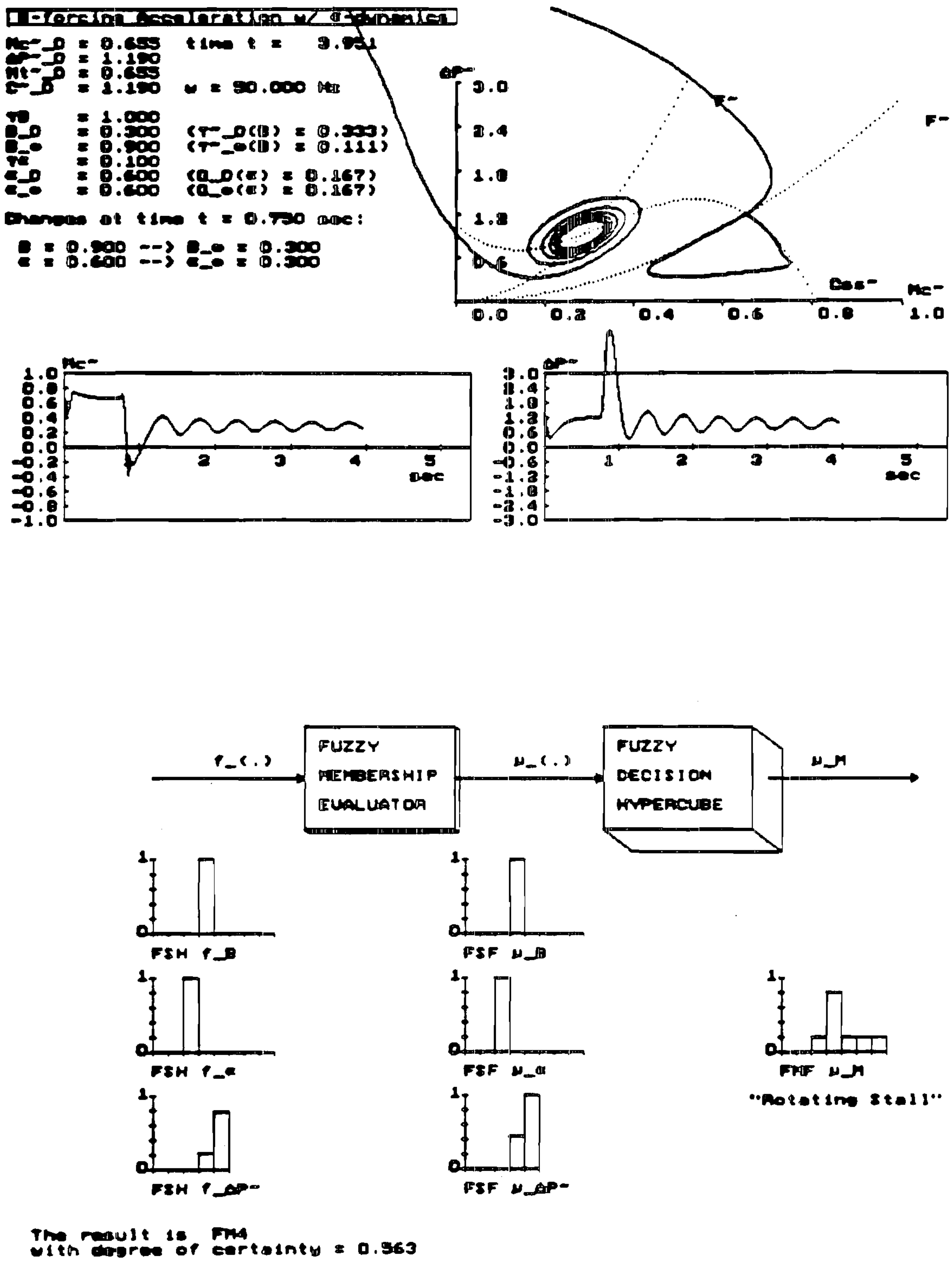


Figure 8: Computer simulation results - Rotating Stall condition

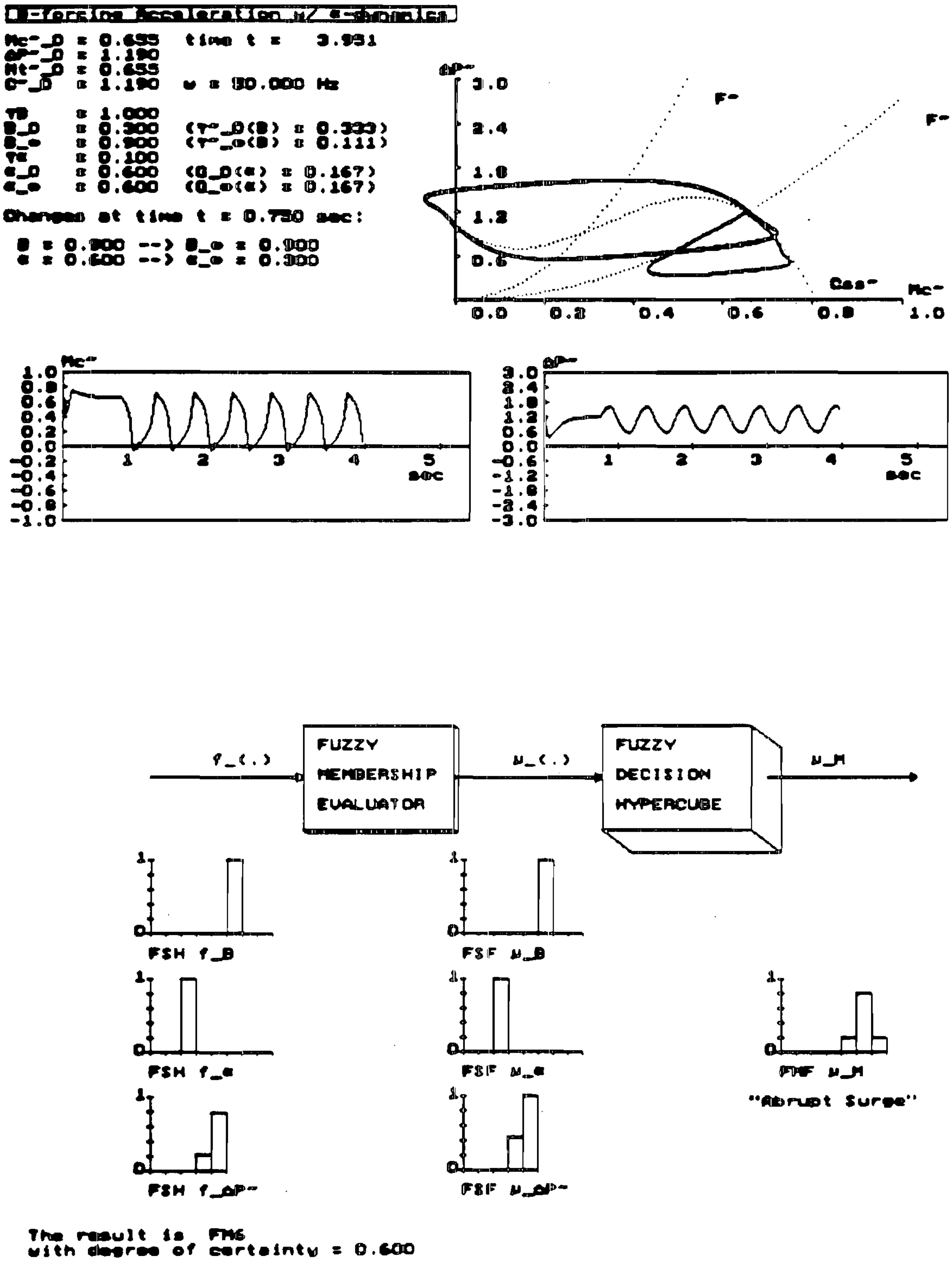


Figure 9: Computer simulation results - Abrupt Surge condition

various FS scenarios have produced similar results. Since the fuzzy decision hypercube and the template vectors use the same fuzzy failure signature distributions, the results are both consistent and complementary. That is, the FDHC decides on the particular FM while managing system uncertainty and the template vector algorithm corroborates this decision and assigns a degree of certainty to it which accounts for incomplete or conflicting evidence in the failure signature information. The combined algorithm thus proves to be an efficient tool for decision making in the presence of uncertainty and ignorance -a situation typified by conditions in an axial flow compressor. A computer package has been developed that integrates the compressor dynamic modeling aspects with the FDI routines. A menu driven procedure allows the user to choose from an available list of parameter values that simulate a given failure condition. The FDI routine provides on-line the corresponding failure mode and degree of certainty. Auxiliary graphics packages display the dynamic evolution of the failure signatures, the fuzzy FS distributions and related histograms. The demonstration package, therefore, is intended to illustrate not only the final failure decision but also the procedural steps leading up to and supporting the decision. In an appropriate implementation environment with suitable I/O devices, the FDI package could become an integral part of the active controls for a combustion engine.

Performance Metrics - Sensitivity and Computational Complexity

The performance of an FDI algorithm may be assessed via a number of metrics that characterize such attributes as robustness, sensitivity, and computational complexity.

The sensitivity of the FDI routine is defined as the change in the numerical value of the degree of certainty as a function of the relative change in fault size, i.e.

$$S = \frac{\Delta \mathrm{DOC}}{\Delta\Theta/\Theta}$$

For our simple motor example, Figure 10 shows the degree of certainty as a function of fault size in the parameter a. As it is expected, the DOC increases with an increase in fault size and the sensitivity function is bounded above and below within a range specified by a maximum and a minimum fault size, respectively.

The second metric relates to computational complexity and the inherent trade-off's between hardware requirements and computing speed. Table 1 illustrates this point by listing the time in seconds required for a typical operating cycle (training/learning and fuzzy reasoning/decision making) and the number of comparators (hardware components) needed to perform these operations. The comparison is made between a Von Neumann machine and a parallel processing architecture implementing the FDHC. As anticipated, the FDHC responds considerably faster than the Von Neumann machine at the expense of increased hardware complexity. For critical processes like the combustion engine, this trade-off offers a welcome opportunity to expedite

$$\textbf{SENSITIVITY} \quad S = \frac{\text{change in degree of certainty}}{\text{relative change in fault size}}$$

$$= \frac{\Delta \text{DOC}}{\Delta\Theta \,/\, \Theta}$$

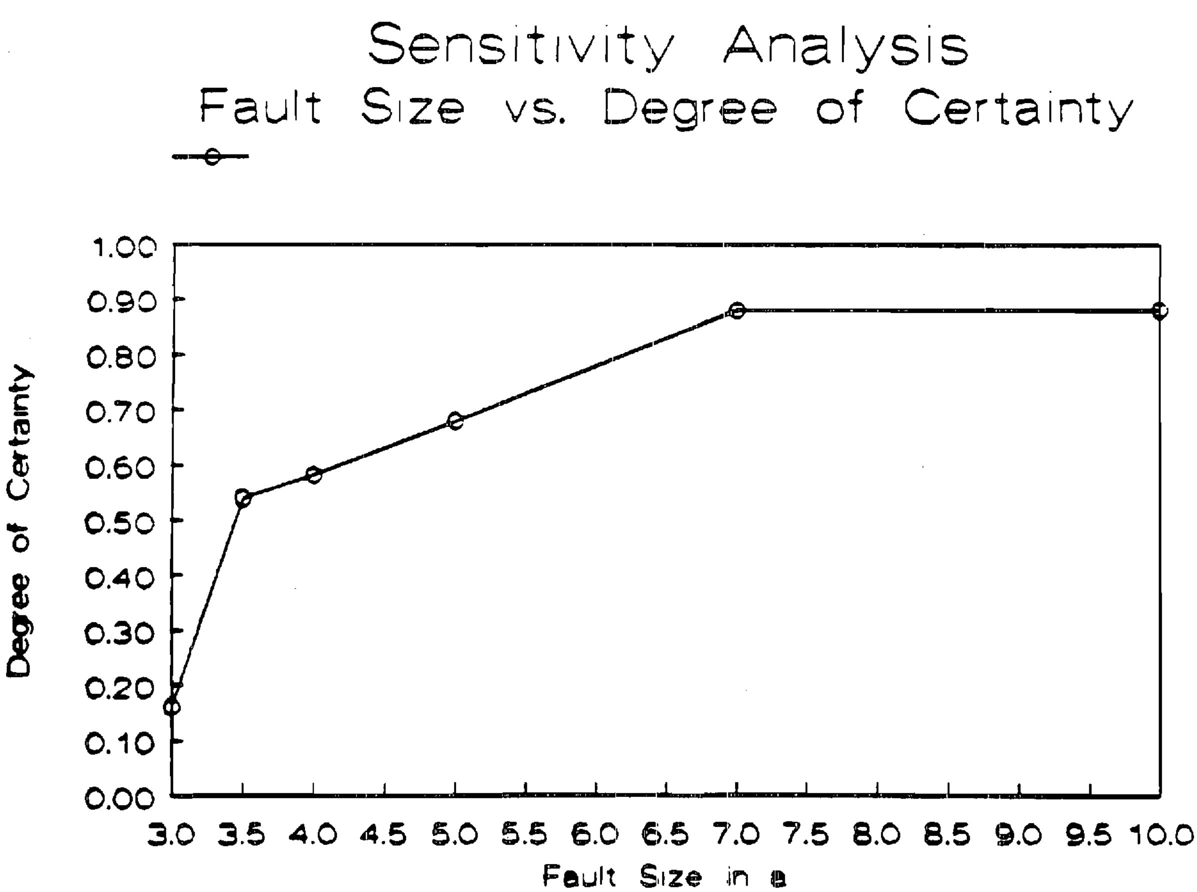


Figure 10: Fault size vs. Degree of Certainty - motor example

failure detection and identification since parallel computer architectures are feasible and cost-effective with present day technology.

The study of FDI performance characteristics will be extended to include a robustness metric. A comparison study will be conducted, as the final task of this effort to evaluate the relative performance of the hybrid FDI vs. other available techniques.

ΔTs = software comparator time delay

ΔTc = multi-input comparator time delay

m = number of FS inputs

n_i = number of the ith FS quantization

n_f = number of FM

Computing Complexity	Training/Learning (each learning cycle)	Fuzzy Reasoning/Decision (one-shot association)
Von Neumann Machine	$2 n_1 n_2 \cdots n_m n_f \Delta Ts$ (seconds)	$n_1 n_2 \cdots n_m (n_f + 1) \Delta Ts$ (seconds)
Fuzzy Decision Hypercube	$2 \Delta Tc$ (seconds)	$2 \Delta Tc$ (seconds)
FDHC Hardware Complexity	$2 n_1 n_2 \cdots n_m n_f$ (comparators)	$n_1 n_2 \cdots n_m (n_f + 1)$ (comparators)

Table 1 Computational complexity tradeoffs.

3 Fault Propagation

We are exploring an alternate methodology to address the issues of fault propagation. A major difficulty associated with qualitative reasoning techniques involves the combinatorics of many of the current approaches. This generally precludes their application for real-time control of complex systems. We are exploiting specialized representational and control strategies to ensure that real-time processing needs are satisfied.

At the representational level we are exploiting the concepts of *hierarchical abstraction* and *clustering* as a basis for developing lumped system models. The use of hierarchical abstractions enables us to defer the analysis of system detail until it has been determined that it is necessary to be examined. We intend to precompile relevant lumped model data where appropriate to facilitate the rapid retrieval of this data are execution time. Clustering logically related components also enables us to constrain the focus-of-attention during fault propagation analysis. These representational strategies can be used in the envisionment process we have used previously as well as supporting the qualitative fault process control techniques mentioned below.

In *qualitative fault process reasoning*, the system's components are depicted not as devices whose *functions* are represented qualitatively, but rather we represent the processes by which the component may *fail* qualitatively. Each device has one or more critical variables (CV) which determine its reliability. These CVs inform as to whether the device will operate nominally in the context of the overall system. When faulty performance occurs, it is manifested by one of several potential qualitative fault states: over-performance, under-performance, no performance, or other states that depend on the nature of the device in question. This fault performance affects the critical variable in some manner which can also be characterized qualitatively.

Analysis of the device's performance serves to provide, via topological constraints (physical connectivity), the relevant fault characteristic input to interconnected devices. As each of these devices is also modeled in a similar manner, a recursive analysis of the system via the interconnections enables a tracing of the potential faulty behavior of the connected components. As the fault(s) transit through the system they can either be absorbed (terminating propagation), transmitted transparently, modulated, or generate new faults. Criteria regarding failure likelihood under given conditions as well as criticality measures of a component in relation to the entire system determine the extent to which a given fault has propagated.

In summary, representational abstractions and topological constraints make this problem more amenable to a real-time solution. Lyapunov methods will also be incorporated to validate the stability of a proposed solution.

Work completed thus far in qualitative fault process reasoning involves the conceptualization of devices and connections as the building blocks of the system. Each device has critical and

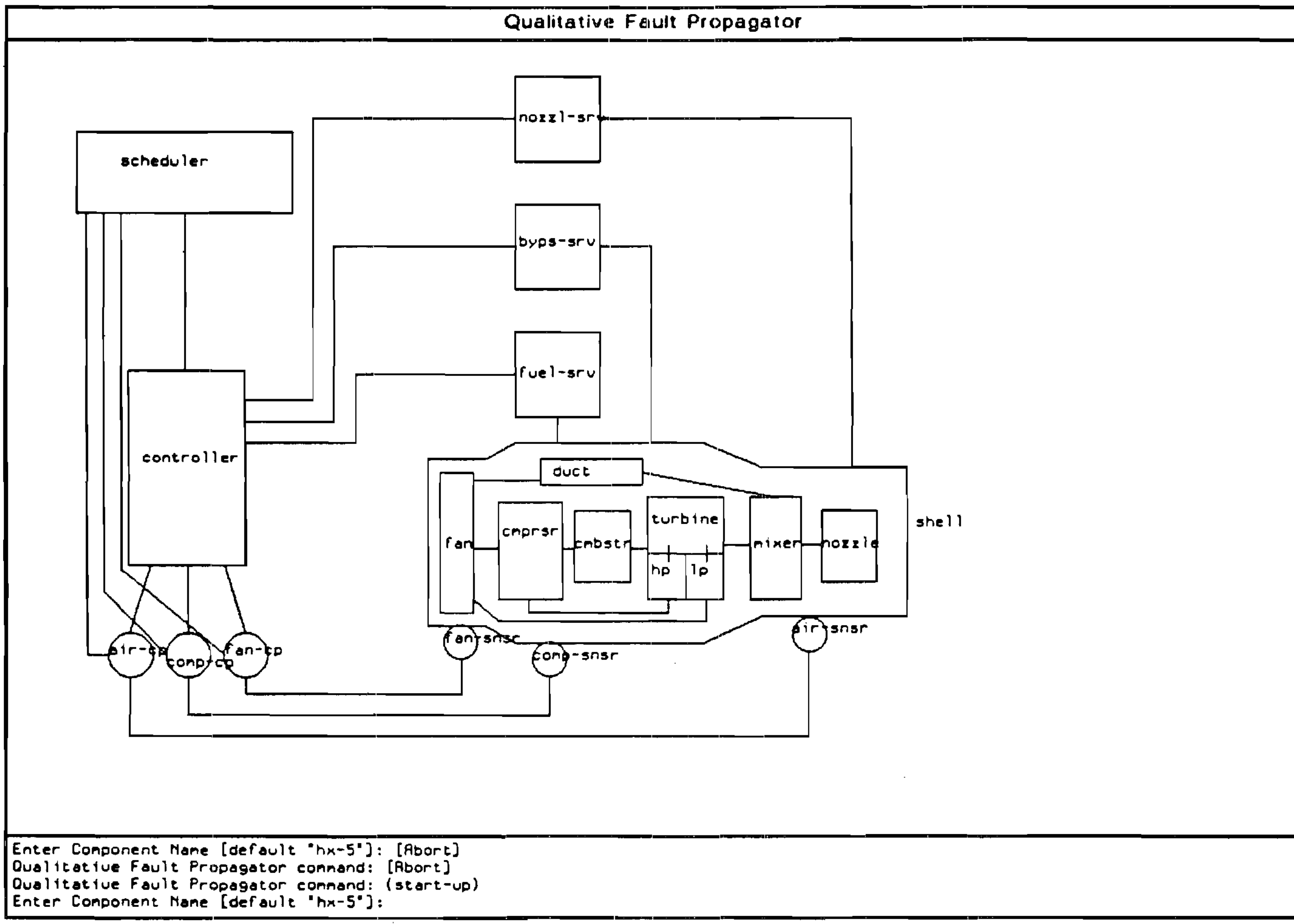


Figure 11: Modular basis for a turbofan engine

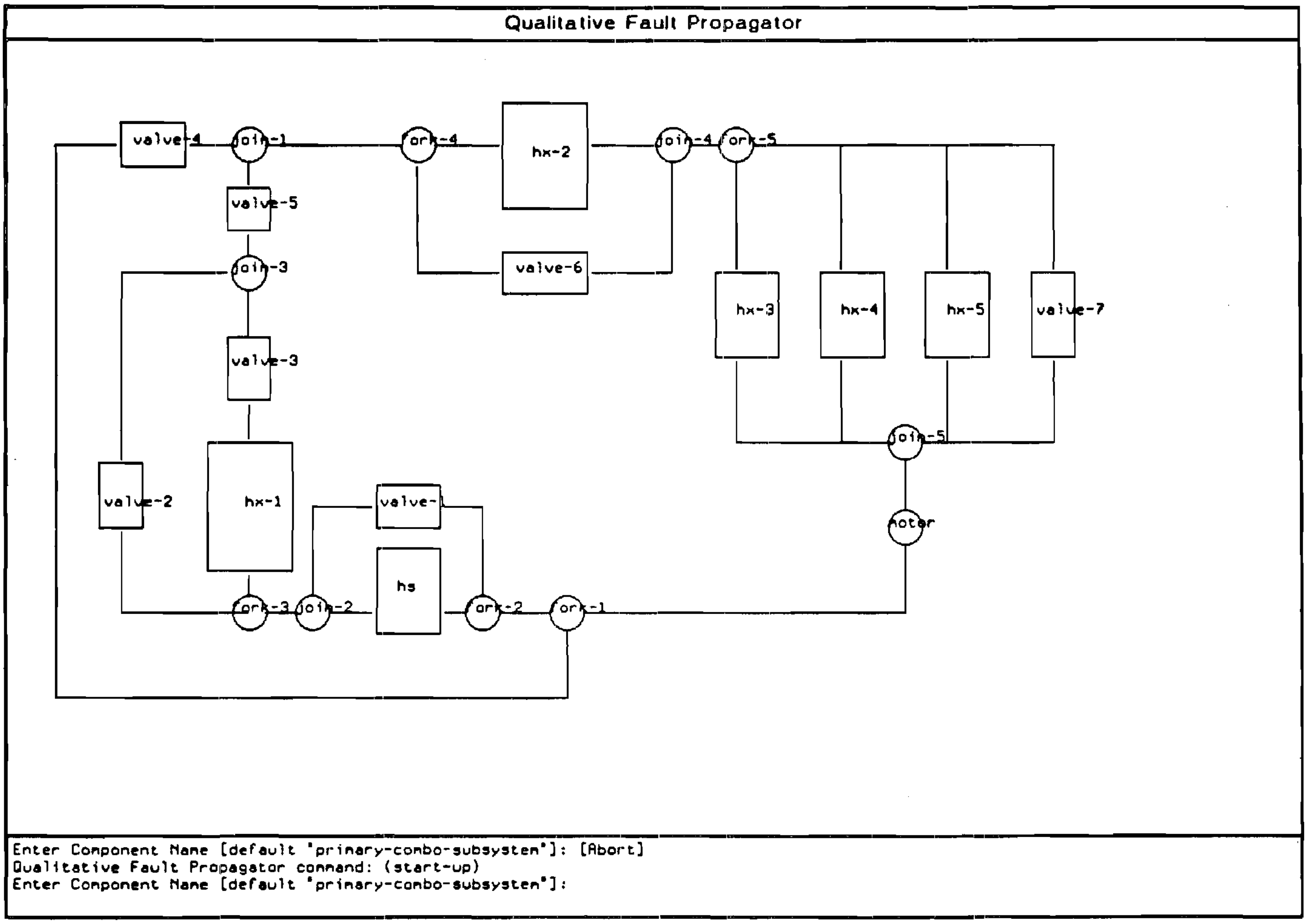


Figure 12: Modular basis for thermal control system of the space station

performance variables, and each performance variable has a performance function for each allowable performance state. Hallmarks denote the transition from one state to the next for each performance variable. A modeling effort has been initiated entailing three major directions: topological models, functional models, and control models. Typical topological models have been constructed on a modular basis for a turbofan engine (Figure 11) and the Thermal Control System of the space station (Figure 12). Future work involves the development of functional and control models and their inter-relationships as well as the topics of control (propagation via locality, local simulation, dynamic abstraction, and termination conditions) and fault induction. This important task of the project is intended to produce an efficient algorithm for fault propagation that will assist the overall fault-tolerant control scheme in determining, in a timely manner, the state of healthy system components in the event of a failure and in assessing the effectiveness of control reconfiguration strategies.

Presentations and Technical Contributions

1. In September 1989, a presentation was given to the staff of NASA's MSFC at Huntsville, Alabama by Drs. Vachtsevanos and Arkin on possible applications of fault-tolerant control technology to the space station program.
2. On 11/9/89, Dr. G. Vachtsevanos was invited to present a seminar to the technical staff of Rockwell International Corp., the Science Center at Thousand Oaks, California on fault-tolerant control techniques. He held technical discussions with fuzzy logic and the application of fault-tolerant control techniues to aerospace systems.
3. In November 1989, discussions were held and a statement on fault- tolerant control was mailed to Nothrop Corp.
4. Interest on our fault-tolerant control activities has been expressed by the Federal Aviation Administration. A position paper on this subject was mailed to FAA research staff.
5. In June 1989, Dr. Vachtsevanos chaired an invited session on Intelligent Control at the 1989 Automatic Control Conference and presented a paper entitled "Fault Tolerant Control of Dynamical Large Scale Systems". A copy of the paper is attatched to this report.
6. In December 1989, Dr. Vachtsevanos presented a paper, co-authored by Dr. H. Kang on "Model Reference Fuzzy Control" at the 28th IEEE Conference on Decision and Control, Tampa, Florida.
7. A paper entitled "Stability and Robustness of a Nonlinear Fuzzy Controller Based on the Phase Portrait Assignment Algorithm" co-authored with H. Kang has been submitted for publication to the IEEE Trans. on Systems, Man and Cybernetics.
8. Dr. Vachtsevanos presented a seminar on Fault-Tolerant Control Systems to the faculty of Simon Fraser University on February 21, 1990.
9. Dr. Vachtsevanos presented a paper at the Sensors 90 Conference in Long Beach, California on March 14, 1990.
10. Presentations to staff of McDonnel Douglas Corporation were made in April of 1990.
11. In May 1990, Dr. Vachtsevanos presented a paper at the Automatic Control Conference in San Diego, California.
12. In September 1990, Dr. Vachtsevanos organized and chaired an invited session on "New Methods for Fault-Tolerant Control System Design" at the 5th IEEE International Symposium on Intelligent Control, Philadelphia, Pennsylvania.
13. Dr. Arkin was a member of the Program Committee of the 5th IEEE International Symposium on Intelligent Control, Philadelphia, Pennsylvania.
14. Dr. Arkin will co-chair an invited session on "Explorations in Qualitative Reasoning for Control" at the 29th IEEE Conference on Decision and Control to be held in Honolulu, Hawaii, in December 1990.

The following technical papers authored or co-authored by members of the research team were published during the reporting period.

1. G. Vachtsevanos, Y.T. Kim, and M. Christodoulou, "Fault-Tolerant Control of Dynamical Large Scale Systems," Proceedings of the 1989 ACC, Pittsburgh, Pennsylvania, pp. 355-360, 1989. (invited).

2. H. Kang and G. Vachtsevanos, "Model Reference Fuzzy Control," Proceedings of the IEEE Conference on Decision and Control, pp. 751-752, 1989.

3. H. Kang, D. Dawson, F. Lewis, and G. Vachtsevanos, "A Robust Adaptive Controller for Rigid Robots," Proceedings of the IEEE Conference on Decision and Control, pp. 2619-2620, 1989.

4. G. Vachtsevanos, K. Davey, J. Cheng, and S. Sobczynski, "An Integrated Intelligent Approach to Lime Kiln Monitoring and Control," Proceedings of the Sensors Expo West, March 13-15, 1990.

5. B.G. Mertzios, F.L. Lewis, G.J. Vachtsevanos, and M.A. Christodoulou, "Analysis of Bilinear Systems Using Orthogonal Functions," IEEE Transactions on Automatic Control, vol. 35, no. 2, pp. 119-123, 1990.

6. H. Kang, G. Vachtsevanos, and F.L. Lewis, "Lyapunov Redesign for Structural Convergence Improvement in Adaptive Control," IEEE Transactions on Automatic Control, vol. 35, no. 2. pp. 250-253, 1990.

7. G. Vachtsevanos and B. Verriest, "An Analytical/Intelligent Approach to Sensor Fusion for Manufacturing Systems," Proceedings of the Capteurs 89, Paris, France, pp. 476-481, 1989.

8. G. Vachtsevanos, H. Kang, I. Kim and J. Cheng, "Managing Ignorance and Uncertainty in System Fault Detection and Identification," Proceedings of 5th IEEE Intern. Symposium on Intelligent Control 1990, pp. 558-563, Philadelphia, Pennsylvania, 1990.

9. B.H. Wang and G. Vachtsevanos, "Fuzzy Associative Memories: Identification and Control of Complex Systems," Proceedings of 5th IEEE Intern. Symposium on Intelligent Control 1990, pp. 910-915, Philadelphia, Pennsylvania, 1990.

10. R.C. Arkin and G. Vachtsevanos, "Techniques for Robot Survivability," Proc. 3rd International Symposim on Robotics in Manufacturing, Vancouver, BC, 1990.

11. R.C. Arkin and G. Vachtsevanos, "Qualitative Fault Propagation in Complex Systems," to appear in Proc. 29th IEEE Conference on Decision and Control, Honolulu, Hawaii, Dec. 1990.

12. A. Goel, R. Arkin, L. Barsalou, D. Billman, R. Catrambone, K. Eiselt, J. Kolodner, D. Lawton, and A. Ram, "Basic Research on Artificial Intelligence and Cognitive Science at the Georgia Institute of Technology," submitted to AI Magazine.

A HYBRID ANAYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN
CONTRACT NO: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

Project Participants

The parincipal Investigator of this project is

Dr. George Vachtsevanos, Professor of Electrical Engineering at the Georgia Institute of Technology, Atlanta, Georgia.

Dr. Ronald Arkin, Associate Professor at the College of Computing, the Georgia Institute of Technology, serves as a co-investigator to the project.

Dr. Hoon Kang, a recent Ph.D. graduate at the Georgia Institute of Technology, is participating in project activities as a post-doctoral fellow.

Three graduate students in the School of Electrical Engineering at Georgia Tech are currently receiving partial support from this project.

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Annual Letter Report for FY 91

A HYBRID ANALYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN

(Contract No: N00014-89-J-3113)

Sponsor: **Office of Naval Research**
Applied Research Technology Directorate

Principal Investigator: **Dr. George Vachtsevanos**
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Tel: (404)894-6252

September 15, 1991

A Hybrid Analytical/Intelligent Approach to Fault-Tolerant Control System Design
Contract No: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

In September 1989, the Office of Naval Research awarded contract No. N00014-89-J-3113 to the Georgia Institute of Technology to develop fault-tolerant control strategies for large scale dynamical systems. Specifically, the technical issues under investigation are: Development of a structure-based modeling methodology of large scale systems which possesses features of structure flexibility, i.e. a component or subsystem may be deleted in a simple way that does not substantially disturb the global control strategy; development of robust failure detection and fault identification algorithms for single and multiple faults that are maximally sensitive to true failure conditions but insensitive to noise thus reducing the possibility of false alarms; a technique to restructure the system dynamics by isolating the faulty components; a method that will lead to a structural control law which reconfigures the original system controller in order to meet the primary performance objective of guaranteed stability while the system is operating in a degraded mode; finally, these algorithmic developments are to be demonstrated on an actual physical system to be designated jointly by ONR and Georgia Tech.

The technical approach pursued to meet these objectives is outlined as follows: The underlying factor of the fault-tolerant methodology capitalized upon structural features of the large scale system and employs a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. In the modeling area, the topological description of many complex dynamical processes may be cast in the form of structurally interconnected subsystems. In every subsystem, a control law is applied which consists of a local and a global feedback term, thus resulting in a two-level hierarchical control strategy. We examine first large scale systems which are described by linear state equations and exhibit this two-level hierarchical structure. The analysis will be extended eventually to nonlinear systems of a similar structure. A methodology for fault diagnosis is introduced which is based upon a combination of signal redundancy and detection/estimation procedures. An expert system, consisting of a multivalued rule base and an appropriate inferencing mechanism, assesses the aggregate of fault symptoms and determines the sensitivity of a failure condition. An innovative approach to multiple failure detection uses qualitative simulation techniques to decide on the impact of a component failure on other (healthy) system components. The proposed fault detection and identification scheme incorporates such additional functionalities as fault trending and the estimation of the "best" value of critical variables and parameters under failure states.

Finally, we propose the development of a two-level structural dynamic hierarchical approach to address the control reconfiguration problem of a faulted large scale system. The approach uses a structural state model, the Block Arrow Structure, which consists of N independent linear subsystems interconnected with a control common linear dynamic system. The objective is to fine a time-invariant decentralized feedback controller which minimizes a quadratic cost functional and has the features of (1) utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.
On January 1991, an additional task was added to this project. With partial funding from NAVSEA/DARPA, we are currently addressing the fault-tolerant design of an integrated shipboard electric power system. Through appropriate modeling and simulation techniques, we

anticipate to assist the David Taylor Research Center in the design and control of a reliable shipboard electric power system.

Research Accomplishments

During this reporting period 9/1/90-9/1/91, the research team focused attention on the following issues:

- Development of a full state model for a single spool turbine jet engine.
- Design and refinement of a hybrid fault detection and identification method to address engine failure events.
- Continuation and completion of work to detect axial flow compressor instabilities; conceptualization of a fuzzy active/adaptive control strategy to extend the operating envelope of a jet engine and to avoid dynamic instabilities.
- Development of generic algorithms for fault-tolerant control, i.e. system restructuring and controller reconfiguration routines to maintain system survivability in the event of a failure.
- Basic research into sensor fusion routines to accommodate maximum availability of sensor data for the jet engine.
- Implementation issues relating to the hybrid analytical/intelligent nature of the fault-tolerant strategies; development of novel parallel processing architecture to provide accurate and timely identification and control information.
- Finally, conceptualization of a fault-tolerant control methodology for an integrated shipboard electric power system and some preliminary simulation studies to demonstrate proof-of-concept feasibility.

Major accomplishments during this period include the extension of the hybrid fault detection and identification algorithm to be applied to a third order surge capable single spool turbine engine model; the development of a theoretical framework for sensor fusion and fault-tolerant control design; the design of novel and efficient implementation structures; and the conceptualization of a fault-tolerant control methodology for shipboard electric power systems.

We anticipate that during the upcoming final period of this project, we will integrate the fault tolerant control modules and apply the strategy to an experimental engine (in collaboration with Dr. C. Nett of the School of Aerospace Engineering at the Georgia Institute of Technology); we will also pursue the design of fault detection and controller reconfiguration routines for the shipboard power system.

1 THE SIXTH ORDER SINGLE SPOOL GAS TURBINE ENGINE MODEL

We have developed a systematic unified failure detection and identification methodology based on possibility theory [1] and Dempster-Shafer theory [2]. The algorithm has been applied to a motor speed control problem and to the multi-stage axial flow compressor to detect and identify aerodynamic instabilities. The algorithm has been verified through computer simulation studies. An axial flow compressor model was adopted to develop the FDI algorithm from an experimental model constructed at MIT to study aerodynamic instabilities of a gas turbine engine. The experimental facility consists of a slow speed motor driven by a variable plenum volume axial flow compressor. In order to fully investigate the aerodynamic instabilities of the turbo jet engine we need to apply the developed FDI algorithm to a real jet engine so that we may validate the routines. The main focus of our research effort is to solve the real gas turbine engine aerodynamic instabilities problem.

At Georgia Tech's School of Aerospace Engineering, a research team headed by Dr. Carl Nett has recently completed the construction of a single-spool, centrifugal compressor, gas turbine engine. An engine model [3] has been developed based on the experimental stall capable gas turbine engine. The engine can be operated at a speed of 60,000 rpm which is much higher than that of Greitzer's compressor model [4]. The gas turbine engine available at the School of Aerospace Engineering is equipped with a high bandwidth fuel flow, nozzle area, and compressor discharge bleed area servos. The model developed for the experimental engine is based on engine component steady state performance maps and unsteady quasi one-dimensional flow equations. The model has three control inputs, three states, and incorporates the dynamic linkage of the compressor and turbine through the spool. The three states are compressor mass flow rate, plenum pressure and speed of spool.

Our main concern is to apply the FDI routines to an appropriate engine model. For this reason, we borrow C. Nett's basic third order dynamic engine model and expand it to a 6th order engine model. The fuel flow rate, bleed valve, and nozzle dynamics are added to the original third order dynamic model. The objective in increasing the order of the dynamics is primarily to study the system parameter effects on engine instabilities. In Greitzer's compressor model [4], the compressor instabilities are caused by perturbations of the compressor rotor rotating speed and the nozzle area. It is well accepted that engine instabilities are generally caused by the same dynamic variations. The final goal is to apply the hybrid FDI algorithm to the GT experimental engine [3] in order to demonstrate its real-time feasibility on a complex dynamic engineered system - the gas turbine engine. The following section details the development of the 6th order dynamic model for GT's single spool, single stage centrifugal compressor engine.

1.1 The sixth order Single Spool Turbo Jet Engine Model:

Figure 1.1 depicts, in a flow diagrammatic form, this sixth order quasi one-dimensional engine model. The model can be described mathematically in terms of the following ten sections:

1.1.1 Inlet: At the inlet, there is no flow rate, so the inlet pressure P_{t2} and the inlet temperature T_{t2} are the same as ambient pressure P_0 and ambient temperature T_0, respectively,

$$P_0, \quad T_0, \quad given, \quad P_{t2} = P_0, \quad T_{t2} = T_0 \tag{1.1}$$

1.1.2 Compressor : the compressor dynamics are described by the steady state performance map, $C(\dot{m}_c, \omega, P_{t2}, T_{t2})$ which is shown in Figure 1.2. $\dot{m}_c$ is the flow rate in the compressor, ω is the rotating speed of the spool, η_c is the efficiency of the compressor, P_3 is the pressure in the compressor duct, γ is the air specific constant, P_{t3} is the plenum total pressure, and T_{t3} is the plenum total temperature, thus

$$P_{t3} = P_{t2}\, C\,(\dot{m}_c\,,\ \omega\,,\ P_{t2}\,,\ T_{t2}) \tag{1.2}$$

$$T_{t3} = T_{t2}\,\{1 + \frac{1}{\eta_c}\,[\,(\frac{P_3}{P_{t2}})^{\frac{\gamma-1}{\gamma}} - 1\,]\,\} \tag{1.3}$$

1.1.3 Compressor Duct: The flow rate change in the compressor can be described as the additional effects of the pressure difference between the compressor inlet P_3, outlet P_{31} and the axial flow velocity difference between inlet u_3 and outlet u_{31}. A_c is the average area of the compressor duct and L_c is the length of the compressor duct,

$$\frac{d\dot{m}_c}{dt} = \frac{A_c}{L_c}\,(P_3 - P_{31}) + \frac{\dot{m}_c}{L_c}\,(u_3 - u_{31}) \tag{1.4}$$

1.1.4 Plenum : The pressure change in the plenum can be described as mass flow balance and adiabatic compression, where $\dot{m}_b$ is the mass flow of the bleed valve, $\dot{m}_T$ is the flow rate out of

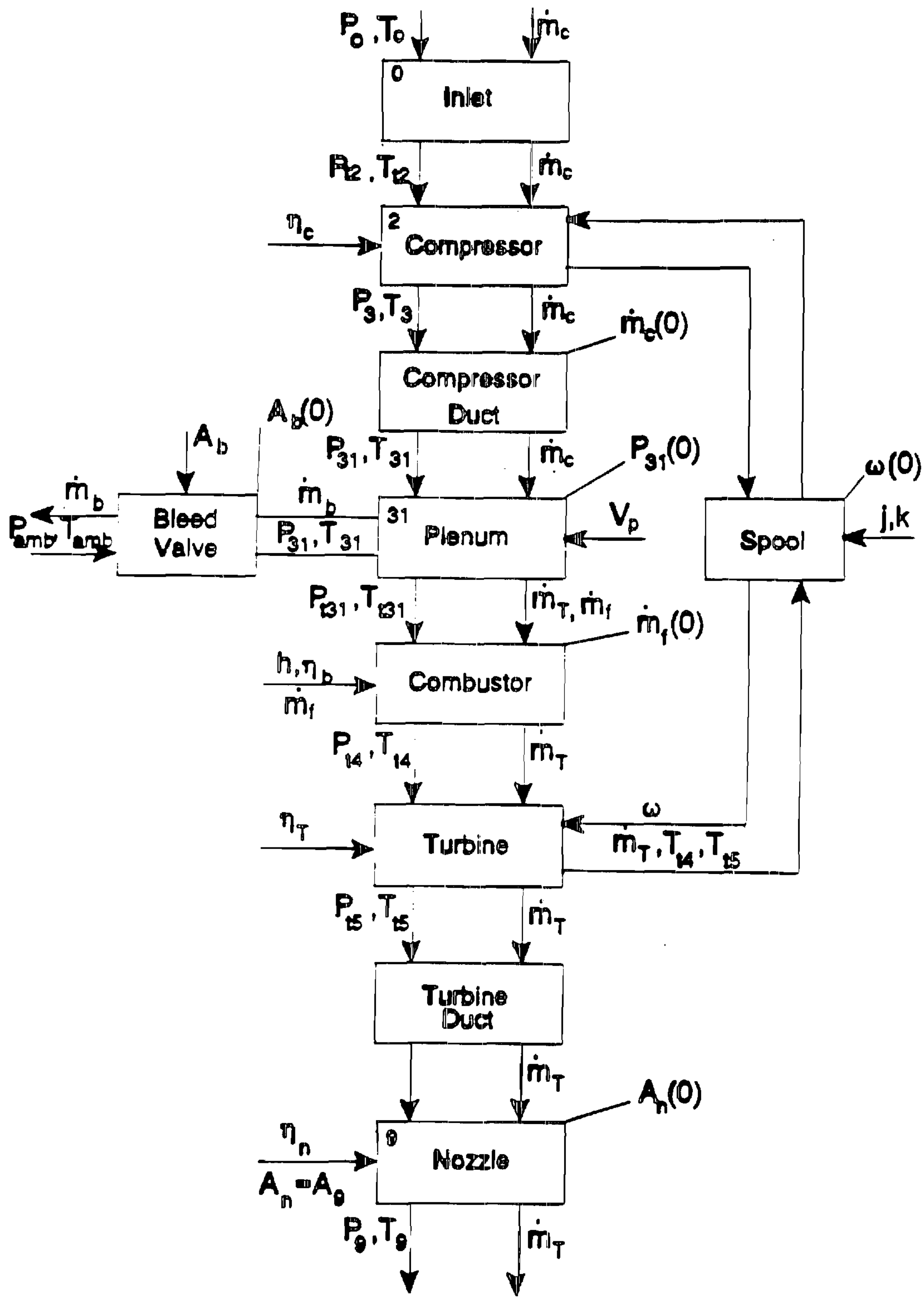


Figure 1.1 Block Diagram of Engine Model

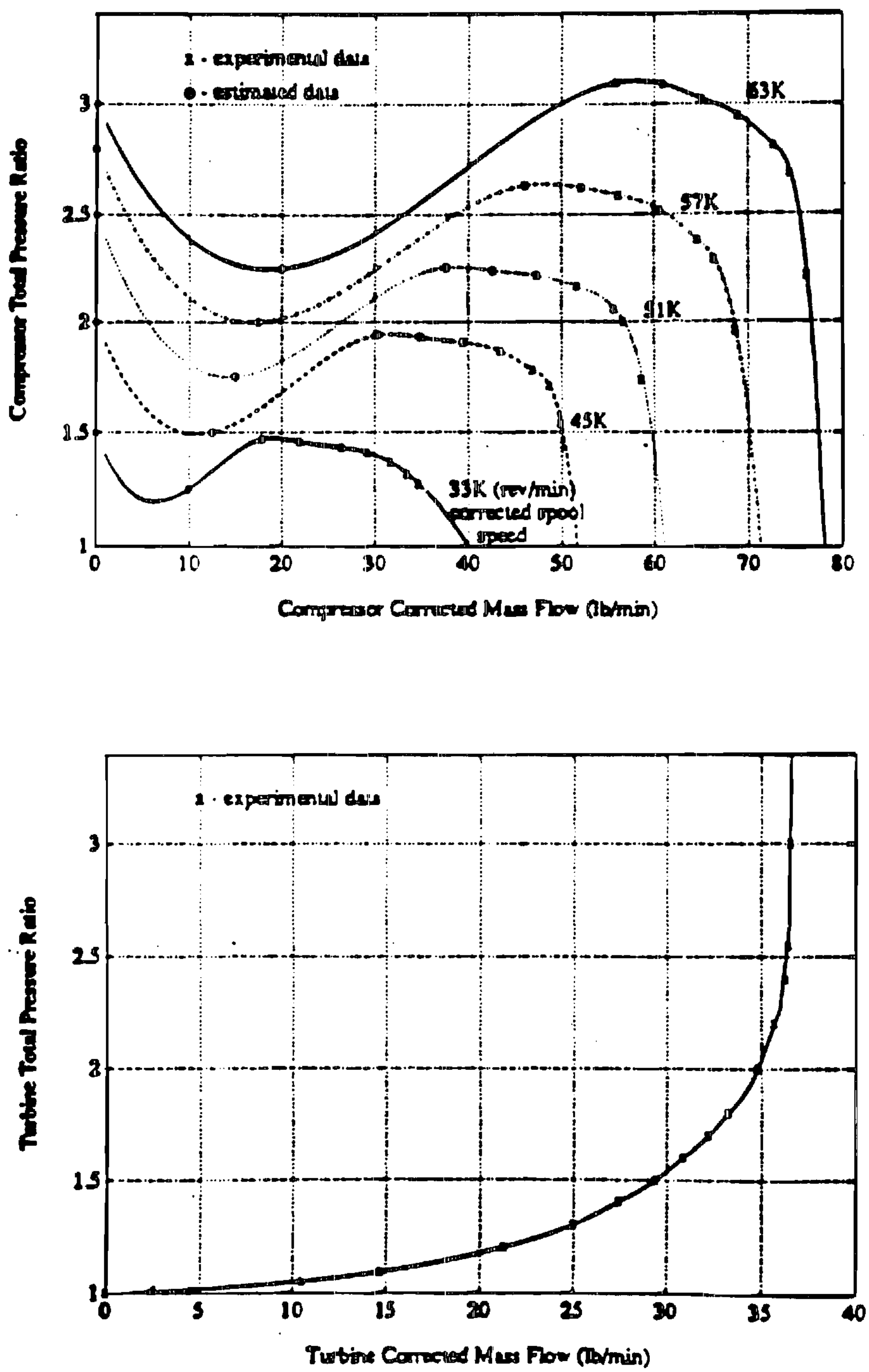


Figure 1.2 Compressor Map

the plenum, $\dot{m}_f$ is the flow rate of the fuel, P_{t31} is the total pressure in the plenum, T_{t31} is the total temperature of the plenum, R is the air constant, and V_p is the volume of the plenum. The relevant equations are :

$$\frac{dp_{31}}{dt} = \frac{\gamma R T_{31}}{V_p}(\dot{m}_c - \dot{m}_b - \dot{m}_T + \dot{m}_f) \tag{1.5}$$

$$\frac{T_{31}}{T_3} = (\frac{P_{31}}{P_3})^{\frac{\gamma-1}{\gamma}} \tag{1.6}$$

$$P_{t31} = P_{31}, \qquad T_{t31} = T_{31} \tag{1.7}$$

1.1.5 Bleed Valve : This valve is used as a damping device to increase the damping of the system dynamics. The device can be described as a simple orifice. P_{amb} is the ambient pressure, T_{amb} is the ambient temperature, A_b is the opening area of the bleed valve, then

$$\dot{m}_b = A_b \sqrt{\frac{2P_{amb}}{RT_{amb}}(P_{31} - P_{amb})} \tag{1.8}$$

1.1.6 Combustor : The combustor section can be modeled as a constant pressure process with a simple heat balance. The temperature change in the combustor is the heat release of the fuel in the combustion process. C_p is the specific heat of air in the constant pressure process, η_b is the efficiency of the combustor, h_f is the enthalpy of the fuel, and T_{t4} is the total temperature at the combustor, thus

$$T_{t4} = \frac{\dot{m}_T C_p T_{t31} + \eta_b \dot{m}_f h_f}{\dot{m}_T C_p} = T_{t31} + \frac{\eta_b \dot{m}_f h_f}{\dot{m}_T C_p} \tag{1.9}$$

$$P_{t4} = P_{t3} \tag{1.10}$$

1.1.7 Turbine : The turbine section is modelled as an adiabatic expansion process. The turbine is described as a steady state performance map $F(\dot{m}_T, P_{t4}, T_{t4})$. The following two equations depict the characteristics of the turbine. P_{t5} is the total pressure at the turbine outlet, T_{t4} is the total temperature at the turbine outlet, and η_T is the efficiency of the turbine,

$$P_{t5} = \frac{P_{t4}}{F(\dot{m}_T, P_{t4}, T_{t4})} \tag{1.11}$$

$$T_{t5} = T_{t4}\{1 - \eta_T[(\frac{P_{t4}}{P_{t5}})^{\frac{\gamma-1}{\gamma}} - 1]\} \tag{1.12}$$

1.1.8 Turbine Duct or (Throttle Duct) : This section is simply modelled as an insulated conducting duct. The pressure P_{t8} and temperature T_{t8} are kept constant through the duct, i.e.

$$P_{t8} = P_{t5} \tag{1.13}$$

$$T_{t8} = T_{t5} \tag{1.14}$$

1.1.9 Nozzle : The nozzle is separated into an unchoked process and a choked process.

- Unchoked process:

$$\frac{P_9}{P_{t8}} > \frac{P_{crit}}{P_{t8}} \quad \sim \quad P_9 > P_{crit} \tag{1.15}$$

$$\dot{m}_T = \frac{A_9}{R}\frac{P_9}{P_{t8}}\frac{P_{t8}}{\sqrt{T_{t8}}}\sqrt{2C_p\eta_n[1-(\frac{P_9}{P_{t8}})^{\frac{\gamma-1}{\gamma}}\;][\frac{1}{1-\eta_n[1-(\frac{P_9}{P_{t8}})^{\frac{\gamma-1}{\gamma}}]}]} \tag{1.16}$$

- Choked process:

$$\frac{P_9}{P_{t8}} < \frac{P_{crit}}{P_{t8}} \quad \sim \quad P_9 < P_{crit} \tag{1.17}$$

$$\dot{m}_T = \frac{P_{t8}}{\sqrt{T_{t8}}}\sqrt{\frac{2\gamma R}{\gamma+1}}\;\frac{A_9}{R}\;[1 - \frac{1}{\eta_n}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}\;\frac{\gamma+1}{2} \tag{1.18}$$

where A_9 is the opening area of the nozzle, and η_n is the efficiency of the nozzle.

1.1.10 Spool : The spool is the mechanical linkage between the compressor and the turbine. The compressor is driven by the turbine, and the inertia dynamics are expressed as

$$J\omega\frac{d}{dt}\omega = C_p\dot{m}_T(T_{14} - T_{15}) - C_p\dot{m}_c(T_{13} - T_{12}) - k\omega^2 \tag{1.19}$$

$$\frac{d}{dt}\omega = \frac{1}{J\omega}[C_p\dot{m}_T(T_{14} - T_{15}) - C_p\dot{m}_c(T_{13} - T_{12}) - k\omega^2] \tag{1.20}$$

ω is the rotational speed of the spool, k is the friction coefficient of the mechanical parts, and J is the rotational inertia of the spool.

1.1.11 State Equations :

The 6th order state equations can be described as follows

$$\frac{d\dot{m}_c}{dt} = \frac{A_c}{L_c}(P_3 - P_{31}) + \frac{\dot{m}_c}{L_c}(u_3 - u_{31}) \tag{1.21}$$

$$\frac{dP_{31}}{dt} = \frac{\gamma R T_{31}}{T_{31}}(\dot{m}_c - \dot{m}_b - \dot{m}_T + \dot{m}_f) \tag{1.22}$$

$$\frac{d\omega}{dt} = \frac{1}{J\omega}[C_p\dot{m}_T(T_{14} - T_{15}) - C_p\dot{m}_c(T_{13} - T_{12}) - k\omega^2] \tag{1.23}$$

$$\frac{d\dot{m}_f}{dt} = \frac{1}{\tau_f}(\dot{m}_{fcd} - \dot{m}_f) \tag{1.24}$$

$$\frac{dA_b}{dt} = \frac{1}{\tau_b}(A_{bcd} - A_b) \tag{1.25}$$

$$\frac{dA_n}{dt} = \frac{1}{\tau_n}(A_{ncd} - A_n) \tag{1.26}$$

1.1.12 State Equations : The six state equations can be rearranged in the following form:

$$\dot{x}(1) = \frac{A_c}{L_c}(P_3 - x(2)) + \frac{x(1)}{L_c}(u_3 - u_{31}) \tag{1.27}$$

$$\dot{x}(2) = \frac{\gamma R T_{31}}{V_p}(x(1) - \dot{m}_b - \dot{m}_T + x(4)) \tag{1.28}$$

$$\dot{x}(3) = \frac{1}{Jx(3)}[C_p \dot{m}_T (T_{t4} - T_{t5}) - C_p x(1)(T_{t3} - T_{t2}) - kx^2(2)] \tag{1.29}$$

$$\dot{x}(4) = \frac{1}{\tau_f}(\dot{m}_{f\infty} - \dot{m}_f) \tag{1.30}$$

$$\dot{x}(5) = \frac{1}{\tau_b}(A_{b\infty} - A_b) \tag{1.31}$$

$$\dot{x}(6) = \frac{1}{\tau_n}(A_{n\infty} - A_n) \tag{1.32}$$

$$\frac{P_{t3}}{P_{t2}} = C(\dot{m}_c, \omega, P_{t2}, T_{t2}) \tag{1.33}$$

where τ_f is the time constant of the Fuel Mass Flow, τ_b is the time constant of the Bleed Valve, τ_n is the time constant of the nozzle, $A_{b\infty}$ is the Bleed Valve Opening Command, $\dot{m}_{f\infty}$ is the Fuel Flow Rate Command, and $A_{n\infty}$ is the Nozzle Opening Command.

We have developed an appropriate computer simulation for this sixth order engine model. Simulation studies are currently being conducted to test model validity and its sensitivity to parametric changes. This engine simulation platform will be linked with the fault detection and identification and control routines. The FDI routines are described in the following section of this report.

2 THE HYBRID FAULT DETECTION AND IDENTIFICATION PROCEDURE

FDI techniques are considered for sensor, actuator, and component failures. Both single and multiple faults are treated. The research objective is to develop a systematic and thorough FDI procedure that is maximally sensitive to failures while avoiding false alarms. The approach is systematic because it relies on a modular architecture to (1) efficiently trigger FDI routines, (2) validate sensor data, (3) combine failure evidence from such diverse sources as analytic redundancy, detection/estimation theory, and limit checking, (4) utilize expert system tools and Dempster-Shafer evidential theory to manage uncertainty and assess the symptomatic evidence, i.e. detect and identify faulty components for monitoring and control purposes. Figure 2.1 is a functional block diagram of the proposed fault diagnostics approach.

Fault triggering is based on a deviation of system performance from its planned trajectory. Sensor data validation involves the detection of sensor failures through a combined parity space (redundancy-based) and detection/estimation algorithm. The combined FDI architecture is depicted schematically in Figure 2.2.

The validation procedure compares each reading to all other like readings. The "best" estimate from a set of good measurements is defined as that value which gives the minimum of a particular function of the measurements. Where more than the minimum number of measurements is available, the "best" estimate is taken in the least squares sense, i.e., it is that value which minimizes the square of the length of the measurement error vector.

Fault detection and identification uses validated sensor and analytic data to analyze the status of the system components. Fault diagnosis involves the incremental accumulation of evidence in the form of symptoms to determine the "health" of the system components. Parity space representation and analytic redundancy [5-10]. in addition to limit checking and statistical estimation techniques may be applied to accumulate the required evidence. Parity space representation transforms an array of redundant measurements into a new array, called a parity vector, in such a way that the true value of the underlying variable is suppressed, leaving only components of the measurement errors as elements of the parity vector [6]. In parity space, the parity vector remains near zero when the redundant measurements are consistent, i.e., when no faults are present. When a fault occurs, the parity vector grows in magnitude, and its direction of growth is uniquely associated with the faulty measurement.

The fault detection part of the algorithm capitalizes upon the behavior of the magnitude of the parity vector to detect the presence of a fault. The magnitude of the parity vector $\boldsymbol{P}$ is,

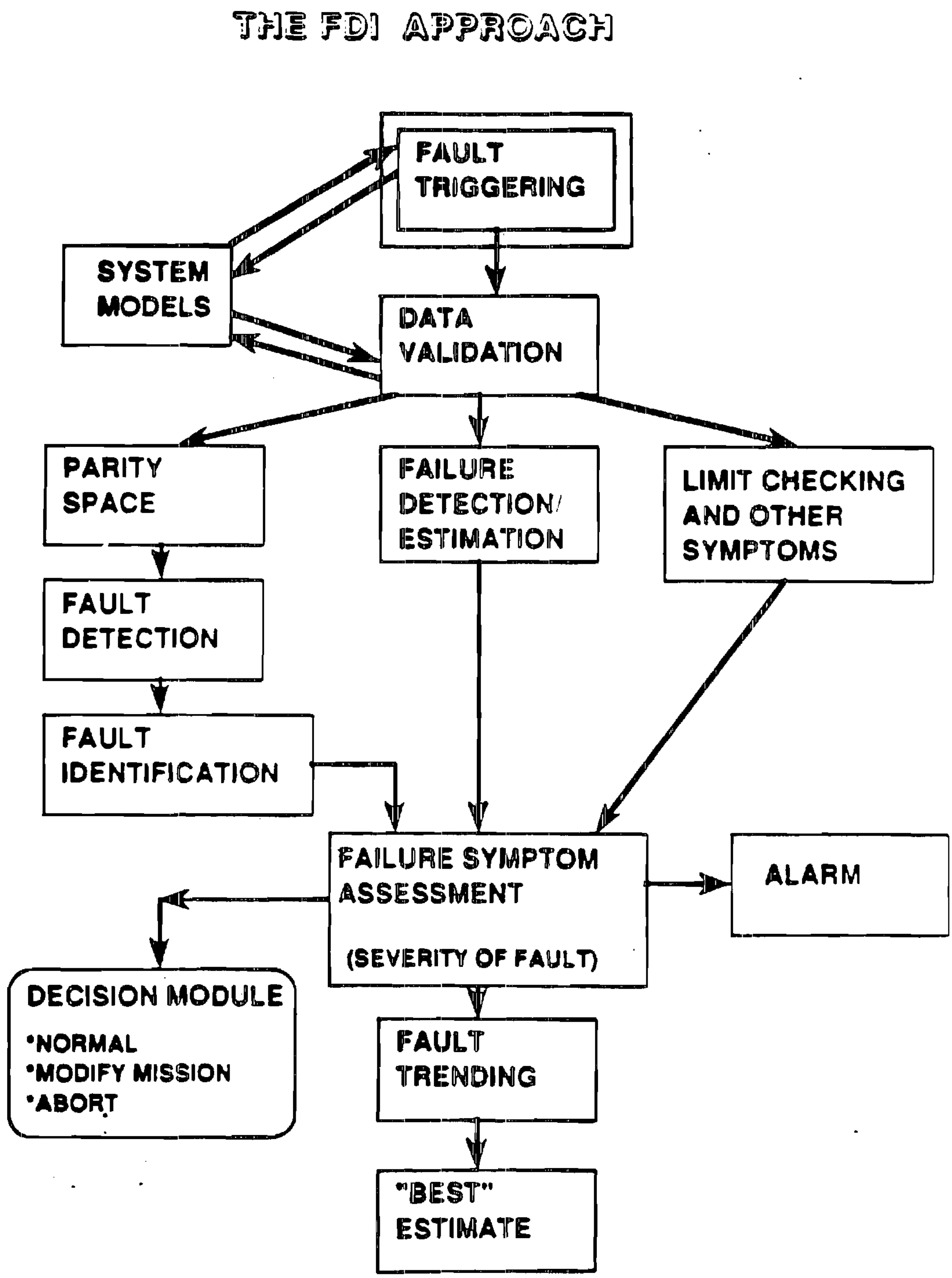


Figure 2.1 The Failure Detection and Identification Approach

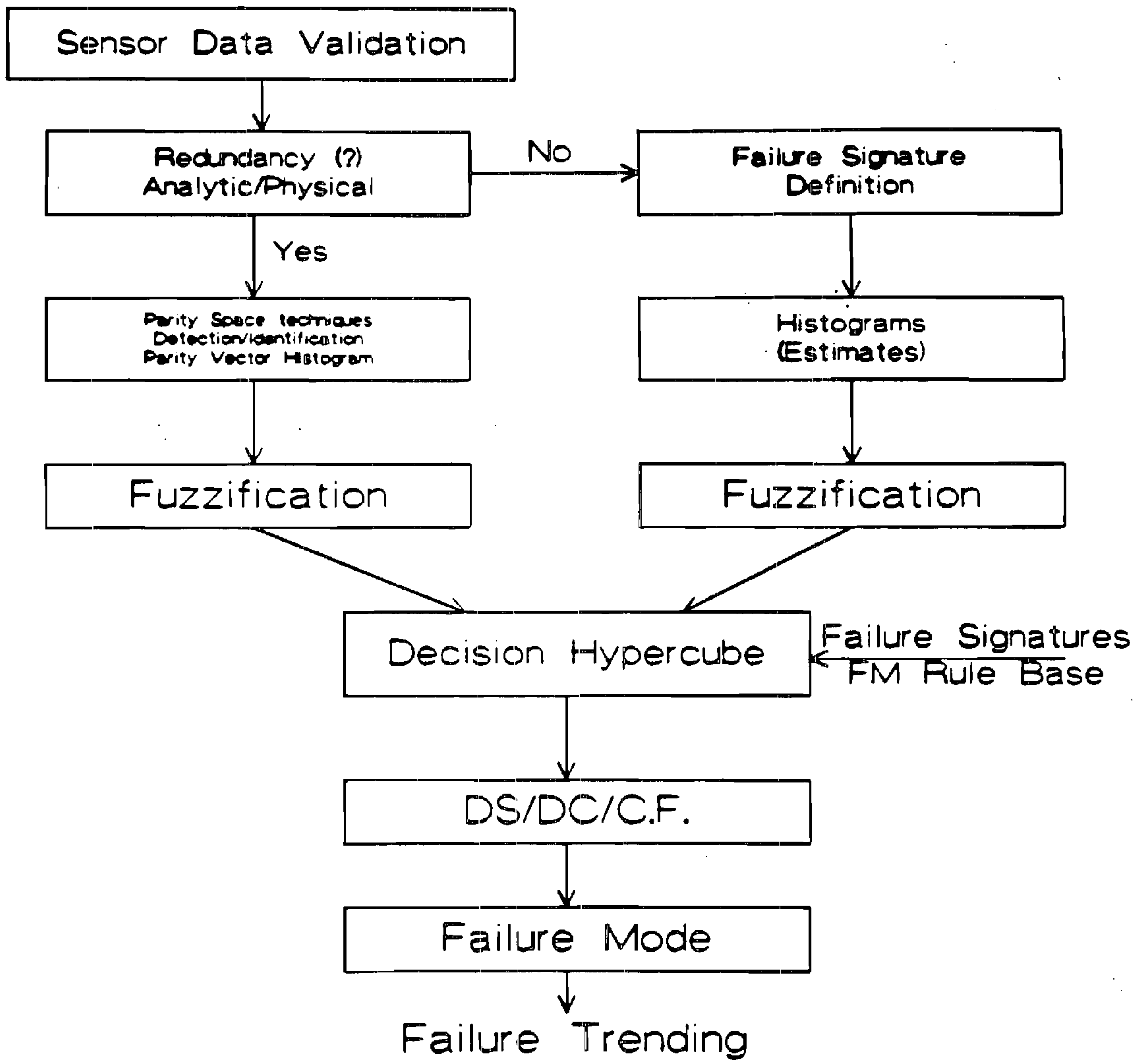


Figure 2.2 The combined FDI architecture

in general

$$P^T P = \sum_{i=1}^{l} \varepsilon_i^2 - \frac{1}{l} \left(\sum_{i=1}^{l} \varepsilon_i\right)^2 \tag{2.1}$$

$$\delta_l = \max(P^T P) = \sum_{i=1}^{l} b_i^2 - \frac{1}{l}\left(\sum_{i=1}^{l} b_i\right)^2 \tag{2.2}$$

If the sensor noise probability density function is uniform and if the error in each measurement for an unfailed component is assumed to be bounded, i.e., $|\epsilon_i| < b_i$, then A search algorithm is used to locate a minimum of Σb_i and, finally, δ_l is obtained from (2.2).

Fault detection is based upon the relative magnitude od the maximum allowable error bound compared to the length of the parity vector, i.e., δ_l/P^TP. A threshold condition is reached when $P^TP=\delta_l$. In general, both δ_l and P^TP are not known accurately and, therefore, a fuzzy representation of the quantity δ_l/P^TP is most appropriate. The compositional rule of inference [11] is used to infer the degree of severity of the fault condition.

The fault identification part of the program is intended to identify which component is faulted. Following [6], we propose a technique based upon the concurrent checking of the relative consistency of small size subsets of measurements. In the absence of the redundant data, a scheme based upon stochastic estimation is called to perform the fault detection and identification function.

The stochastic estimation scheme consists of two processing units. First, fault triggering plays the role of a failure sensitive filter at the learning state. A recursive parameter estimation technique is used to determine the direction and size of the parameter changes in real-time from the failure signature. Once the FDI system is alert, the histograms of parameter residues are evaluated and transformed into the corresponding fuzzy sets as decision samples at the tracking stage. In the second step, the corresponding fuzzy input vectors are applied to fuzzy relational matrices which undertake the identification of the failure mode utilizing the concept of possibility theory. Thus, multiple failures can be identified using the knowledge base. The combined analytical/intelligent approach relaxes many assumptions raised in other FDI schemes such as the conditions of a linear time-invariant system [12] and left invertibility from the failure mode to the output [11] as well as the requirement of a bank of Kalman filters [13][14]. Figure 2.3 shows the systematic stages of FDI required for potential system failures.

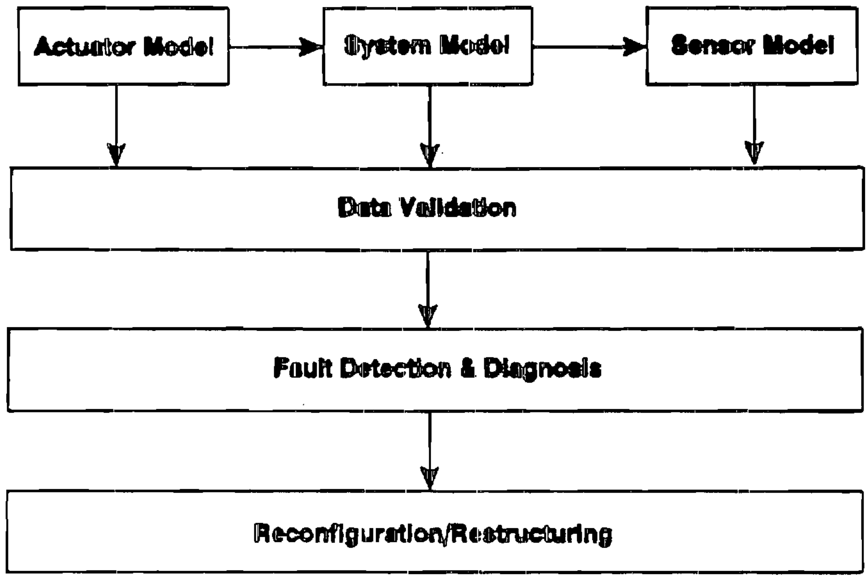


Figure 2.3 Block Diagram of FDI modules

The advantages of knowledge based approaches to FDI are flexibility of failure information, intelligence in the failure representations, and direct description of failure modes. When uncertainty is inevitable, Zadeh's possibility theory [1] can be applied to FDI systems as one of the most efficient methods of coping with uncertainty. Fuzzy semantics can be applied to the intelligent part of FDI where soft bodies of evidence are used for detecting and identifying a potential failure. Let us limit the FDI scheme to the case of completely known failure modes.

We define by "failure" the deviation of some parameters that are physically responsible for the abnormal behavior of the system. The FDI scheme can be expressed as a stochastic parameter estimation problem. A failure mode is defined as a one-to-one mapping relationship with a failure symptom such as "the impulse line is leaking" or "the motor inertia is very large". In fact, the failure mode is directly related to the system parameters and can be additively incorporated in a failure model. Therefore, we can easily interpret a failure mode as a physical representation of a failure signature is defined as a time history of each failure mode.

The actual system may be described by a state space representation in terms of discrete nonlinear vector equations of the form,

$$x(t+1) = f(x(t),\theta(t),u(t)) \tag{2.3}$$

$$z(t) = h(x(t),\theta(t)) \tag{2.4}$$

where $u(t) \in U$ and $z(t) \in Z$ are the scalar input and output, respectively; $\theta(t) \in \Theta$ is the time-varying system parameter vector; $x(T) \in \chi$ denotes the state vector; and f, h represent the nonlinear vector fields for the state and output vectors, respectively. A nonlinear model can be obtained for a class of nonlinear systems where the corresponding equations are linear in the parameter vector $\theta(t) \in \Re^n$ such as

$$z(t) = \phi(t,\bar{\theta})^T\,\theta(t) \tag{2.5}$$

where $\bar{\theta}$ is the nominal solution for the parameters - $\bar{\theta}$ may be a partially known set of $\theta(t)$ that does not require the estimation process - during the <u>time window</u> $J_T=[t_0,t_0+t]$ and $\phi(t,\bar{\theta}) \in \mathbf{R}^{p\times n}$ is the nonlinear regressor associated with z(t-i) for $i=1\ldots r_1$ and u(t-d-j) for $j=0\ldots r_2-1$ (d is an integer designating the system delay and $n=r_1+r_2$). In the linear case, it is easy to derive the above relation from an auto-regressive moving-average (ARMA) model,

$$A(q^{-1},\theta(t))z(t) = q^{-d}B(q^{-1},\theta(t))u(t) \tag{2.6}$$

where q is the shift operator; $A(q^{-1},\theta)$ is a monic matrix polynomial; and $B(q^{-1},\theta)$ is a matrix polynomial. The transfer function $H(q^{-1},\theta)$ is

$$H(q^{-1},\theta) = q^{-d}A(q^{-1},\theta)^{-1}B(q^{-1},\theta) \tag{2.7}$$

in the linear multivariable case.

The failure model can be represented by

$$\theta(t+1) = \theta(t) + \omega(t) + \xi(t) \tag{2.8}$$

$$z(t) = \phi(t)^T\,\theta(t) + \nu(t) + \eta(t) \tag{2.9}$$

where $\omega(t) \in \mathbf{R}^n$ and $\nu(t) \in \mathbf{R}^p$ are white gaussian parametric and measurement disturbances, respectively; $\phi(t)=\phi(t,\bar{\Theta}) \in \mathbb{R}^n$ is the regression vector; $\xi(t) \in \mathbf{R}^n$is the system plus actuator failure signature while $\eta(t) \in \mathbf{R}^p$ represents the sensor failure signature. For initial conditions, it is required that $\theta_0 = E[\theta(0)$ and $P_0 = Cov[\theta(0) = \theta_0)$.

We assume that failure modes are unique and the associated failure signatures do not occur simultaneously in one parameter. It is required that the failure should occur inside the time-window $J_T = [t_0, t_0 + T]$ in order to get the nominal parameter values

$$\bar{\theta} = \lim_{\theta \to t_f^-} \theta(t) \tag{2.10}$$

where t_f^- denotes the time just before the failure. Let the parameter residue be $\theta(t) \in \tilde{\theta}$. $\theta(t+1) = \theta(t) + \omega(t)$ when there is no failure in $(t < t_f)$,

$$\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta} \tag{2.11}$$

where $\hat{\theta}(t)$ is the estimate of the parameter $\theta(t)$. The parameter residues are quantized in a finite number of bins, q_i, with respect to the i-th element of $\tilde{\theta}(t)$ in order to generate a histogram of the parameter residues by counting the frequency of each bin $\tilde{\theta}_k$, $f^i = \{f^i\}Q_i$ for $k = 1 \ldots q_i$ and for $i = 1 \ldots n$ and θ_k is the bin symbol with a total number of q_i bins. Here, we call the vector $f^i \in \mathbf{R}^{q_i}$ the failure signature histogram (FSH) of the i-th system parameter. The time-window function W_T is introduced such that

$$W_T : \tilde{\Theta} \rightarrow Q \tag{2.12}$$

In the normal case, these failures are

$$\xi(t) = 0, \quad \eta(t) = 0. \tag{2.13}$$

<u>Failure model operation</u>: Let us assume that either one of $\xi(t)$ or one of $\eta(t)$ is non-zero and let $\xi(t) \neq 0$ for simplicity. The steady state assumption assures that

$$\hat{\theta}(t) \rightarrow \bar{\theta} \;\; as \;\; t \rightarrow \infty \tag{2.14}$$

but if the failure occurs at $t = t_f < T$,

Therefore, the parameter residue $\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta}$ plays an important role in representing the unique failure signature by the certainty equivalence principle. This failure signature can be obtained

$$\hat{\theta}(t) \rightarrow \bar{\theta} + \sum_{r=t_f}^{T} \xi(r) \quad as \quad t \mapsto \infty \tag{2.15}$$

by the windowing function W_T that stores the residue $\tilde{\theta}(t)$ in J_T into the histogram f^θ. Note that $\xi(t)$ of the impulse function shows a biased failure signature in the parameter while $\xi(t)$ of the step function corresponds to a parameter drift. The block diagrams of the FDI scheme are shown in Figures 2.4 and 2.5.

The failure identification method is based on a rulebase implemented by a multi-dimensional decision making system called "Fuzzy Decision Hyper-Cube" (FDHC). Given a FSH f^θ, there exists a transformation from the probabilities to the corresponding fuzzy sets μ^θ which is based on possibility theory. Thus, each failure mode is represented uniquely by a failure signature fuzzy set (FSF). We also define a priori failure mode fuzzy sets (FMF) which correspond to each crisp failure mode.

The fuzzy hyper-cube is constructed along each rule on the basis of a priori knowledge of FSF and FMF. A matching or inferencing process is used to derive the resultant and FMF output which is excited by FSF vector μ^θ. There may be several rules fired by the matching process, but only one failure mode is obtained. It is suggested that the number of rules for each parameter FSF may exceed the number of failure modes for almost perfect recall.

If there are more than one FSF available, then we combine the fuzzy output vectors (FMF's) together in order to make certain that we arrive at the true failure mode using Dempster's rule of combination of evidence.

We demonstrate the proposed approach with a simple example. A motor speed control system is considered with a two-dimensional parameter vector $\theta(t)=[\alpha,\kappa]$ describing possible failure modes such as "the motor has stalled". The fuzzy relational matrix is constructed from statements of the form "if κ is high and α is low, the motor has stalled". "κ is high" is an FSF derived from the failure signature histogram. "The motor stalls" is a failure mode fuzzy set (FMF). If inferencing declares the failure motor "motor stalls", then Dempster-Shafer theory assigns a certainty factor to this failure mode. A decision on the occurrence of a specific failure mode is made only when the certainty factor exceeds a predetermined threshold limit. Thus, the scheme accounts for both ignorance (limited data base) and uncertainty (noise, unmodeled dynamics, parameter variations) resulting in a reliable and robust fault detection method.

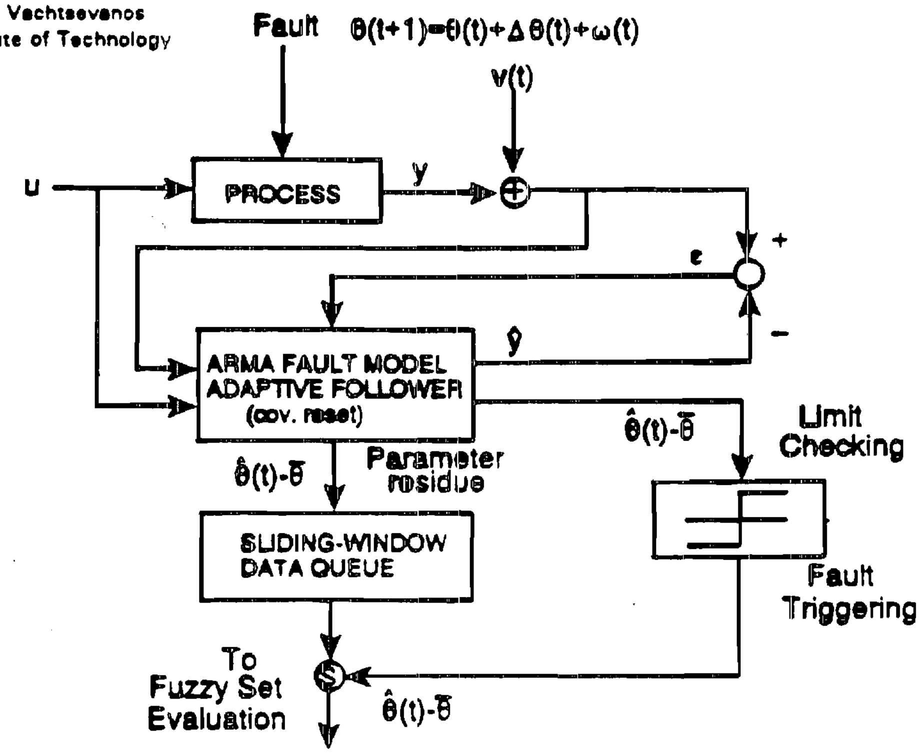


Figure 2.4 Analytic part of FDI

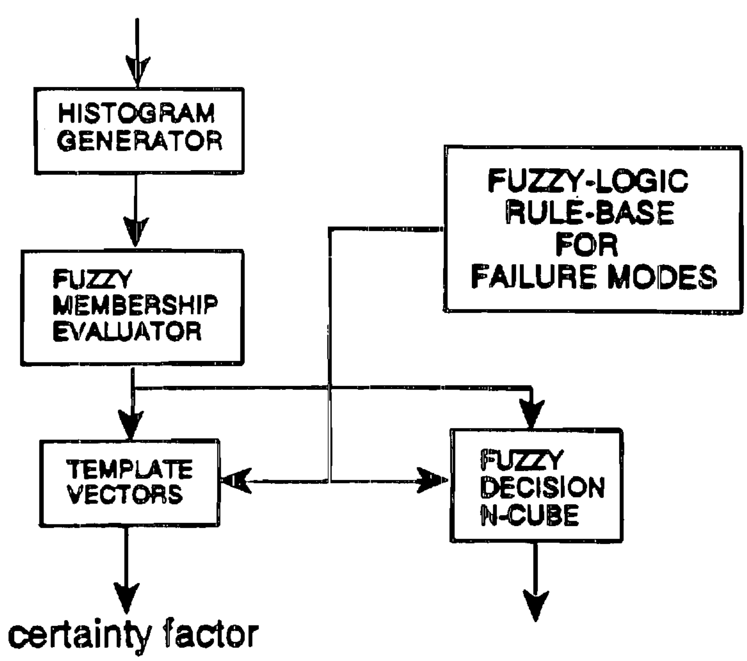


Figure 2.5 Intelligent part of FDI

Figures 2.6 and 2.7 depict a block diagram of the FDI modules and computer results that illustrate the fuzzy relational matrix for this example, fuzzy membership functions, the failure signature detection (single-fault, multiple-faults, and false alarm cases) and the failure mode identification algorithm. Figures 2.8, 2.9 and 2.10 show typical computer simulation results for an impending surge, a rotating stall, and an abrupt surge instability condition in an axial flow compressor. Since the fuzzy decision hyper-cube and the template vector use the same fuzzy failure signature distributions, the results are both consistent and complementary.

Our research team has developed a unified methodology to the fault-tolerant control problem that addresses not only FDI issues, as discussed above, but also such basic concerns as fault propagation, system restructuring and controller reconfiguration. These concerns are essential in the design of control logic for autonomous systems that demand a high degree of survivability.

For large scale complex dynamical systems, such as the jet engine or the integrated shipboard electric power system, efficient application of FDI and control strategies requires the accurate and timely processing of data from multiple sensors. It is essential, therefore, for us to consider the basic problem of integrating or fusing and processing reliably real-time monitored information for intelligent decision-making processes. We examine this topic briefly in the next section. Sensor fusion has been an active area of our research endeavor and we are currently developing efficient algorithms to accomplish this objective.

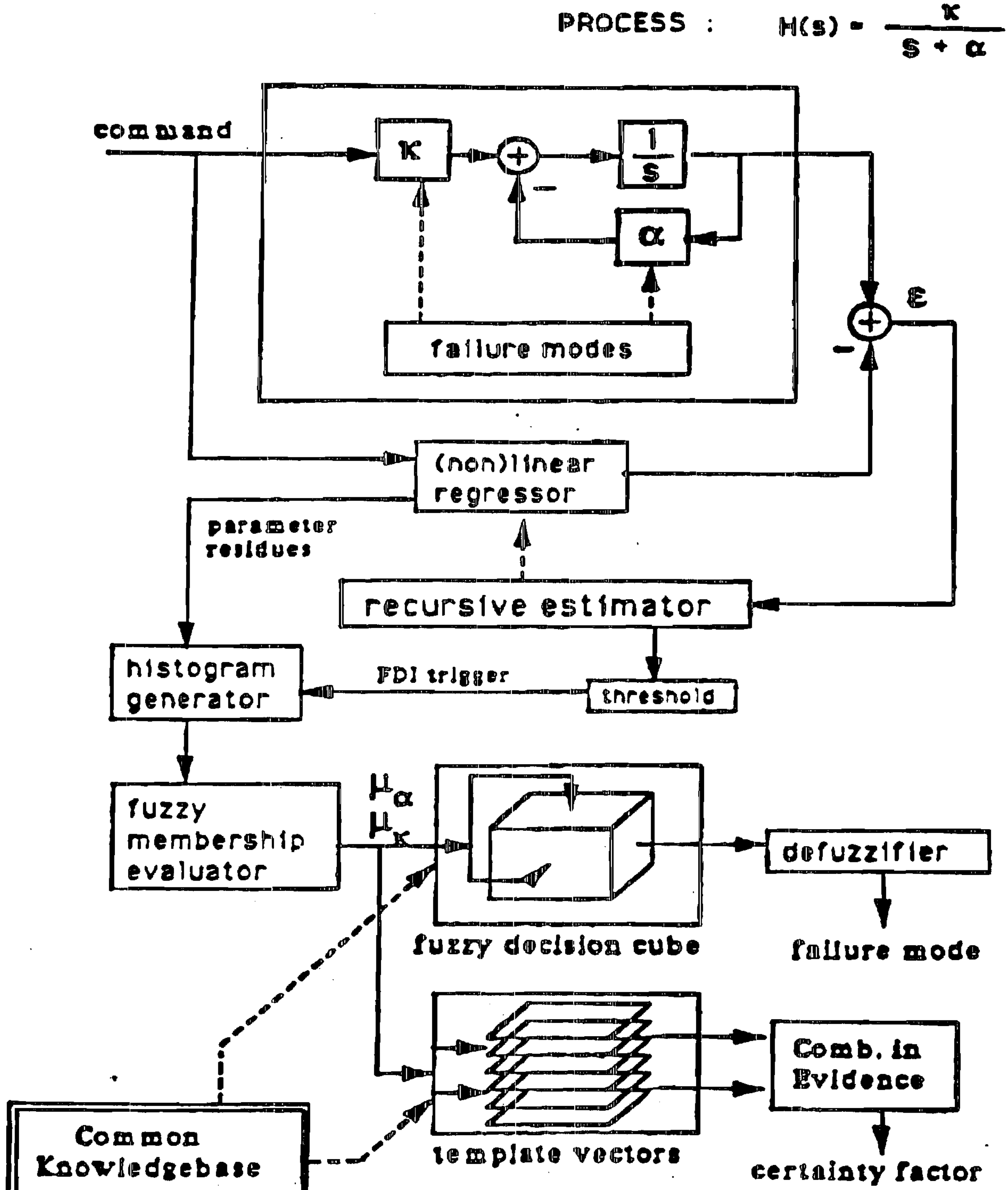


Figure 2.6 FDI structure for the Demonstration Example

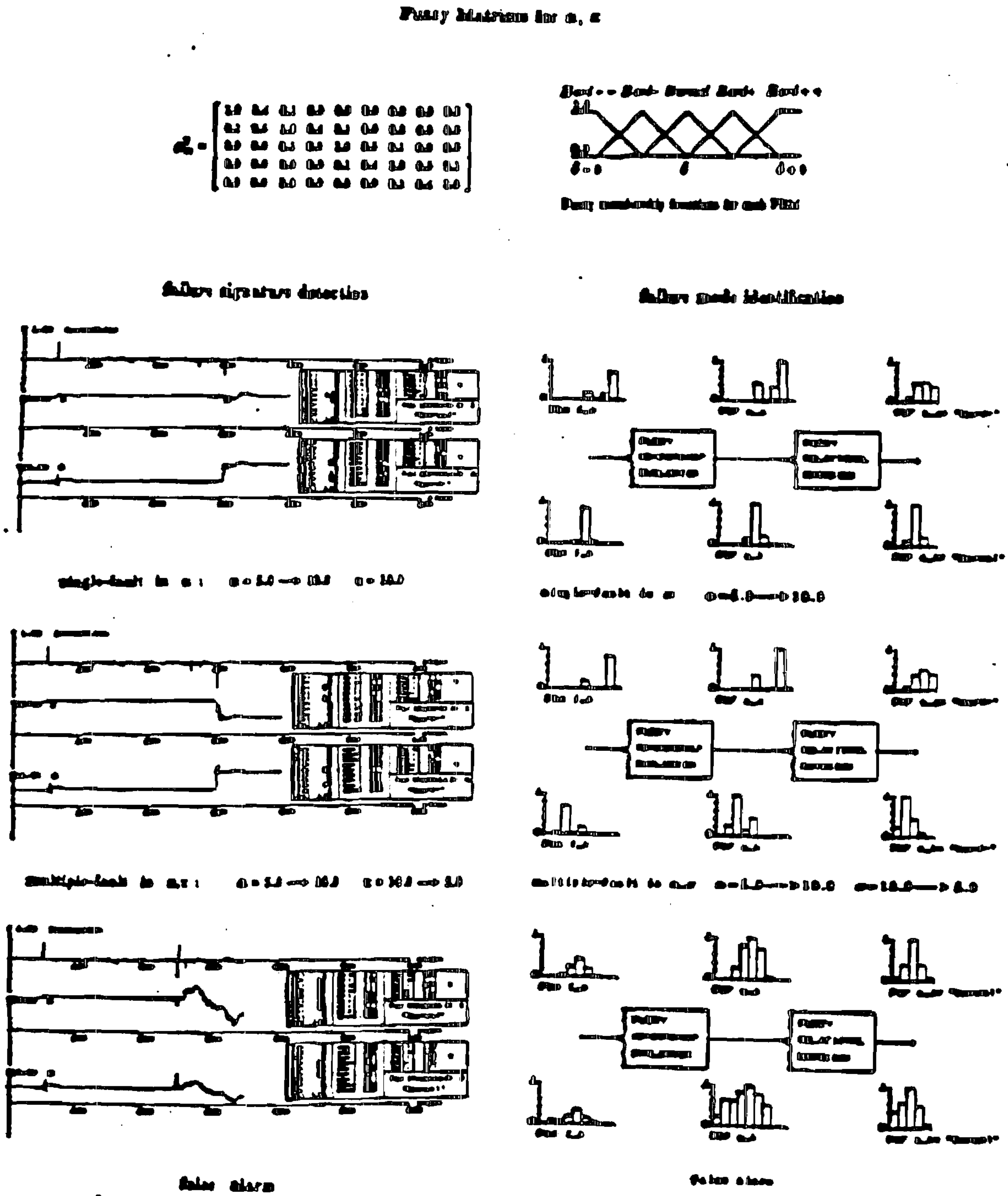


Figure 2.7 Simulation Results for Demonstration Example

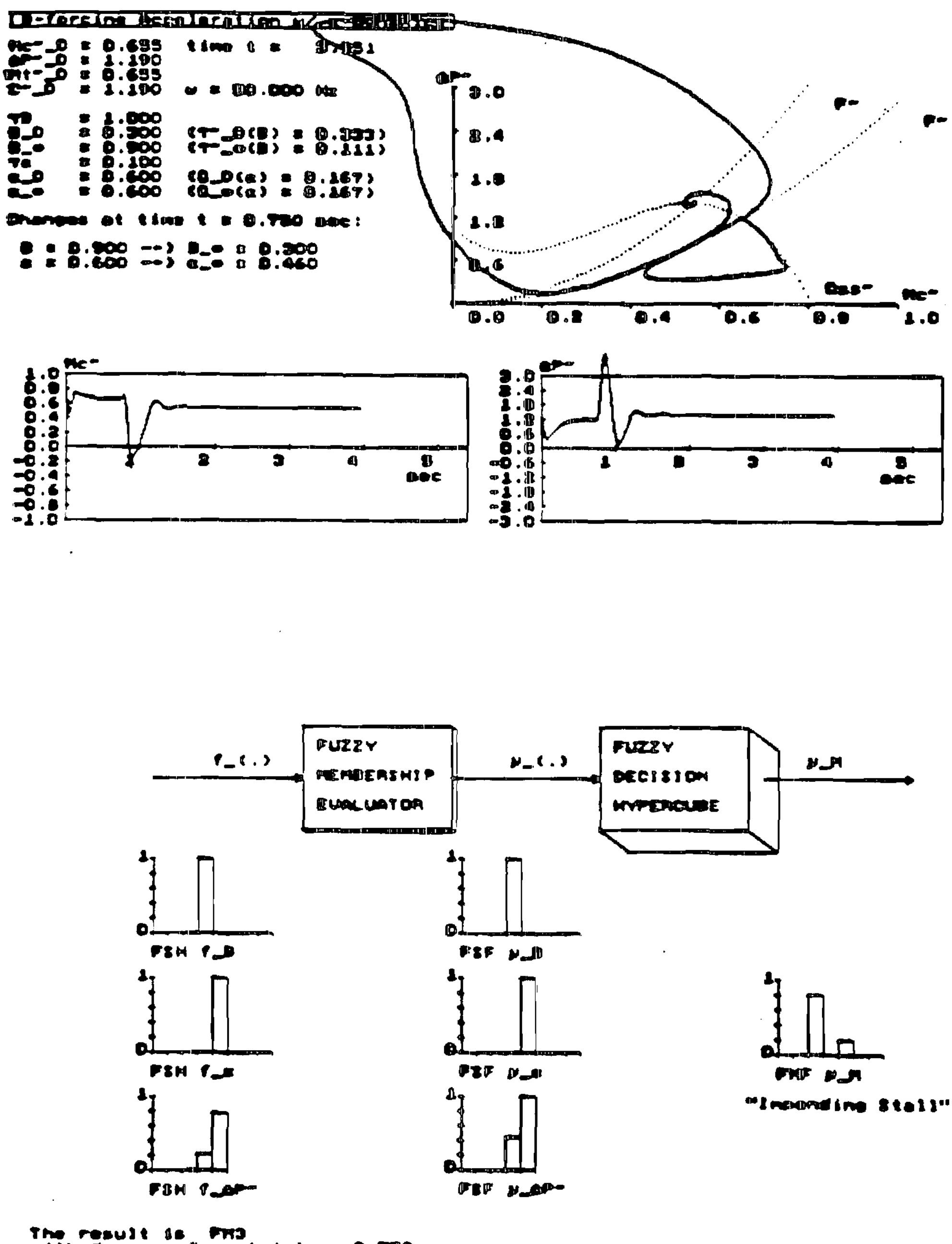


Figure 2.8 Simulation Result for an Impending Surge Instability

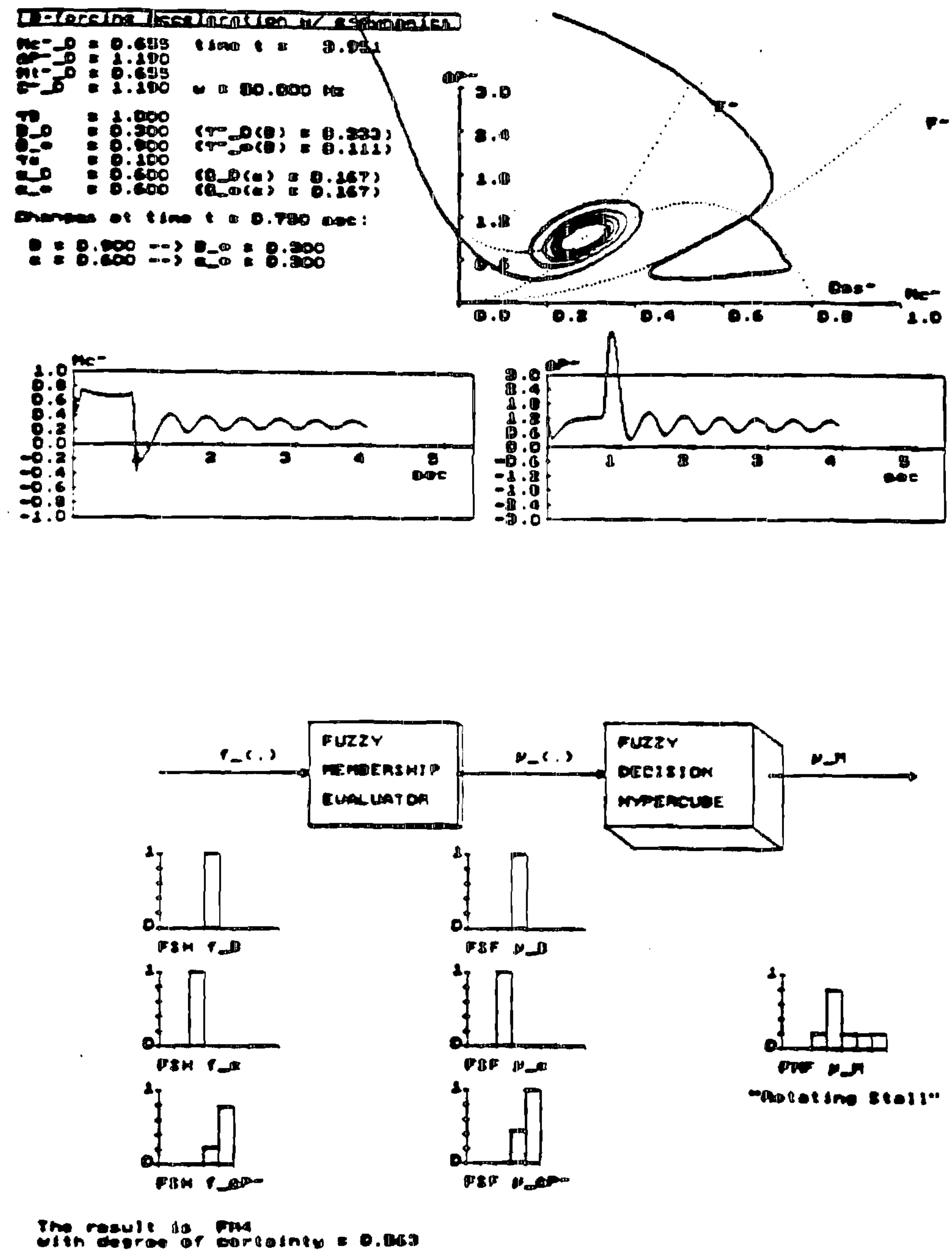


Figure 2.9 Simulation Result for a Rotating Stall Instability

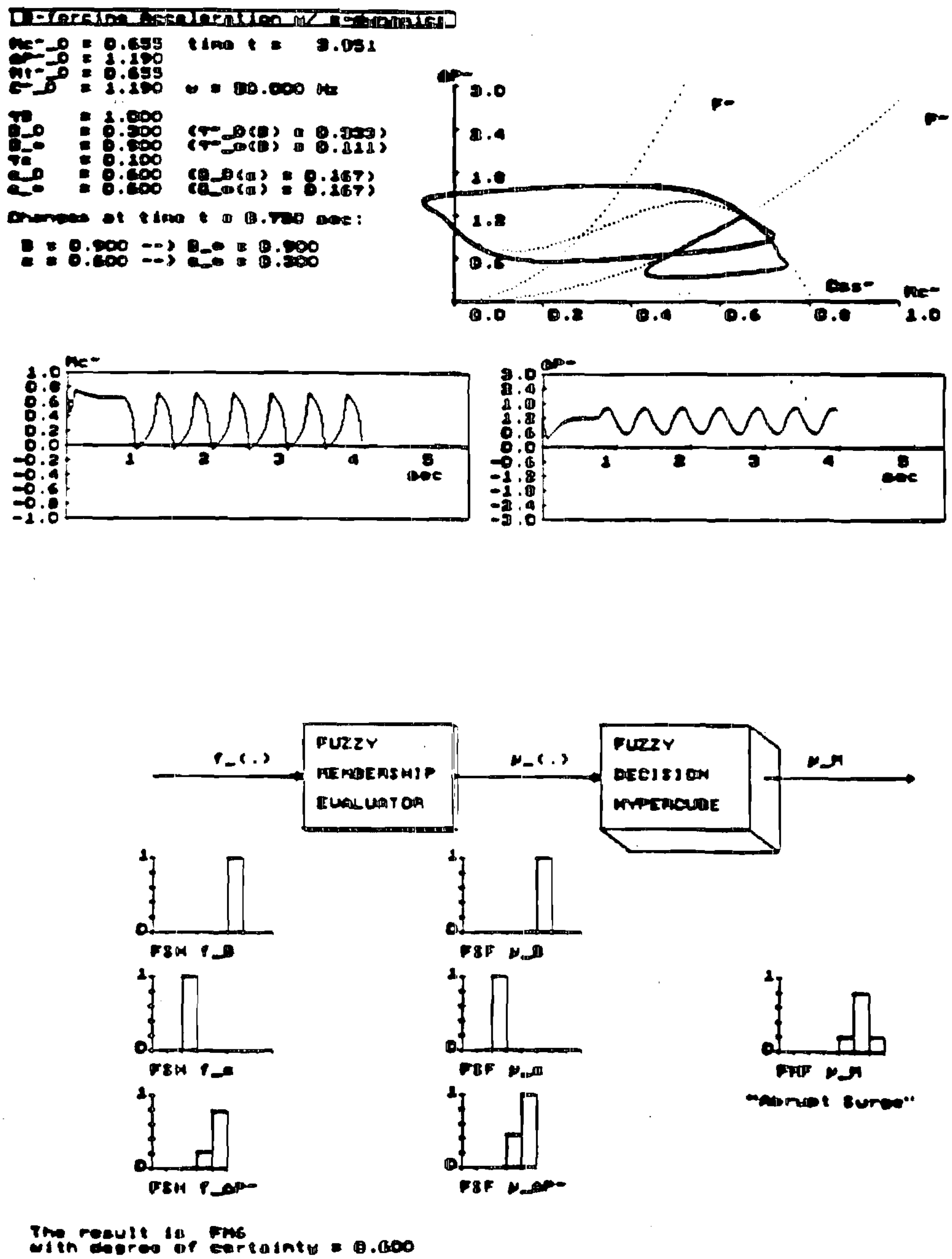


Figure 2.10 Simulation Result for an Abrupt Surge Instability

3 SENSOR FUSION

The concept of sensor fusion can be applied to sensor data from two viewpoints: sensor data validation and noise reduction. It is of course assumed that the sensors are somehow dependent on each other such that a measurement of a specific sensor can be obtained from those of other sensors through analytic relations. This technique is known as analytic redundancy which, in contrast to physical redundancy, is an alternative method to generate redundant data.

The fusion of data from several sensors, whether they constitute analytic redundancy, or physical redundancy, may bring about a number of advantages. Sensor data validation and noise reduction tend to improve the accuracy and the efficiency of the FDI and control routines.

3.1 Sensor data validation

The FDI routine assumes that sensors are operating in a normal state and the mechanism of inferencing is based on the same assumption. It is very important to ascertain whether sensors function properly, because any sensor failure may result in a catastrophic event.

The FDI routine makes a decision when the degree of certainty(DOC) is above a preset threshold. Failure on a sensor produces a low DOC value which prolongs the decision making process. Accordingly, it is not possible to make a timely decision and the system under observation will be at the risk of deteriorating to a catastrophic state. In order to exclude data from a failed sensor, it is necessary to check the validity of the sensor data.

We may rely on it to derive the available FDI rule base to assess the validity of sensor data in terms of conflicting factors between active sensors. The conflicting factor, κ, is obtained by

$$\kappa = \sum_{A \cap B = \emptyset} m_1(A)\, m_2(B) \tag{3.1}$$

where A and B are any focal elements of sensors 1 and 2, respectively.

[example]
Assume that there are three sensors employed for a measurement. The result in terms of basic assignments is the following:

$m_1(A)=0.5$	$m_1(A,B)=0.3$	$m_1(A,B,C)=0.2$
$m_2(C)=0.6$	$m_2(C,A)=0.3$	$m_2(C,A,B)=0.1$
$m_3(A)=0.7$	$m_3(A,B,C)=0.3$	

where A, B, and C indicate the failure modes. Then the conflicting factors κ's are given by (3.1). Since the common number of subscripts for κ is 2, it is suspected that sensor 2 has failed.

One of the most important reasons for using Dempster-Shafer theory is to resolve conflicting information as shown in the above example. It is difficult to distinguish whether the conflict occurs due to a sensor failure or noise in the corrupted signal, but using a rule of thumb, a larger value of the conflicting factor typically implies a sensor failure. For the efficient use of sensors, it is recommended that specific sensors, whose conflicting factors are large, are excluded from consideration, otherwise the faulty data may procrastinate the decision making process.

3.2 Noise reduction

The most primitive form of sensor fusion can be found in averaging measurements from sensors to derive an accurate signal representation without noise,

$$x(t) = s(t) + n(t)$$

where $x(t)$, $s(t)$ and $n(t)$ are the measurement, signal and noise, respectively. It is generally assumed that the statistics satisfy a Gaussian distribution with a zero mean. Therefore, it can be claimed that for large n, the average of n measurements is close to the actual signal level. Thus, for the purpose of obtaining more accurate data, data fusion or simply averaging data from several analytic redundant sources, may be considered.

4. Fault Tolerant Control-Reconfiguration

Suppose that a failure occurs in a subsystem or component of a complex large scale dynamic system. The FDI routine, described in the previous sections of this report, identifies the failure (or impending failure) and reports this event to the control logic. The failure(s) may impair the system's operational capability or may even lead to a catastrophic event, if left unattended. It is desirable, therefore, to assess first the impact of the fault on other healthy system units and to restructure and/or reconfigure the system and its control authority to assure survivability. This section of the report describes a systematic methodology that we have developed to address the fault-tolerant control problem.

With the fault information provided by the fault detection and identification scheme, the faulted large scale system can remain in a survivable state via system restructuring and/or controller reconfiguration. Typical possible transitions, from a fault tolerance point of view, of a large scale system are shown in Figure 4.1, where

S: Normal large scale system
S^f: Faulty large scale system
S^{rs}: Restructured large scale system
S^{rc}: Reconfigured large scale system

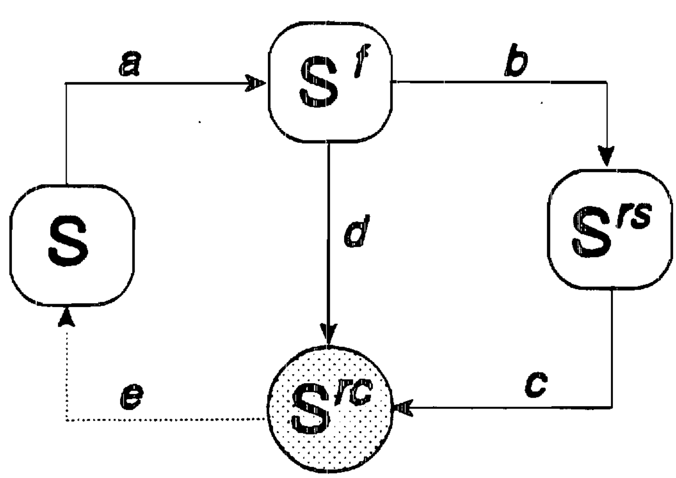


Figure 4.1 Transition of Large Scale System under failure

and the fault-tolerant issues are classified as

Path (a) $S \Rightarrow S^f$ Fault Diagnosis
Path (b) $S^f \Rightarrow S^r$ Restructuring
Path (c) $S^r \Rightarrow S^{rc}$ Reconfiguration
Path (d) $S^f \Rightarrow S^{rc}$ Reconfiguration without restructuring
Path (e) $S^{rc} \Rightarrow S$ Repairing

Our focus is on path (b), path (c), and path (d), which are described below.

<u>Restructuring Problem</u>: Determine which subsystem(s) should be isolated in order for the overall system to be controllable with the available control authority and to block the effect of any failure on other healthy subsystems.
<u>Reconfiguration Problem</u>: Reconfigure the controllers so that the overall (restructured) system is stable (survivable) with possibly maximum performance.

Control of a complex large scale system typically involves three distinct but interrelated modules:

- Mission Planner (MP)
- Decision Making (DM)
- Control Action (CA)

The mission planner sets a mission plan based on the functional capability of the system and the conditions of the environment in which the system operates. The functional capability of the LSS is described in terms of various operational constraints. The mission plan is modified on the basis of the environmental changes and the limits of various operational variables. The task of estimating the limits of the operational variables is undertaken by DM. Obviously, these limits are quite dependent on the system status. Based on the modified mission plan, the next main task of DM is to determine the proper controller-reconfiguration and to command the CA to perform the specfied action.

In practice, a failure event may cause failure(s) of healthy system units sequentially, and eventually the LSS may enter a catastrophic state. In this respect, analysis of the impact of the failure on healthy system units is required in order for the reconfiguration action to be determined and completed accurately and efficiently. With fault information from the Fault Propagation module, DM re-estimates the limits of various operational variables and sends such information to MP. Next, the mission planning module sets a modified possible mission plan under the occurred failure. Figure 4.2 shows the functional block diagram of the proposed fault tolerant control philosophy. Before detailing the reconfiguration algorithm, some useful definitions need to be stated.

Definition 1 (Survivability, Fault-Tolerance) The large scale system is said to be survivable with respect to the *i*th subsystem if the system has the capability to perform the certain critical functions without the *i*th subsystem; it is then said to be fault tolerant with respect to the *i*th subsystem.

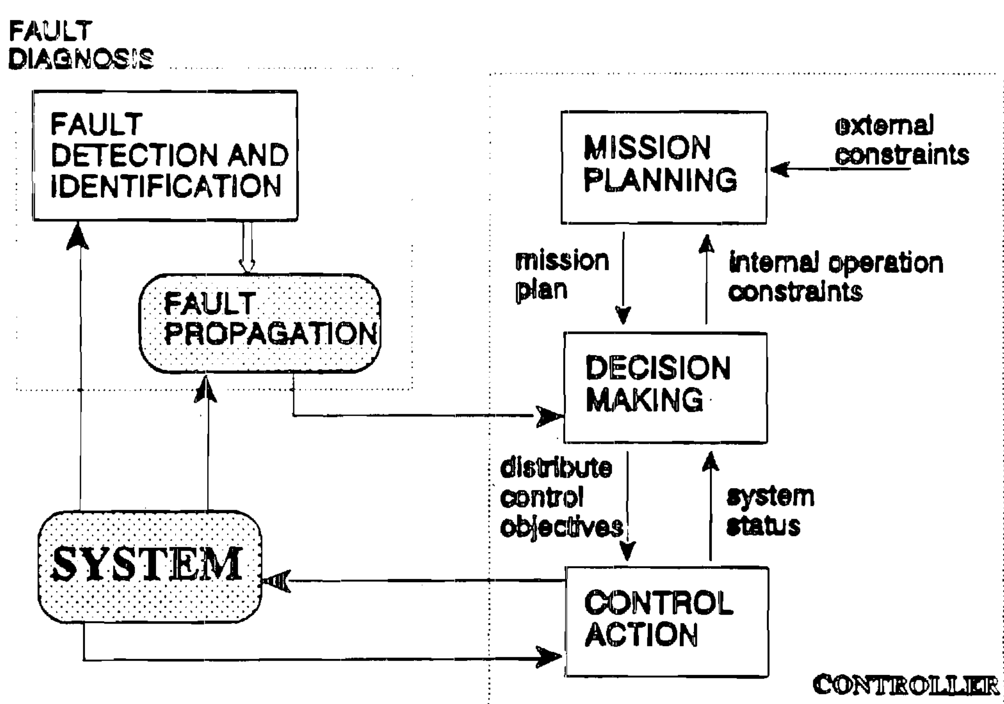


Figure 4.2 Block Diagram of Fault Tolerant Control System

Definition 2 (Restructurability, Reconfigurability, Restructurable System)

- The large scale system is said to have the property of restructurability with respect to the *i*th subsystem if the *i*th subsystem can be isolated.
- The large scale system is said to have the property of reconfigurability with respect to the *i*th subsystem if the restructured system with respect to the *i*th subsystem is structurally controllable, and stabilizable.
- The large scale system is said to be restructurable with respect to the *i*th subsystem, if the large scale system has both properties of restructurability and reconfigurability with respect to the *i*th subsystem.

Definition 3 (Critical Units)

The set of subsystems and appropriate interconnections which are required to absolutely perform those functions that allow the whole system to be in a survivable state are called critical units of the system. The removal of a critical unit results in a catastrophic event.

As for the faulted large scale system, the specific tasks required to perform the proper fault accommodating action are as follows:

Based on the failure information provided by the FDI module,

1. Analyze the impact of the failure on other healthy subsystems.
2. If necessary, restructure the system by isolating system units.
3. Determine (modify) the control objectives.
4. Perform a properly reconfigured control action to meet the (modified) control objectives.

Although the proposed fault accommodating philosophy promises to accomplish an effective control action, it is necessary that the large scale system possess certain structural properties for restructurability and reconfigurability. We have investigated a structured system which possesses the required structural properties. Block triangular structure large scale systems (BTS LSS) are considered in detail in the following subsection.

4.1 BLOCK TRIANGULAR STRUCTURE LARGE SCALE SYSTEM

Consider the unforced large scale system with l subsystems described as

$$\begin{aligned} \dot{X} &= F(X,t) \\ \dot{x}_i &= f_i(x_i,t) + h_i(X,t) = f_i(x_i,t) + \sum_{j=1}^{l} h_{ij}(x_j,t) \end{aligned} \tag{4.1}$$

It may be rewritten in matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_l \end{bmatrix} = \begin{bmatrix} f_1(x_1,t) & h_{12}(x_2,t) & \cdots & \\ h_{21}(x_1,t) & f_2(x_2,t) & & \\ \vdots & & \ddots & \\ h_{l1}(x_1,t) & \cdots & & f_l(x_l,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{4.2}$$

With respect to (4.2), the *interconnection matrix (E)* is determined as

$$E = [e_{ij}] \quad where \quad e_{ij} = \begin{cases} 1, & if\ h_{ij} \neq 0 \\ 0, & otherwise \end{cases}$$

The structure of the system is dictated by the interconnection matrix E. A class of systems is said to have block triangular structure (BTS), if $e_{ij}=0$ $(\forall\ i<j)$. The BTS LSS is described in matrix form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_l \end{bmatrix} = \begin{bmatrix} f_1(x_1,t) & 0 & \cdots & 0 \\ h_{21}(x_1,t) & f_2(x_2,t) & & \\ \vdots & & \ddots & \\ h_{l1}(x_1,t) & \cdots & & f_l(x_l,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{4.3}$$

As shown in Figure 4.3, the subsystems in a BTS LSS are interconnected in a uni-directional form. In general, sparsity is one of the structural properties of a large scale system. Many engineered systems can be realized as block triangular structure large scale systems (BTS LSS) by rearranging their subsystems via a square permutation matrix, P, with only one non-zero element exactly equal to 1 in each row and in each column,

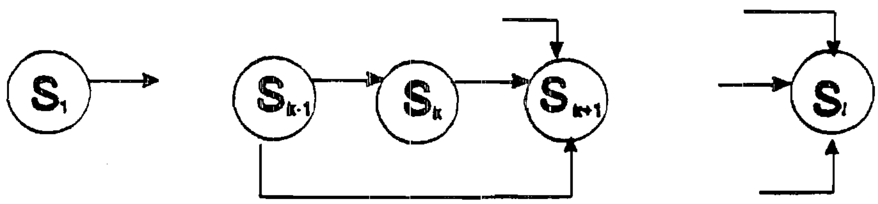


Figure 4.3 Block Triangular Structure (One-way interconnected) Large Scale System.

$$PEP^T = \hat{E}$$
$$\hat{E}=[\hat{e}_{ij}] \quad where \quad \hat{e}_{ij}=0 \quad \forall \ i<j \tag{4.4}$$

Similarly, large scale systems can be realized in BTS form by acyclic decomposition employing graph theoretic techniques. The following example illustrates how to realize a given large scale system as BTS LSS.

(Example 1) [BTS realization]

Consider the large scale system with 5 subsystems described by

$$\dot{X} = F(X,t), \quad or$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} f_1(x_1,t) & h_{12}(x_2,t) & h_{13}(x_3,t) & 0 & h_{15}(x_5,t) \\ 0 & f_2(x_2,t) & 0 & 0 & h_{25}(x_5,t) \\ 0 & h_{32}(x_3,t) & f_3(x_3,t) & 0 & h_{35}(x_5,t) \\ h_{41}(x_1,t) & 0 & 0 & f_4(x_4,t) & h_{45}(x_5,t) \\ 0 & 0 & 0 & 0 & f_5(x_5,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.5}$$

The direct graph (digraph) of the given system is shown in Figure 4.4(a). Even though it may appear complicated, there are no cycles or loops. Thus, it can be realized in BTS form by rearranging the subsystems, i.e.,

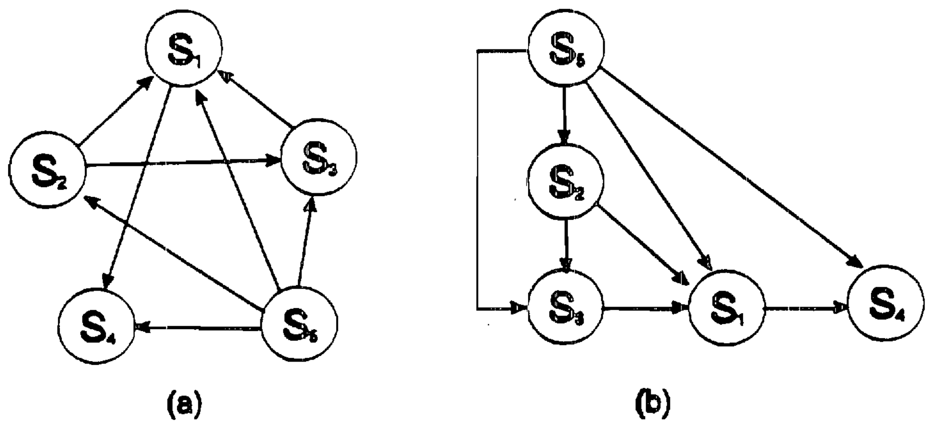


Figure 4.4 Digraphs of Example 1

$$\bar{X} = P\,X, \quad \bar{E} = PEP^T$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \bar{X} = \begin{bmatrix} x_5 \\ x_2 \\ x_3 \\ x_1 \\ x_4 \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Then (4.5) becomes

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_1 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} f_5(x_5,t) & 0 & 0 & 0 & 0 \\ h_{25}(x_5,t) & f_2(x_2,t) & 0 & 0 & 0 \\ h_{35}(x_5,t) & h_{32}(x_2,t) & f_3(x_3,t) & 0 & 0 \\ h_{15}(x_5,t) & h_{12}(x_2,t) & h_{13}(x_3,t) & f_1(x_1,t) & 0 \\ h_{45}(x_5,t) & 0 & 0 & h_{41}(x_1,t) & f_4(x_4,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{4.6}$$

and its digraph is shown in Figure 4.4(b). ††

As illustrated, a large scale system is BTS realizable if there is no cyclic path among its subsystems. A large scale system with cyclic connections between some of its subsystems may also be BTS realizable by aggregating such subsystems into one subsystem. The following example illustrates this case.

(Example 2) [BTS realization]

Consider the system (4.5) in the previous example. Suppose that there is one more path from S_4 to S_1, as shown in Figure 4.5(a). There is only one cycle in the digraph, consisting of S_1 and S_4. By aggregating S_1 and S_4 into one subsystem, the given system can be realized in BTS form as shown in Figure 4.5(b).

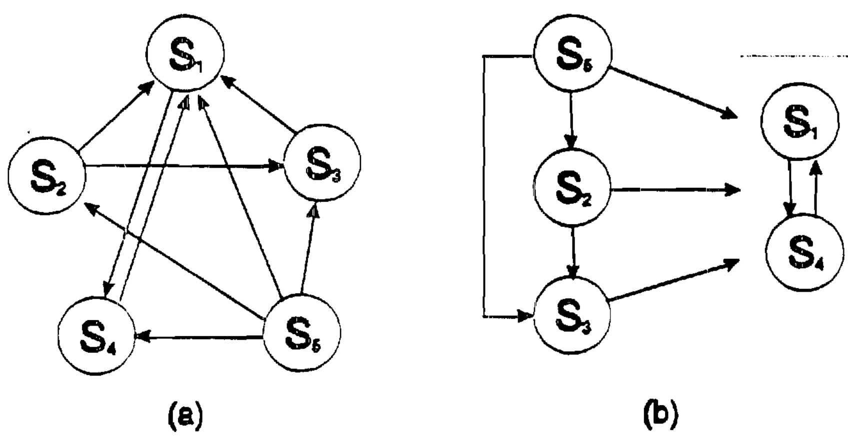


Figure 4.5 Digraphs of Example 2

The resulting system equations in BTS form are

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_1 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} f_5(x_5,t) & 0 & 0 & 0 & 0 \\ h_{25}(x_5,t) & f_2(x_2,t) & 0 & 0 & 0 \\ h_{35}(x_5,t) & h_{32}(x_2,t) & f_3(x_3,t) & 0 & 0 \\ h_{15}(x_5,t) & h_{12}(x_2,t) & h_{13}(x_3,t) & f_1(x_1,t) & h_{14}(x_4,t) \\ h_{45}(x_5,t) & 0 & 0 & h_{41}(x_1,t) & f_4(x_4,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{4.7}$$

For convenience, let us consider the linear LSS described by

$$\begin{aligned}\dot{X} &= AX + BU \\ Y &= CX\end{aligned} \tag{4.8}$$

where

$$X \in \Re^{n},\quad U \in \Re^{m},\quad Y \in \Re^{p}$$
$$A \in \Re^{n \times n},\ B \in \Re^{n \times m},\ C \in \Re^{p \times n}$$

Then the ith subsystem is

$$\begin{aligned}\dot{x}_i &= A_i x_i + B_i u_i + \sum_{j=1}^{l} A_{ij} x_j \\ y_i &= C_i x_i\end{aligned} \tag{4.9}$$

where

$$X_i \in \Re^{n_i},\quad U_i \in \Re^{m_i},\quad Y_i \in \Re^{p_i}$$
$$A_i \in \Re^{n_i \times n_i},\ B_i \in \Re^{n_i \times m_i},\ C_i \in \Re^{p_i \times n_i}$$
$$\sum_{i=1}^{l} n_i = n,\quad \sum_{i=1}^{l} m_i = m,\quad \sum_{i=1}^{l} p_i = p$$

The interconnection matrix (E) is

$$E = [e_{ij}] \quad where \quad e_{ij} = \begin{cases} 1, & if\ A_{ij} \neq 0 \\ 0, & otherwise \end{cases}$$

if there exists a permutation matrix (P) described by (4.9), then the following transformed system has block triangular structure

$$\frac{d\tilde{x}}{dt} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}$$
$$where\ x = P^T\tilde{x},\ u = P^T\tilde{u},\ \tilde{A} = PAP^T,\ \tilde{B} = PBP^T$$

Suppose that the controller consists of a local and a global controller, i.e.

$$u_i = u_i^l + u_i^g$$

then (4.9) is rewritten as

$$\dot{x}_i = A_i x_i + B_i \mu_i^l + \sum_{j=1}^{l} A_{ij} x_j + B_i \mu_i^g \tag{4.10}$$

the ith isolated subsystem is described by

$$\dot{x}_i = A_i x_i + B_i \mu_i^l$$

With the state feedback controller as

$$u_i^l = -k_i x_i, \quad u_i^g = -\sum_{i=1, i \neq j}^{l} k_{ij} x_{ij}$$

The following theorem gives a sufficient condition for transforming an LSS into BTS form, if physically feasible.

<u>Theorem 1</u> (Transformation to BTS) A linear LSS (4.8)-(4.10) is restructurable (reconfigurable) to a BTS LSS, by applying a global controller, if the following condition is satisfied,

$$\text{Range } A_{ij} \in \text{Range } B_i \quad \forall \ i<j \quad (i,j = 1, \ldots, l)$$

(proof) We can prove this by finding appropriate global feedback gains such that

$$A_{ij} - B_i k_{ij} = 0 \qquad \forall \ i<j$$
$$E = [e_{ij}] \quad \text{where } e_{ij} = 0 \quad \forall \ i<j$$

Since Range $A_{ij} \in$ Range B_i,

$$A_{ij} = B_i L_{ij}$$

Thus

$$B_i L_{ij} - B_i k_{ij} = 0 \quad \forall \ i<j$$

$$k_{ij} = L_{ij}$$

Then, the resulting system has block triangular structure

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_l \end{bmatrix} = \begin{bmatrix} A_1 - B_1 k_1 & 0 & \cdots & 0 \\ A_{21} - B_2 k_{21} & A_2 - B_2 k_2 & & \vdots \\ \vdots & & \ddots & 0 \\ A_{l1} - B_l k_{l1} & \cdots & & A_l - B_l k_l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_l \end{bmatrix} \tag{4.11}$$

4.1.1 Stability of BTS LSS

In general, it is difficult to check the stability of an LSS because of its high dimensionality and complex interconnections. However, for the BTS LSS, this is not difficult when local stability (the stability of ISS) can be verified with ease. The following theorem reveals the relation between the stability of LSS and that of ISS.

Theorem 2 (Stability of BTS LSS) If all of the subsystems are stable, then the linear BTS LSS is stable. In other words, if the linear LSS with l subsystems has the block triangular structure, the overall system is stable if all of its subsystems are stable, i.e.

$$\text{stable } S_i \ \ \forall\, i \rightarrow \text{ stable } S$$

(proof) Consider the linear LSS described in (4.11). The ith isolated subsystem is

$$\dot{x}_i = (A_i - B_i k_i) x_i$$

From the stability of S_i, let us assume that

$$A_i - B_i k_i = \Lambda_i = diag\left\{ \begin{bmatrix} -\sigma_1^i & \omega_1^i \\ -\omega_1^i & -\sigma_1^i \end{bmatrix}, \cdots, \begin{bmatrix} -\sigma_p^i & \omega_p^i \\ -\omega_p^i & -\sigma_p^i \end{bmatrix}, -\sigma_{p+1}^i, \cdots, -\sigma_{n_i-p}^i \right\}$$

Then, an appropriate Lyapunov function for the ith isolated subsystem is chosen as

$$V_i = (x_i^T H_i x_i)^{\frac{1}{2}}, \qquad V_i\colon R^n \to R_+$$

where

$$\Lambda_i^T H_i + H_i \Lambda_i = -G_i$$

$$G_i = 2\alpha_i diag\{\sigma_1^i, \sigma_1^i, \cdots, \sigma_p^i, \sigma_p^i, \sigma_{p+1}^i, \cdots, \sigma_{n_i-p}^i\}$$
$$H_i = \alpha_i I_i$$

Since the α_i's are arbitrary positive constants, choose $\alpha_i = 1$ for simplicity. Then the relations

$$V_i = \|x_i\|$$
$$\dot{V}_i \le -\rho_i \|x_i\|$$

are satisfied, where

$$\rho_i = \frac{1}{2}\lambda_m(G_i) = \min_j \sigma_j^i$$

With regard to the interconnected system

$$\dot{x}_i = (A_i - B_i k_i)x_i + \sum_{j=1}^{l} (A_{ij} - B_i k_{ij})x_j \tag{4.12}$$

the derivative of the Lyapunov function is described by the following inequality

$$\begin{aligned} \dot{V}_i &\le -\rho_i \|x_i\| + \sum\nolimits_{j=1}^{l} \|(A_{ij}+B_i k_{ij})x_j\| \\ &\le -\rho_i \|x_i\| + \sum\nolimits_{j=1}^{l} \|A_{ij} - B_i k_{ij}\| \, \|x_j\| \\ &\le -\rho_i V_i + \sum\nolimits_{j=1}^{l} \xi_{ij} V_j \end{aligned} \tag{4.13}$$

For the overall system, choose a vector Lyapunov function as

$$V = (V_1, V_2, \cdots, V_l)^T, \qquad V\colon R^n \to R_+^l \tag{4.14}$$

From (4.13) and (4.14), we have the following aggregation form

$$\dot{V} \leq WV$$

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_l \end{bmatrix} \leq \begin{bmatrix} -\rho_1 & 0 & \cdots & 0 \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & 0 \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_l \end{bmatrix} \tag{4.15}$$

where W is a Metzler matrix. By the comparison principle, if W is stable, then the LSS is stable. For stability of a Metzler matrix, necessary and sufficient conditions are expressed by the Sevastyanov-Kotelyanskii theorem as follows

The $n \times n$ Metzler matrix $A = (a_{ij})$ is stable if and only if

$$(-1)^n \ \det \begin{bmatrix} a_{11} & a_{12} & & a_{1n} \\ a_{21} & a_{22} & & \\ \vdots & & \ddots & \\ a_{n1} & & & a_{nn} \end{bmatrix} > 0$$

The aggregation matrix for the BTS LSS, W in (4.15), satisfies the Sevastyanov and Kotelyanskii condition, since

$$(-1)^l \det \begin{bmatrix} -\rho_1 & 0 & \cdots & 0 \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & 0 \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} = (-1)^{2l} \rho_1 \rho_2 \cdots \rho_l \ > 0$$

Thus, the BTS LSS consisting of l stable interconnected subsystems is stable. ††

Theorem 3 (Stabilizability of BTS LSS) For the LSS with BTS, the overall system is stabilizable simultaneously when the local system is stabilizable, i.e.,

$$S_i \text{ is stabilizable } \forall i \rightarrow S \text{ is stabilizable}$$

(proof) It follows from Theorem 2. ††

Theorem 4 (Stabilization of linear LSS) The linear LSS is globally stabilized by stabilizing the local systems using a decentralized multilevel state feedback controller if the following conditions are satisfied,

(i) (A_i, B_i) are completely controllable
(ii) Range $A_{ij} \in$ Range $B_i \quad \forall\, i<j \quad (i,j = 1, \ldots, l)$

(proof) Condition (i) implies stabilizability (pole assignability) in the local subsystem and condition (ii) implies that the given system is restructurable to BTS LSS. For BTS LSS, the whole system is stabilizable simultaneously when the local system is stabilizable (Theorem 3) ††

4.2. System Restructuring and Controller Reconfiguration

As mentioned above, in order for the faulted system to survive a failure event, a new control law, U_P, must be found to stabilize faulted large scale system and to retain its performance as much as possible. Based on the failure information and the available control authority, the failure event is accomodated via one of the following procedures

- Controller reconfiguration
- System restructuring and controller reconfiguration

With the remaining control authority, if the new control law can be achieved, the failure event can be accommodated via controller reconfiguration only (i.e., re-distribute the available local & global control authority to maintain the system within its stable operational envelope). Otherwise, it is required to restructure the system in order for the remaining control authority to save the system. The system restructuring strategy is described as follows,

- If the failed subsystem is unstable and cannot be stabilized, then restructure the faulted system by isolating the unstabilizable subsystem.
- Otherwise, isolate certain subsystem(s) based on their degree of criticality.

The main issue of the proposed fault-tloerant control strategy is how to reconfigure the remaining control authority. Suppose that the remaining control authority is reconfigurable and capable assuring that the system survives the failure event. Then the following proposed procedure can guarantee the stability of the large scale system

<u>Controller Reconfiguration</u>: With the available failure information,

I. <u>Stabilize the overall system</u>: use both the global and local controller *in parallel*,

- <u>local controller reconfiguration</u>: stabilize the local subsystem.
- <u>global controller reconfiguration</u>: restructure the faulted system into Block Triangular Structure form.

II. <u>Improve performance</u> - Recover the interconnection strength as much as possible by reconfiguring the global controller.

For a given LSS, which is stable under normal operating conditions, in case of a failure event, stabilization of the isolated local subsystems can not, in general, guarantee the stability of the overall system. However, if the system has block triangular structure, local stability of all isolated subsystems can, indeed, guarantee the overall stability, as explained in the previous subsection. That is the reason why reconfiguration of the global controller is proposed as a means to restructure the given system into block triangular structure. Thus, the system with both local and global controllers can be reconfigured to guarantee stability.

The performance of the reconfigured system may be degraded because of loss of the interconnection strength among its subsystems. In order to improve the system performance, for the stabilized system, we recover the interconnection strength as much as possible via reconfiguration of the global controller.

For purposes of illustration, let us consider the following linear large scale system with state feedback control laws determined at the design stage,

$$U = -KX, \quad u_i = -k_i x_i - \sum_{j=1, i\neq j}^{l} k_{ij} x_j$$

$$\begin{aligned}
&(LSS\colon S\ ) &&\dot{X} = (A - BK)X \\
&(CSS\colon S_i\ ) &&\dot{x}_i = (A_i - B_i k_i)x_i + \sum_{i=1, i\neq j}^{l} (A_{ij} - B_i k_{ij})x_j \\
&(ISS\colon S_i\ ) &&\dot{x}_i = (A_i - B_i k_i)x_i
\end{aligned}$$

The block diagram of Figure 4.6 shows the *i*th subsystem (composite subsystem). Without loss of generality, suppose that the given system is stable under normal operating conditions, i.e., the following inequality is satisfied,

$$(-1)^l \det \begin{bmatrix} -\rho_1 & \xi_{12} & \cdots & \xi_{1l} \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & \xi_{(l-1)l} \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} > 0, \quad \begin{cases} -\rho_i = \min\limits_j Re[\lambda_j(A_i - B_i k_i)] \quad j=1,\ldots,n_i \\ \xi_{ij} = \lambda_M^{\frac{1}{2}}[(A_{ij} - B_i k_{ij})^T (A_{ij} - B_i k_{ij})] \end{cases}$$

Let us further suppose that an actuator failure in S_m,(i.e., (A_m, B_{mF})) occurs. For the failed subsystem, one of the available stabilization schemes can be applied since each isolated

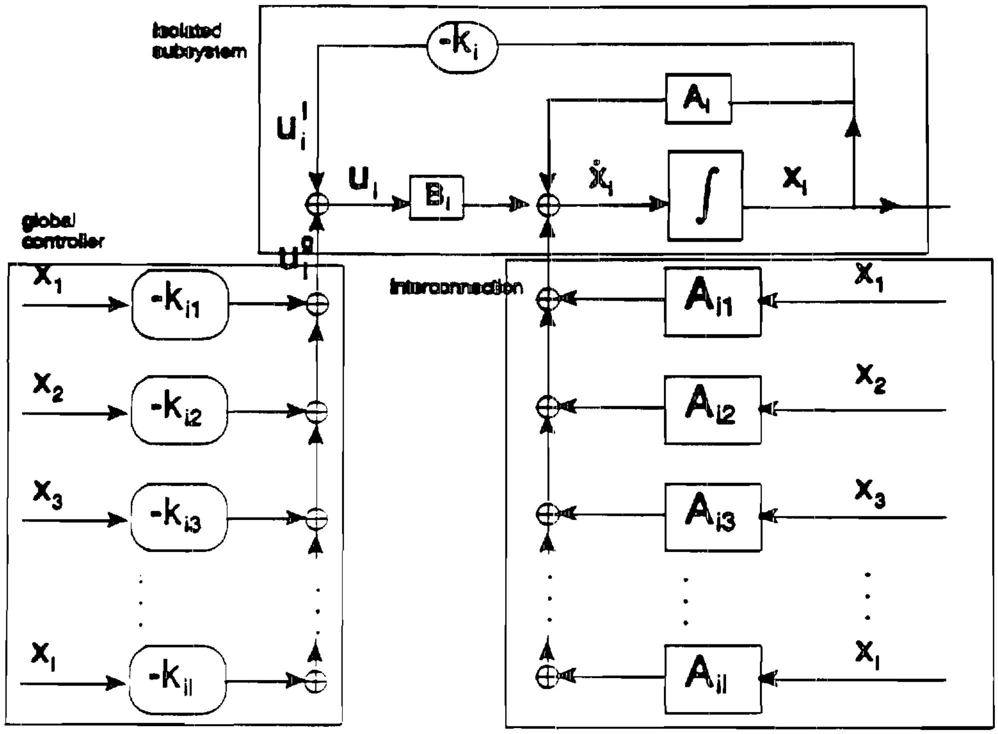


Figure 4.6 Block Diagram of ith composite subsystem

subsystem has a relatively small dimension. Let us consider the reconfiguration scheme based on the Pseudo Inverse method [15,16]

Stabilize the overall system

Find new feedback gains K_F such that $Re[\lambda_j (A-B_F K_F)] < 0$ $(j=1,\ldots, n)$

1. Local controller reconfiguration - local stabilization:

   Find new feedback gains k_{mF} such that

   $$Re[\lambda_j (A_m - B_{mF} k_{mF})] < 0 , (j=1,\ldots, n_i)$$

$$\dot{x}_m = (A_m - B_m k_m)x_m$$
$$\dot{x}_{mF} = (A_m - B_{mF} k_{mF})x_{mF}$$

In robust control,

$$\dot{x} = (A + \Delta A)x$$

$$A \text{ is stable}, \ \|\Delta A\| < \sigma \Rightarrow (A+\Delta A) \text{ is stable}$$

So, the faulted system is stable if

$$\|B_{mF}k_{mF}\| < \sigma_m \Rightarrow \|k_{mF}\| < \hat{\sigma}_{mF}$$

The feedback gain, k_{mF}, is determined from

$$\min \|(A_m - B_m k_m) - (A_m - B_{mF}k_{mF})\|$$

$$k_{mF} = (B_{mF})^{+} B_m k_m \qquad \|k_{mF}\| < \hat{\sigma}_{mF}$$

2. Global controller reconfiguration - Transform to BTS

Apply new global feedback gains k_{ijF} ($i<j$) such that

$$A_{ij} - B_i k_{ijF} = 0 \qquad \forall\ i<j,\ i \neq m$$
$$A_{mj} - B_{mF} k_{mjF} = 0 \qquad \forall\ m<j$$

$$k_{ijF} = (B_i)^{+} A_{ij} \qquad \forall\ i<j,\ i \neq m$$
$$k_{mjF} = (B_{mF})^{+} A_{mj}\ \forall\ \ m<j$$

$$(-1)^l \ \det \begin{bmatrix} -\rho_1 & 0 & \cdots & 0 \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & 0 \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} = (-1)^{2l}\rho_1\rho_2 \cdots \rho_l \ > 0$$

<u>Improve performance</u>: Recover the interconnection dynamics as much as possible.

1. Find a new degree of stability for the local subsystem.

Find the degree of stability of the stabilized subsystem, S_m

$$-\hat{\rho}_m = \min_i Re[\lambda_i(A_m - B_{mF}k_{mF})] < 0$$

2. Reconfigure the global controller to meet the stability condition (i.e., the interconnection strength is less than the degree of local stability).

$$A_{ij} - B_i\hat{k}_{ijF} = A_{ij} - B_ik_{ij} \quad \forall\ i<j,\ i\neq m$$
$$A_{mj} - B_{mF}\hat{k}_{mjF} = A_{mj} - B_mk_{mj} \quad \forall\ m<j$$

$$\hat{k}_{ijF} = k_{ij} \quad \forall\ i<j, i\neq m$$
$$\hat{k}_{mjF} = (B_{mF})^{+}B_{mj}k_{mj} \quad \forall\ \ m<j$$

check the stability condition

$$(-1)^l \ \det \begin{bmatrix} -\rho_1 & \xi_{12} & \cdots & \xi_{1l} \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & \xi_{(l-1)l} \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} > 0$$

As illustrated, the stabilization problem of faulted large scale system is viewed as that of a relatively small scale isolated subsystem. Thus with the proposed reconfiguration strategy, faulted large scale system can maintain its overall stability in a limited time, since it is possible to make use of parallel implementation for each isolated subsystem. Furthermore, this proposed methodology can be applied to a large scale system in which each free subsystem is controlled by the different type of control laws. For many of practical large scale systems, the fault-tolerance can be achieved by applying additional global controller.

5 IMPLEMENTATION ISSUES

Fuzzy reasoning or inferencing in a rule base system is one possible application of fuzzy logic and fuzzy set theory, as introduced first by Zadeh [17,18]. Main concerns in this section are to introduce a hardware structure for implementing a fuzzy rule base system, to demonstrate the compositional rule of inference and to assess the performance of this fuzzy hardware.

An approach to fuzzy inferencing has been reported by Togai [19], but it does not provide flexibility from a hardware viewpoint. Since the same fuzzy rule can be realized in several ways, it is very important to consider a number of aspects which govern the overall system performance. As the number of premises or rules increases, our approach to fuzzy implementation can be realized efficiently and effectively. Important factors may include the processing time, hardware complexity, and flexibility in changing rules or adding a premise part.

The basic mathematics of a fuzzy rule base system employ maximum and minimum operations, it is generally desirable to reduce the number of these operations in order to minimize processing time. Another factor is related to hardware complexity, which determines the storage capacity of the rule base system. Thus, it is important to store data compactly without loss of information. Flexibility is crucial in incrementing or modifying the rules thus avoiding complicated learning processes. Flexibility is gained by making the rule base modular, that is, insertion of a new rule is accomplished by simply augmenting the matrix which stores information not by re-calculating its elements each time.

5.1 Fuzzy Hypercubes

A fuzzy hypercube, H^F, is a fuzzy system characterized by a fuzzy Cartesian product that can store a set of possibilistic if-then propositional rules and which approximately matches the applied stimulus (input) fuzzy sets so that response (output) fuzzy sets may be obtained via fuzzy inferencing. The degree of possibility is modelled in terms of membership functions for each linguistic rule that represents the relative degree of extent. Each rule is a fuzzy implication of an 'IF--THEN' proposition. In a fuzzy hypercube, the k-th rule is of the general form,

rule k: IF (x^k_1) and ... and (x^k_N) THEN (y^k_1) and ... and (y^k_M).

x^k_i, y^k_i are fuzzy I/O vectors for the k-th rule. x^k_i, y^k_i are assumed to satisfy certain convexity and normality conditions [20]. The premises and consequences in this fuzzy relation are, therefore, the inputs (stimuli) and outputs (responses) of H^F.

5.2 Realization of Fuzzy Hypercubes: Split-and-Merge Method

We can decompose a fuzzy hypercube so that only one premise is associated with each rule by using premise matrices and consequence matrices that enable only one consequence from each rule. This method is based on Mamdani's fuzzy inferencing technique [21] as extended from continuous fuzzy sets to discretized fuzzy systems. The premise vector from the i-th premise and the consequence vector from the j-th consequence, in each rule, can be expressed as follows:

$$P_i = \begin{bmatrix} x_i^{1T} \\ \vdots \\ x_i^{KT} \end{bmatrix} , \qquad C_i = \begin{bmatrix} y_j^{1T} \\ \vdots \\ y_j^{KT} \end{bmatrix}$$

where K is the number of rules; x^{pT}_i is the transpose of the i-th premise vector and y^{pT}_j is that of the j-th consequence vector for the p-th rule.

Now we consider the fuzzy inferencing mechanism. The degree of fulfillment $d^i \in [0,1]^K$ for the i-th premise vector is

$$d^i = P_i \circ \tilde{x}_i$$

and this intermediate degree of fulfillment d^i indicates how close the input is to the i-th rule. Combining d^i with each premise yields the resultant degree of fulfillment $d \in [0,1]^K$:

$$d[i] = d^1[i] \otimes \ldots . \otimes d^N[i]$$

where $\otimes$ is the minimum operation. Finally, the j-th fuzzy output vector $\tilde{y}_j \in [0,1]^{n_j}$ is obtained using the max-min operation with consequence matrix C_j,

$$\tilde{y}_j = C_j \circ d \tag{5.1}$$

<u>Defuzzification Strategy</u>

The fuzzy output vector $\tilde{y}$ is converted to the corresponding crisp data j^* using the maximum finder:

Defuzzification Strategy : (the peak detector)

$s^* = s[j^*] = s[\arg\max_j y[j]]$

where j^* stands for the index of y in the quantized bin, and s[j] is the representative value of index j. This is suitable for the above decomposition case in (5.1).

5.3 Implementation: Storage and Timing Requirements

The implementation of fuzzy hypercubes using the SM method is shown in Figure 5.1. Each input is max-mined with its corresponding premise matrix and the intermediate degree of fulfillment(DOF) is calculated. Then the resultant DOF is again max-mined with the consequence matrices and the final result is produced. There are several advantages to employing the SM method in implementing a fuzzy rule base system. For the purpose of comparison, let the direct implementation of a fuzzy hypercube be FDHC and assume that there are K rules, N premises, m_i bins for the i-th premise, M consequences, and n_j bins for the j-th consequence. The most important advantage gained by using the SM method is the saving in memory space required for constructing the rule base. In realizing the SM method, there are two types of matrices: premise and consequence matrices. Since the bins of the i-th premise matrix and that of the j-th consequence matrix are n_i and m_j, respectively, the total space allocation required is $K(\Sigma_{i=1}^{N} m_i + \Sigma_{j=1}^{M} n_j)$. The hardware implementation of the FDHC requires $\Pi^{N}_{i=1} m_i$ memory allocation for each rule for a crisp consequence. Therefore, the total number of memory units is $\Sigma^{M}_{j=1} m_j \, \Pi^{N}_{i=1} n_i$

It should be emphasized that there exists a trade-off between the amount of storage and the processing time in realizing a specific hardware configuration. Thus, using a larger capacity or storage we may reduce the processing time and the reverse is also true. With the storage space as shown above, the processing time for the SM implementation is $\max_j\{[(m_i \Delta_{min}) + (m_j - 1)\Delta_{max}]K + [K \Delta_{min} + (K-1)\Delta_{max}]n_j\}$, whereas the FDHC needs $\Delta_{max} + \Delta_{min}$, where Δ_{max} and Δ_{min} indicate the processing time for the maximum and minimum operations, respectively. Table 5.1 summarizes the comparison between the FDHC and the SM methods, where m_i and n_j are simplified to L and $\Delta_{max} = \Delta_{min} = T$ for convenience. Figure 5.2(a) shows the required memory ratio between the FDHC and the SM, based on Table 5.1, with respect to the number of bins, where it is assumed that $m_i = n_j$ for any i,j and N=4, M=3 and K=10. Figure 5.2(b) shows the same ratio with respect to the number of premises with a fixed number of bins(L=5), consequences(M=3) and rules(K=10). Another advantage of the SM over the FDHC technique is its structural flexibility. In contrast to the FDHC, there is no coupling between premises in the SM structure and the content of a rule is preserved until the last stages, d^i; modification or increment of a rule is achieved by replacing a component of the matrix d^i with a new one, or by augmenting a vector to the matrix. Figure 5.3 is a practical circuit diagram to realize fuzzy hypercubes based on the SM method.

	FDHC	SM
storage	ML^{N+1}	$KL(N + M)$
time	2T	T(4LK - K - L)

Table 5.1 Comparison of FDHC and SM

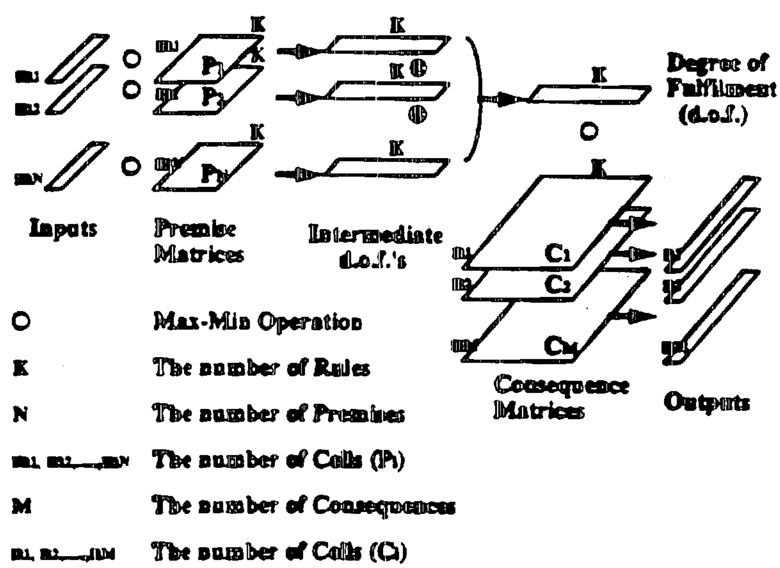


Figure 5.1 Fuzzy Hypercube Inferencing Architecture

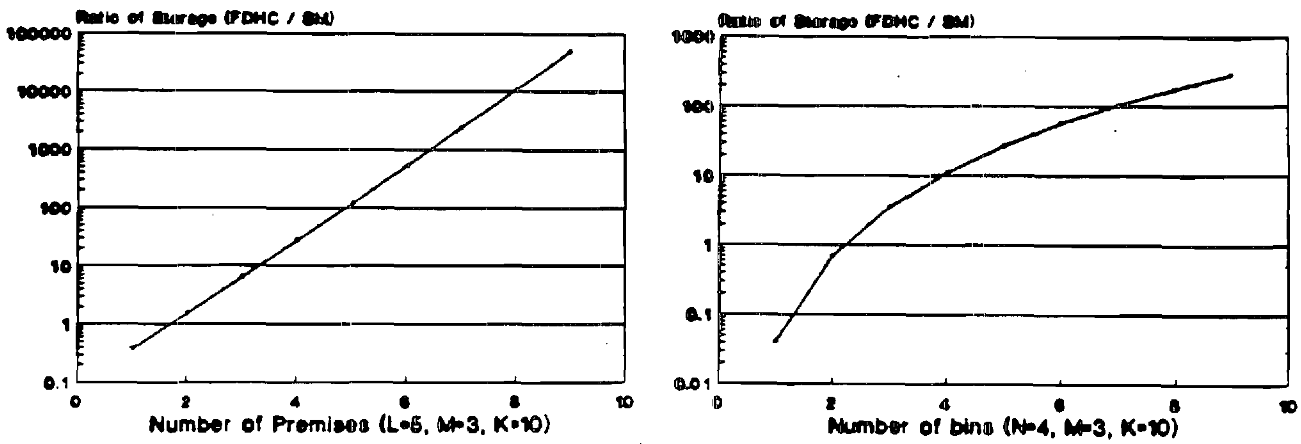


Figure 5.2 The required memory ratio (FDHC/SM) with respect to the number of (a) Premises and (b) Bins

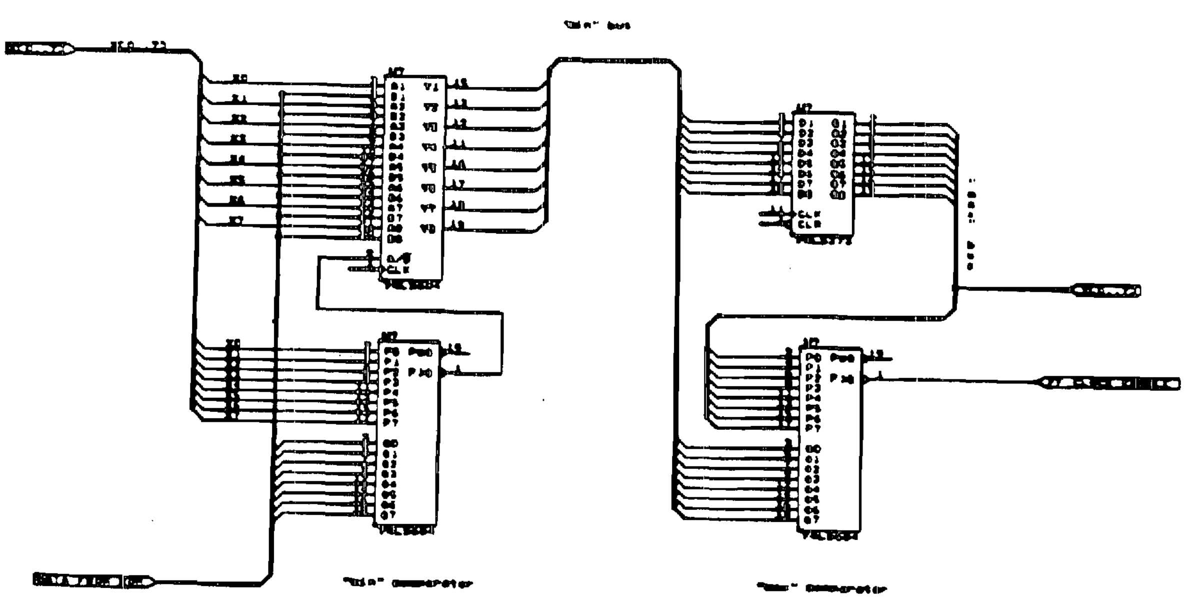


Figure 5.3 The circuit diagram for realization of fuzzy hypercubes

6 THE INTEGRATED SHIPBOARD ELECTRIC POWER SYSTEM

A subproject to contract No. N00014-89-J-3113 was initiated in June 1991 to support work conducted by the David Taylor Research Center and other Navy contractors towards the design of a new electrical distribution system of the integrated shipboard electric power system. Georgia Tech's participation involves the development of fault-tolerant control strategies for the shipboard distribution network. Specifically, in the event of a failure, such as a line short circuit or voltage perturbation, the system is called upon to provide certain critical loads whose uninterrupted operational integrity is essential to the Navy. The reconfiguration strategy addresses those and control actions that will assure system survivability. The detailed work scope of this activity is as follows :

1. Develop a fault-tolerant control strategy. For a particular distribution network topology, we intend to develop a theoretical basis for fault detection and identification, systemrestructuring, and controller reconfiguration. The primary objective of this task is to assist system designers in incorporating fault-tolerant concepts to the shipboard electric power system design.

2. Develop appropriate Fault Detection and Identification routines to be applied to the reconfiguration task and to be used for status monitoring of critical mechanical subsystems. These routines will be demonstrated via simulation studies.

3. Develop an analytical framework for the analysis and control design of the AC/DC interface. Attention may to focused here on the development of an active control strategy to address such problems as power quality and dynamic stability. The power quality problem might be an issue of immediate concern to the Navy. The objective is to damp out potentially harmful oscillations arising from fault conditions or to cancel out harmonic currents.

Our research effort will focus essentially on the reconfiguration task, since it constitutes a high priority item for the DTRC.

6.1 Accomplishments During this Reporting Period

We have been pursuing the definition of a possible topology for the integrated shipboard electric power system that embodies two basic requirements : simplicity and reliability. Both simplicity and cost-effectiveness in terms of minimum parts and interconnections, and an extended life span for critical system units. A preliminary topology that incorporates major system units and aggregates similar functions has been defined and is shown in Figure 6.1. DTRC is currently providing us a new distribution system architecture that is based on a two-bus system. This new architecture will be used as a benchmark to test and validate the reconfiguration and FDI routines.

Our efforts have been focused on two detailed activities : The first one involves the development of a robust FDI algorithm that can be applied to a major subsystem of the shipboard electric power system, and the second is concerned with the selection of an appropriate simulation platform to assist in the development process as well as in testing and validating the FDI and control routines. Here, we have been exploring the possibility of a hybrid simulation environment using C or TURBO-PASCAL to be developed by our research team. Figures 6.2 and 6.3 depict some generic SPICE simulation results for a three phrase power generation results for a three phase power generator and a double tuned transformer, respectively.

The basic FDI architecture is complete and we anticipate to apply this tool to a subsystem of the electrical power system (the propulsion mechanism) for verification purposed. Details of the hybrid FDI algorithm are presented in another section of this report.
Finally, some preliminary work have been conducted on the problem of defining the AC/DC interface requirements and the dynamic stability/control problem.

We anticipate that, over the next reporting period, we will be collaborating closely with DTRC personnel to define fully and to develop the fault-tolerant control laws. Thus, both design and control development activities will take place to optimize the system configuration from a reliability and a cost-effectiveness point of view.

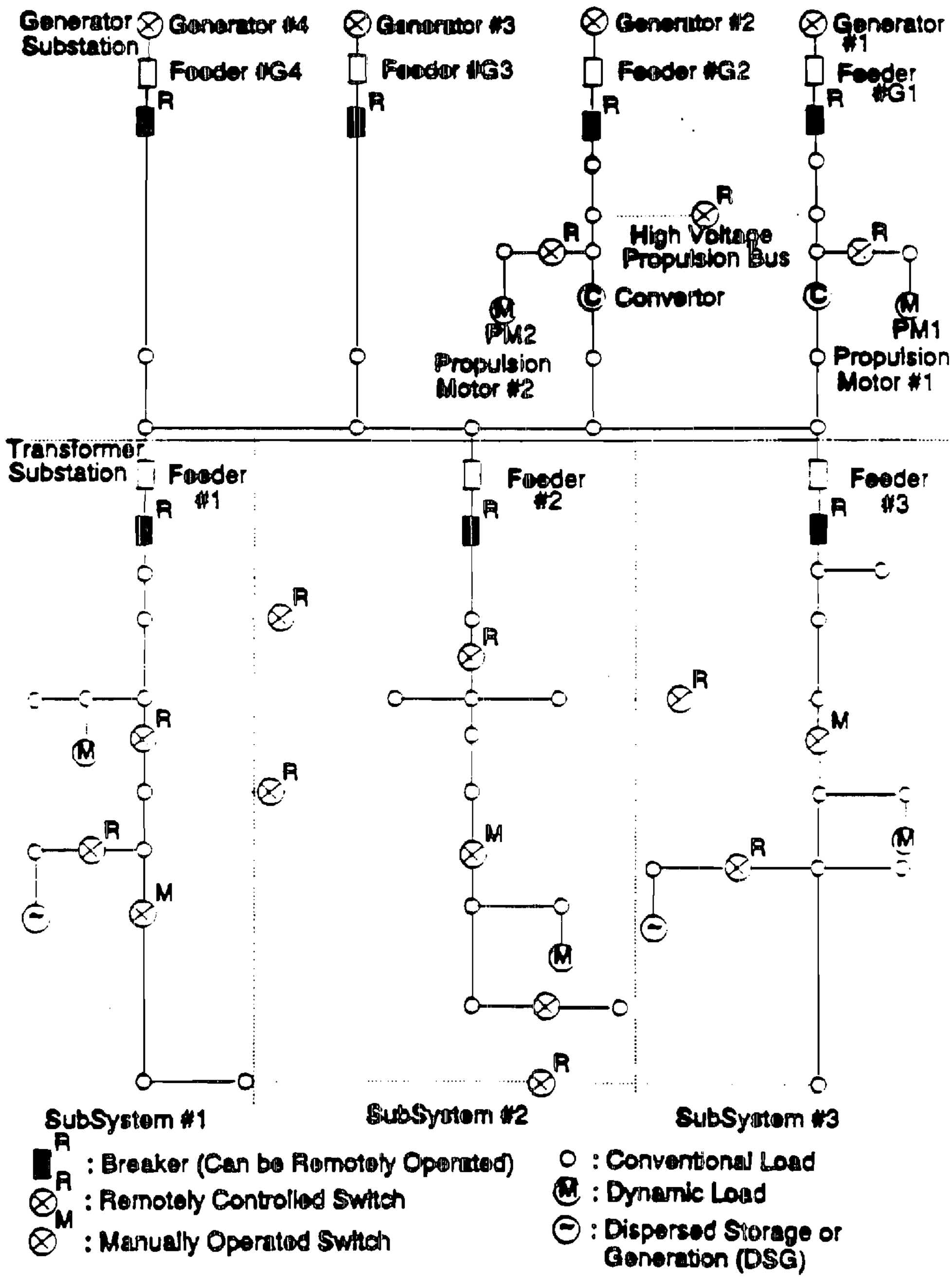
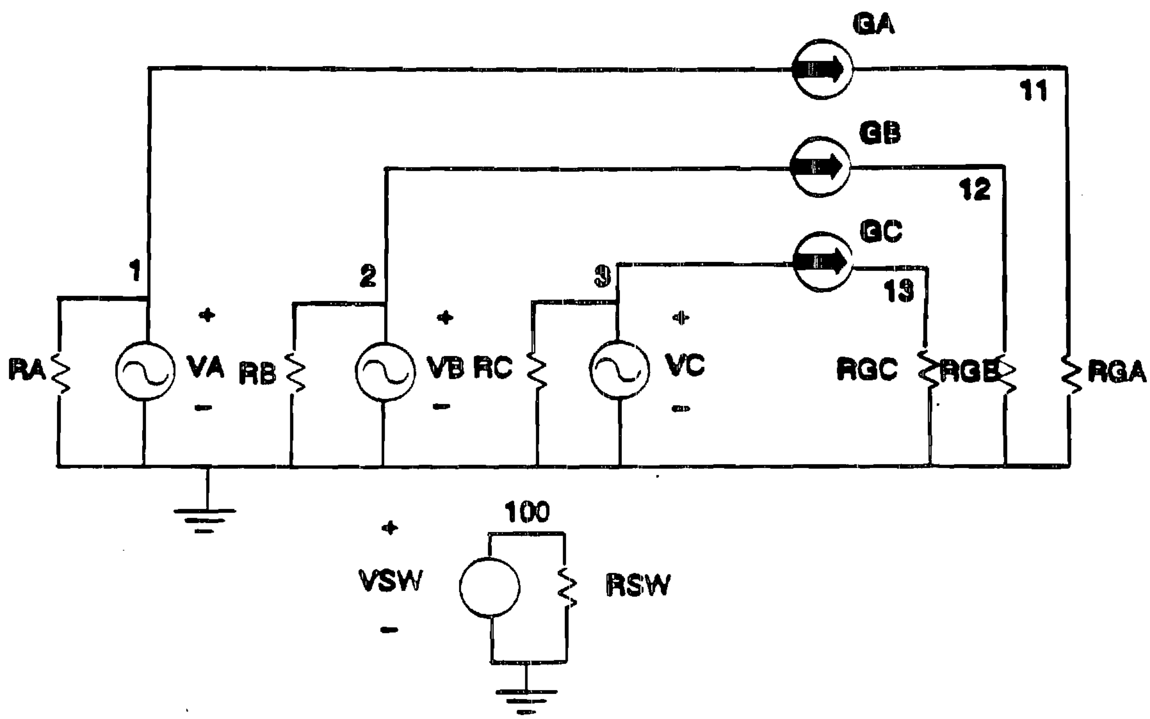


Figure 6.1 Distributed Power System Network



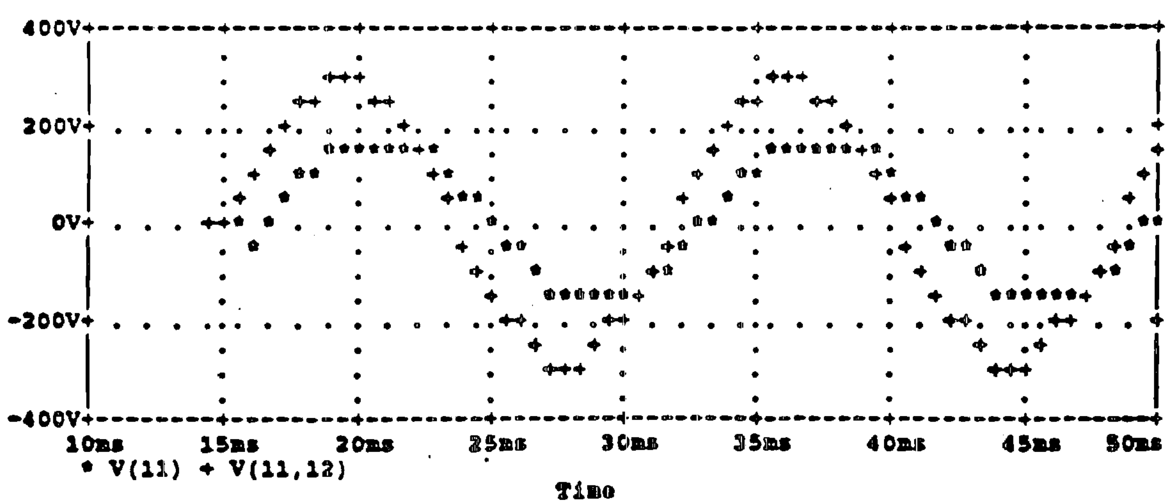


Figure 6.2 Switched Three-phase generator: Example for SPICE simulation

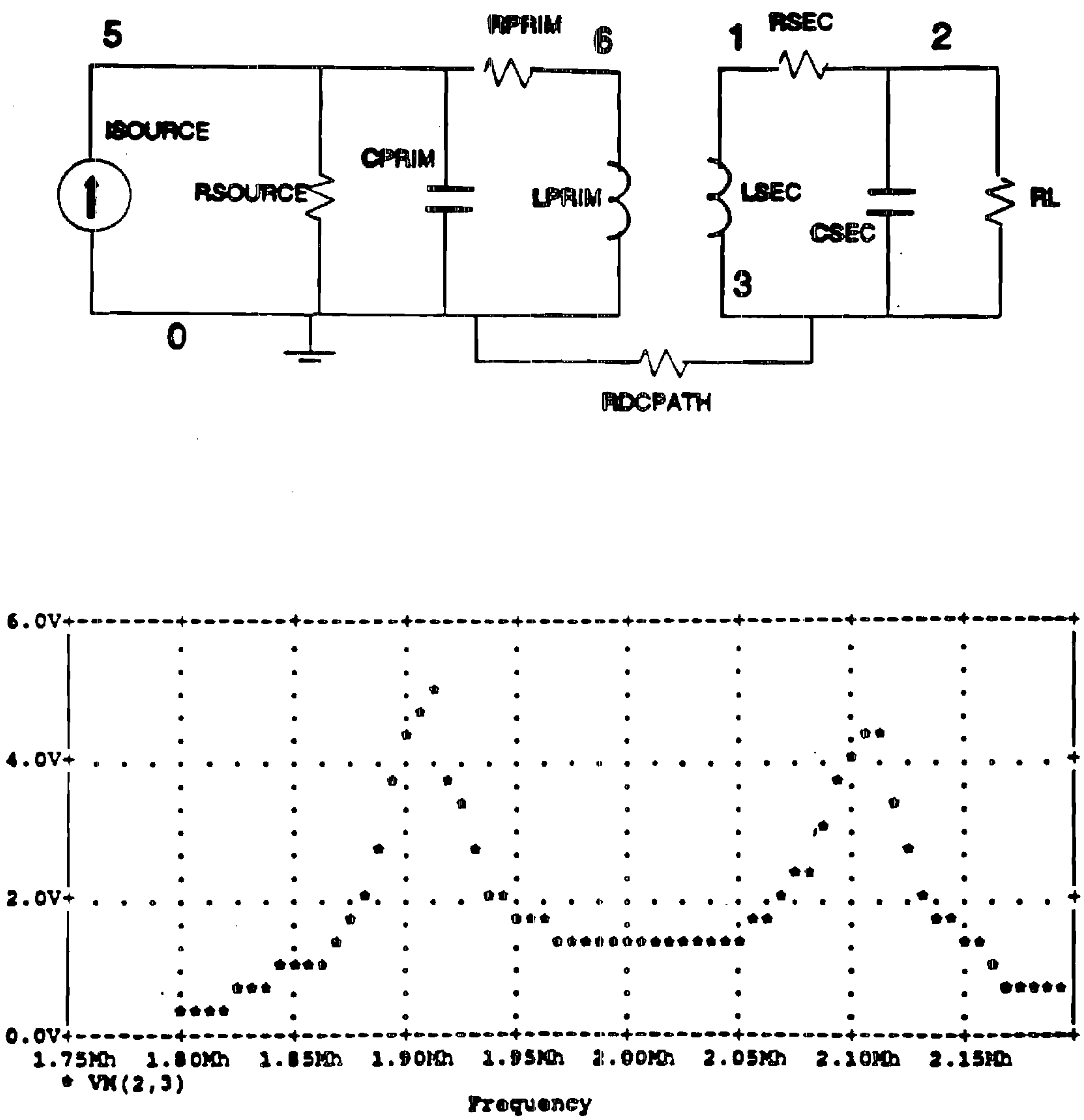


Figure 6.3 Doubly tuned transformer: Example for SPICE simulation

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Presentations and Technical Contributions

1. In October 1990, Dr. Vachtsevanos presented a paper on fuzzy logic control at the Instrument Society of American 1990 Conference in New Orleans, Louisiana.

2. In December 1990, Dr. Vachtsevanos presented two papers at the 29th IEEE Conference on Decision and Control in Honolulu, Hawaii and participated as a panelist in a panel discussion on intelligent control.

3. In January 1991, Dr. Vachtsevanos participated in an Advanced Process Control Workshop at the invitation of Honeywell, Inc. in Phoenix, Arizona.

4. Dr. Vachtsevanos attended the kickoff meeting for the NAVSEA/DARPA project in Washington, DC on 1/30/91 and made a presentation on fault-tolerant control work at Georgia Tech.

5. In April 1991, Dr. Vachtsevanos attended the annual ONR review meeting and presented the results of our research activity.

6. In May 1991, Dr. Vachtsevanos chaired a session on Process Control and presented a paper on fuzzy logic control at the 1991 IEEE Conference on Instrumentation and Measurements Technology.

7. In June 1991, Dr. Vachtsevanos presented a paper at the 1991 Automatic Control Conference.

8. On June 26, 1991, Dr. Vachtsevanos attended the DARPA review meeting at MIT.

9. In June 1991, Dr. Vachtsevanos chaired a technical session on Intelligent Manufacturing and presented a paper at the EURISCON '91 Conference.

The following technical papers authored or co-authored by the research team were published during the reporting period:

1. H. Kang and G. Vachtsevanos, "Nonlinear Fuzzy Control Based on the Vector Fields of the Phase Portrait Assignment Algorithm," **Proceedings of the 1990 American Control Conference**, San Diego, California, pp. 1479-1484, 1990.

2. R. Youngblood, J. Curtis, and G. Vachtsevanos, "Fuzzy Logic Control: New Approaches and Applications," **Proceedings of ISA 90 Conference**, New Orleans,

Louisiana, pp. 1129-1136, 1990.

3. H. Kang and G. Vachtsevanos, "Fuzzy Hypercubes: Linguistic Learning/Reasoning System for Intelligent Control and Identification," to appear in International Journal of Intelligent and Robotic Systems.

4. G. Vachtsevanos, H. Kang, J. Cheng and I. Kim, "On the Detection and Identification of Axial Flow Compressor Instabilities," to appear in American Institute of Aeronautics and Astronautics, Journal of Guidance, Control, and Dynamics.

5. G. Vachtsevanos, "Fuzzy Logic Control Applications in Textile Manufacturing," IEEE Instrumentation and Measurements Technology Conference, IMTC/91, Atlanta, GA, May 1991.

6. H. Kang and G. Vachtsevanos, "An Intelligent Strategy to Robot Coordination and Control," **Proc. of 29th IEEE Conference on Decision and Control**, pp. 2208-2213, 1990.

7. R.C. Arkin and G. Vachtsevanos, "Qualitative Fault Propagation in Complex Systems," **Proc. of 29th IEEE Conference on Decision and Control**, pp. 1509-1510, 1990.

8. B.H. Wang and G. Vachtsevanos, "Fuzzy Logic Control: A Systematic Control Methodology," Proc. of IEEE Conference on Decision and Control, Brighton, England, Dec. 1991.

9. H. Kang and G. Vachtsevanos, "Fuzzy Hypercubes: Linguistic Learning/Reasoning Systems for Intelligent Control and Identification," **Proc. of IEEE Conference on Decision and Control**, Brighton, England, Dec. 1991.

10. B.H. Wang and G.J. Vachtsevanos, "Fuzzy Dynamic Systems: An Application of Fuzzy Associative Memory with an Intermediate Layer," **Proc. of Automatic Control Conference '91**, pp. 12-13, 1991.

11. G. Vachtsevanos and P. Groumpos, "Fault-Tolerant Design Issues for A Manufacturing Cell," European Robotics and Intelligent Systems Conference, June 27-28, 1991.

12. H. Kang, J. Cheng, I. Kim and G. Vachtsevanos, "An Application of Fuzzy Logic and Dempster-Shafer Theory to Failure Detection and Identification," Proc. of IEEE Conference on Decision and Control, Brighton, England, Dec. 1991.

Project Participants

The principal investigator of this project is
Dr. *George Vachtsevanos* Professor of Electrical Engineering at the Georgia Institute of Technology, Atlanta, Georgia.
Dr. *Ronald Arkin*, Associate Professor at the College of Computing, the Georgia Institute of Technology, serves as a co-investigator to the project.
Dr. *Hoon Kang*, a recent Ph.D. graduate at the Georgia Institute of Technology, is participating in project activities as a post-doctoral fellow.

Three graduate students in the School of Electrical Engineering at Georgia Tech are currently receiving partial support from this project.

E21-F49 #3

~~IN CONFIDENCE~~

Annual Letter Report for FY 92

A HYBRID ANALYTICAL/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN

(Contract No: N00014-89-J-3113)

Sponsor: **Office of Naval Research**
Applied Research Technology Directorate

Principal Investigator: **Dr. Goerge Vachtsevanos**
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Tel: (404)894-6252

September 1, 1992

A HYBRID ANALYTICAL/INTELLIGENT
APPROACH TO FAULT-TOLERANT
CONTROL SYSTEM DESIGN
Contract No: N00014-89-J-3113
PI: Dr. George Vachtsevanos
Georgia Institute of Technology

In September 1989, the Office of Naval Research awarded contract No. N00014-89-J-3113 to the Georgia Institute of Technology to develop fault-tolerant control strategies for large scale dynamical systems. Specifically, the technical issues under investigation are: Development of a structure-based modeling methodology of large scale systems which possesses features of structure flexibility, i.e. a component or subsystem may be deleted in a simple way that does not substantially disturb the global control strategy; development of robust failure detection and fault identification algorithms for single and multiple faults that are maximally sensitive to true failure conditions but insensitive to noise thus reducing the possibility of false alarms; a technique to restructure the system dynamics by isolating the faulty components; a method that will lead to a structural control law which reconfigures the original system controller in order to meet the primary performance objective of guaranteed stability while the system is operating in a degraded mode; finally, these algorithmic developments are to be demonstrated on an actual physical system to be designated jointly by ONR and Georgia Tech.

The technical approach pursued to meet these objectives is outlined as follows: The underlying factor of the fault-tolerant methodology capitalized upon structural features of the large scale system and employs a blend of numerical and symbolic manipulations, thus combining concepts of control theory and artificial intelligence. In the modeling area, the topological description of many complex dynamical processes may be cast in the form of structurally interconnected subsystems. In every subsystem, a control law is applied which consists of a local and a global feedback term, thus resulting in a two-level hierarchical control strategy. We examine first large scale systems which are described by linear state equations and exhibit this two-level hierarchical structure. The analysis will be extended eventually to nonlinear systems of a similar structure. A methodology for fault diagnosis is introduced which is based upon a combination of signal redundancy and detection/estimation procedures. An expert system, consisting of a multivalued rule base and an appropriate inferencing mechanism, assesses the aggregate of fault symptoms and determines the sensitivity of a failure condition. An innovative approach to multiple failure detection uses qualitative simulation techniques to decide on the impact of a component failure on other (healthy) system components. The proposed fault detection and identification scheme incorporates such additional functionalities as fault trending and the estimation of the "best" value of critical variables and parameters under failure states.

Finally, we propose the development of a two-level structural dynamic hierarchical approach to address the control reconfiguration problem of a faulted large scale system. The approach uses a structural state model, the Block Arrow Structure, which consists of N independent linear subsystems interconnected with a control common linear dynamic system. The objective is to find a time-invariant decentralized feedback controller which minimizes a quadratic cost functional and has the features of (1) utilizing the interconnections, (2) parallel implementation, and (3) structure flexibility in the sense that the addition and/or deletion of a subsystem does not require redesigning the problem from the beginning.
On January 1991, an additional task was added to this project. With partial funding from NAVSEA/DARPA and in collaboration with the David Taylor Research Center, we were charged

with the responsibility of developing a fault-tolerant design methodology for a shipboard electric power distribution network. Through appropriate modeling and simulation techniques, the work scope we are addressing includes reliability studies aimed at the "best", from a fault-tolerant standpoint, network topology, the design of protection and control reconfiguration strategies to guarantee maximum power availability to vital sensitive loads and the development of appropriate modeling/simulation tools to assist in the design and validation phases of the project. The methodologies adopted to address these issues are again a hybrid of analytical and intelligent techniques that offer optimum performance trade-offs.

Research Accomplishments

During this reporting period 9/1/91-9/1/92, the research team focused attention on the following issues:

- Completed the development of a generic fault detection and identification methodology; the technique capitalizes upon analytic redundancy and stochastic estimation and combines concepts from control theory and artificial intelligence; the algorithms have been verified on a simulation basis using typical sensor and actuator failures as test cases.
- A unified fault-tolerant control methodology has been developed to address complex large scale systems that are subjected to large-grained disturbances; a unique feature of the approach is the ability to address the control reconfiguration problem even when one or more of the system's constituent components is nonlinear; verification of the effectiveness and viability of the proposed algorithms has been carried out on a simulation basis.
- A new fault-tolerant control methodology has been developed for an integrated shipboard electric power system; the research activity here included system modeling techniques, a reliability analysis to identify the optimum network topology, a method to detect and isolate failures and, finally, a hybrid analytical/intelligent procedure has been developed to reconfigure the network in the event of a failure and maintain power continuity to vital sensitive loads.

Major accomplishments during this period included the fault detection and identification algorithm that may be applied to many critical processes of interest to the Navy. In combination with the fault-tolerant control strategies, these tools can enhance the autonomy and improve the reliability of such complex systems as AUV's, missiles, aircraft, etc. We anticipate that during the upcoming final period of this project, we will integrate the fault tolerant control modules and apply the strategy to an experimental engine (in collaboration with Dr. C. Nett of the School of Aerospace Engineering at the Georgia Institute of Technology).

We summarize below the approaches developed during the reporting period and the advances in understanding that have been achieved.

1. The Hybrid Fault Detection and Identification Procedure

FDI techniques were considered for sensor, actuator, and component failures. Both single and multiple faults are treated. The research objective is to develop a systematic and thorough

FDI procedure that is maximally sensitive to failures while avoiding false alarms. The approach is systematic because it relies on a modular architecture to (1) efficiently trigger FDI routines, (2) validate sensor data, (3) combine failure evidence from such diverse sources as analytic redundancy, detection/estimation theory, and limit checking, (4) utilize expert system tools and Dempster-Shafer evidential theory to manage uncertainty and assess the symptomatic evidence, i.e. detect and identify faulty components for monitoring and control purposes.

Fault detection and identification uses validated sensor and analytic data to analyze the status of the system components. Fault diagnosis involves the incremental accumulation of evidence in the form of symptoms to determine the "health" of the system components. Parity space representation and analytic redundancy, in addition to limit checking and statistical estimation techniques, may be applied to accumulate the required evidence. Parity space representation transforms an array of redundant measurements into a new array, called a parity vector, in such a way that the true value of the underlying variable is suppressed, leaving only components of the measurement errors as elements of the parity vector. In parity space, the faults are present. When a fault occurs, the parity vector grows in magnitude, and its direction of growth is uniquely associated with the faulty measurement.

The stochastic estimation scheme consists of two processing units. First, fault triggering plays the role of a failure sensitive filter at the learning stage. A recursive parameter estimation technique is used to determine the direction and size of parameter changes in real-time from the failure signature. Once the FDI system is alert, the histograms of parameter residues are evaluated and transformed into the corresponding fuzzy sets as decision samples at the tracking stage. In the second step, the corresponding fuzzy input vectors are applied to fuzzy relation matrices which undertake the identification of the failure mode utilizing the concept of possibility theory. Thus, multiple failures can be identified using the knowledge base. The combined analytical/intelligent approach relaxes many assumptions raised in other FDI schemes such as the conditions of a linear time-invariant system and left invertibility from the failure mode to the output as well as the requirement of a bank of Kalman filters.

2. The Sixth Order Single Spool Gas Turbine Engine Model

We have developed a systematic unified failure detection and identification methodology based on possibility theory and Dempster-Shafer theory. The algorithm has been applied to a motor speed control problem and to the multi-stage axial flow compressor to detect and identify aerodynamic instabilities. The algorithm has been verified through computer simulation studies. An axial flow compressor model was adopted to develop the FDI algorithm from an experimental model constructed at MIT to study aerodynamic instabilities of a gas turbine engine. The experimental facility consists of a slow speed motor driven by a variable plenum volume axial flow compressor. In order to fully investigate the aerodynamic instabilities of the turbo jet engine we need to apply the developed FDI algorithm to a real jet engine so that we may validate the routines. The main focus of our research effort is to solve the real gas turbine engine aerodynamic instabilities problem.

At Georgia Tech's School of Aerospace Engineering, a research team headed by Dr. Carl

Nett has recently completed the construction of a single-spool, centrifugal compressor, gas turbine engine. An engine model has been developed based on the experimental stall capable gas turbine engine. The engine can be operated at a speed of 60,000 rpm which is much higher than that of Greitzer's compressor model. The gas turbine engine available at the School of Aerospace Engineering is equipped with a high bandwidth fuel flow, nozzle area, and compressor discharge bleed area servos. The model developed for the experimental engine is based on engine component steady state performance maps and unsteady quasi one-dimensional flow equations. The model has three control inputs, three states, and incorporates the dynamic linkage of the compressor and turbine through the spool. The three states are compressor mass flow rate, plenum pressure and speed of spool.

Our main concern is to apply the FDI routines to an appropriate engine model. For this reason, we borrow C. Nett's basic third order dynamic engine model and expand it to a 6th order engine model. The fuel flow rate, bleed valve, and nozzle dynamics are added to the original third order dynamic model. The objective in increasing the order of the dynamics is primarily to study the system parameter effects on engine instabilities. In Greitzer's compressor model [4], the compressor instabilities are caused by perturbations of the compressor rotor rotating speed and the nozzle area. It is well accepted that engine instabilities are generally caused by the same dynamic variations. The final goal is to apply the hybrid FDI algorithm to the GT experimental engine in order to demonstrate its real-time feasibility on a complex dynamic engineered system - the gas turbine engine.

3. Fault-Tolerant Control Reconfiguration

Suppose that a failure occurs in a subsystem or component of a complex large scale dynamic system. The FDI routine identifies the failure (or impending failure) and reports this event to the control logic. The failure(s) may impair the system's operational capability or may even lead to a catastrophic event, if left unattended. It is desirable, therefore, to assess first the impact of the fault on other healthy system units and to restructure and/or reconfigure the system and its control authority to assure survivability. We outline a systematic methodology that we have developed to address the fault-tolerant control problem.

With the fault information provided by the fault detection and identification scheme, the faulted large scale system can remain in a survivable state via system restructuring and/or controller reconfiguration.

For a faulted system to survive a failure event, a new control law must be found to stabilize the faulted large scale system and to retain its performance as much as possible. Based on the failure information and the available control authority, the failure event is accommodated via one of the following procedures

- Controller reconfiguration
- System restructuring and controller reconfiguration

With the remaining control authority, if the new control law can be achieved, the failure event can be accommodated via controller reconfiguration only (i.e., re-distribute the available local & global control authority to maintain the system within its stable operational envelope). Otherwise, it is required to restructure the system in order for the remaining control authority to save the system. The system restructuring strategy is described as follows,

- If the failed subsystem is unstable and cannot be stabilized, then restructure the faulted system by isolating the unstabilizable subsystems.
- Otherwise, isolate certain subsystem(s) based on their degree of criticality.

The main issue of the proposed fault-tolerant control strategy is how to reconfigure the remaining control authority. Suppose that the remaining control authority is reconfigurable and capable of assuring that the system survives the failure event. Then the following proposed procedure can guarantee the stability of the large scale system.

Controller Reconfiguration: With the available failure information,

I. Stabilize the overall system: use both the global and local controller in parallel,
- local controller reconfiguration: stabilize the local subsystem.
- global controller reconfiguration: restructure the faulted system into Block Triangular Structure form.

II. Improve performance: recover the interconnection strength as much as possible by reconfiguring the global controller.

The fault-tolerant control methodology capitalizes upon structural properties of the large-scale system and uses such functional characteristics as stability in order to arrive at a robust control strategy. The methodology is demonstrated using a thermal control system as the test bed. The basic elements of the proposed approach are briefly described below.
An innovative methodology was introduced to address the fault tolerant control problem. A new fast and effective system restructuring and controller reconfiguration strategy has been developed with the assumption that the failure can be detected and identified in real time. A large scale system with the fault-tolerant structural property of restructurability and reconfigurability can be made safe in the event of a failure, by applying the control reconfiguration strategy. For the purpose of ensuring the speed and effectiveness of the proposed scheme, some fundamental issues are considered at the design stage: the relation between the system units and survivability conditions as well as the degree of criticality of each system unit. This information enhances the effectiveness of the fault accommodating action. Furthermore, for those system units that have been verified to be critical in some sense, the system can be designed to tolerate a failure in such system units at the design stage. On the other hand, in order to guarantee the effectiveness of the fault accommodating action, a fault propagation strategy accompanies the FDI scheme to analyze the failure effect on other "healthy" system units. We are exploiting qualitative analysis techniques from the area of AI and cognitive sciences in order to achieve the fault propagation

objective. The main advantage of qualitative analysis of a large scale system is that it provides an effective and rapid means to determine the future state of the system without solving a set of differential equations. Hence, a decision to restructure the faulted large scale system can be made very quickly allowing more time to perform the proper control action. Applying the failure information provided from the FDI module as the initial states of the qualitative variables, the possible qualitative behaviors are generated via qualitative simulation. In order to circumvent the major problem in employing qualitative simulation techniques, which is nonuniquness of the generated qualitative behaviors, additional constraints are applied. A Lyapunov function is used as an energy-type constraint, and the qualitative description of the operational envelope is augmented.

The control reconfiguration strategy makes use of such desirable structural properties of the system as the block triangular structure. For a large scale system with only linear interconnections among its subsystems, a sufficient condition to realize the given system as a block triangular structure system is provided. Beginning with stable subsystems, the BTS LSS is shown to be connectively stable. Based on the assumption of the stability of the isolated (free) subsystems, a Lyapunov stability criterion is considered. Since we are dealing with engineered systems, it is not difficult to define an appropriate Lyapunov function for the isolated systems. A vector Lyapunov function for the large scale system is defined next whose elements consist of the Lyapunov functions of the isolated subsystems. The derivative of the Lyapunov function along the composite system is evaluated and the comparison system is constructed.

The failure event can be accommodated by applying both a local and a global reconfiguration strategy. The local controller is reconfigured to stabilize the faulted subsystem while the stability of the remaining modules is addressed via the global controller reconfiguration. The stability of the remaining system is verified by the comparison system of the remaining modules. An aggregation matrix is determined by eliminating the corresponding column and row from the aggregate matrix of the original system. If the restructured system obtained by isolating the faulted subsystem can not be guaranteed to be stable, then we choose a one-way interconnection which is relatively weak. This is then neutralized via the global controller reconfiguration. If the global controller can neutralize the desired interconnections perfectly, then the system has a block triangular structure and the stability of the remaining system is guaranteed. Otherwise, the aggregation matrix is evaluated with the reconfigured interconnections. If it satisfies the stability condition, then the remaining interconnections are neutralized. The procedure guarantees the stability of the remaining system.

If the faulted subsystem can be stabilized by the local controller reconfiguration, the performance of the restructured system can be improved by re-inserting the stabilized subsystem, and by recovering the interconnection dynamics which are the same as those of the original system.

The control reconfiguration strategy was extended to a class of large scale systems consisting of linear interconnections and nonlinear subsystems, for which the aggregation matrix can be constructed. As a test case, a feedback linearizable nonlinear subsystem has been considered. Utilizing the assumption of the boundedness of the diffeomorphism, the comparison system was constructed. For purposes of illustration, a large scale system consisting of one nonlinear feedback linearizable subsystem and two linear subsystems was considered.

In general, the stabilization problem of a faulted large scale system is viewed as the problem of stabilizing a relatively small scale isolated subsystem. Thus, with the proposed reconfiguration strategy, a faulted large scale system can maintain its overall stability in some sense, since it is possible to make use of parallel implementation techniques for each isolated subsystem. Furthermore, the proposed methodology can be applied to a large scale system in which each free subsystem is controlled by a different control law. For many practical large scale systems, fault-tolerance can be achieved by applying an additional global controller.

4. Fuzzy Active/Adaptive Control

The Fault Detection and Identification scheme may be further exploited as the identification module of a fuzzy active/adaptive control algorithm. The controller is formulated as a fuzzy expert system and its rule base is designed using a phase portrait method, i.e. projections of the state trajectories unto phase planes of the state variables taken two at a time. An exhaustive search procedure is used to derive the control law that minimizes a multi-objective criterion. The control signal is employed next to activate such "active" control devices as a loudspeaker, a bleed valve, or a throttle actuator. In a computer simulation, both pressure and flow rate return to their normal state upon application of the active/adaptive control law. Fuzzy hypercubic constructs have been designed to implement efficiently the control rules that exhibit attributes of parallelism and pipelining. The attractive feature of this fuzzy adaptive/active approach is that it treats large-grained uncertainty effectively while it addresses robustly the control of complex nonlinear systems.

5. The Integrated Shipboard Electric Power System

A subproject to contract No. N00014-89-J-3113 was initiated in June 1991 to support work conducted by the David Taylor Research Center and other Navy contractors towards the design of a new electrical distribution system of the integrated shipboard electric power system. Georgia Tech's participation involves the development of fault-tolerant control strategies for the shipboard distribution network. Specifically, in the event of a failure, such as a line short circuit or voltage perturbation, the system is called upon to provide certain critical loads whose uninterrupted operational integrity is essential to the Navy. The reconfiguration strategy addresses those control actions that will assure system survivability. The detailed work scope of this activity is as follows :

1. Develop a fault-tolerant control strategy. For a particular distribution network topology, we developed a theoretical basis for fault detection and identification, system structuring, and controller reconfiguration. The primary objective of this task is to assist system designers in incorporating fault-tolerant concepts to the shipboard electric power system design.

2. Develop appropriate Fault Detection and Identification routines to be applied to the reconfiguration task and to be used for status monitoring of critical

mechanical subsystems. These routines are demonstrated via simulation studies.

Our research effort focuses essentially on the reconfiguration task, since it constitutes a high priority item for the DTRC.

The primary objective is to "Reconfigure" a faulted system using intelligent control strategies (or actions) and concepts from fuzzy logic control, artificial intelligence, search techniques, and expert systems. The "Hybrid" analytical/intelligent approach is applied to the shipboard network reconfiguration problem. The general philosophy attempts to extract maximum information possible by using the analytical tools available, within the given time constraints, and then by employing an intelligent strategy to reconfigure the system. The analytical tools (such as FDI, FEAT etc.) address the problem of fault identification. These analytical tools extract and use such properties of the system as structure, function, etc. The intelligent methodology gathers the information provided by the analytical tools and uses decision making algorithms like search techniques (A^* ...), Fuzzy Logic Control, Dempster Shafer theory, and an Expert System paradigm, etc. to reconfigure the system.

The reconfiguration methodology proceeds along the following steps:

Step I: A reliability analysis based on the program FEAT provides information regarding the best network topology (Full, Loop, Ring...) under different operating conditions (battle or normal ...). Assuming that the system is operating under normal operating conditions and then under battle conditions the topology is changed to Full - this solution is provided by FEAT.

Step II: The network topology is fed as the primary input to a search technique. The system then generates the trees for the shipboard power system. The tree structure is modelled off-line and could be combined into a single structure or could be divided into "sub-trees" based on each load center (or load). The tree is formed by using the load center as the root or starting node and by following all possible branches or connections (or inter-connections). The generators form the goal nodes. This technique provides us with flexibility (due to the number of alternative path solutions generated by the search technique for the reconfigured system) and accuracy.

Step III: After all the trees (for the respective load centers) are constructed, a search is carried out. The search is initiated by the occurrence of a fault in the system (and possibly loss of load/s). The search technique used here is A^* - an optimal search procedure used extensively in AI.

A^* Search Technique: The A^* -algorithm always finds the optimum solution. Basically, the A^* search procedure is an extension of the A- algorithm and hence incorporates all the properties of the A -algorithm.

The search technique is carried out on-line because it is based on faults occurring in the system.

It might be possible to account for a large number of faults in case of the scaled test model but for a larger, more complex system it is generally very difficult to carry out a thorough analysis of all possible faulted conditions and hence our decision to proceed on-line for this part. The search technique is carried out by assigning certain cost functions to all the inter-connections.

Step IV The cost functions are generated based on fuzzy logic. The reason to use fuzzy logic is that the configuration and constraints or parameters of the system are constantly changing and hence a great deal of uncertainty is incorporated in the system description.

Step V Finally, the results of the search technique (Step III) and the cost functions (Step IV) are combined and all possible solutions are analyzed based on the value attributed to each path. The path with the minimum cost is chosen - as that is the optimum solution.

Project Participants

The principal investigator of this project is
Dr. *George Vachtsevanos*, Professor of Electrical Engineering at the Georgia Institute of Technology, Atlanta, Georgia.
Dr. *Ronald Arkin*, Associate Professor at the College of Computing at the Georgia Institute of Technology, serves as a co-investigator to the project.
Dr. *Miroslav Begovic*, Assistant Professor of Electrical Engineering at the Georgia Institute of Technology, serves as a co-investigator to the project.
Dr. *Hoon Kang*, a recent Ph.D. graduate at the Georgia Institute of Technology, has participated in project activities as a post-doctoral fellow.
Three graduate students in the School of Electrical Engineering at Georgia Tech are currently receiving partial support from this project.

Presentations and Technical Contributions

1. H. Kang and G. Vachtsevanos, "Fuzzy Hypercubes: Linguistic Learning/Reasoning System for Intelligent Control and Identification," to appear in **International Journal of Intelligent and Robotic Systems**.

2. G. Vachtsevanos, H. Kang, J. Cheng and I. Kim, "On the Detection and Identification of Axial Flow Compressor Instabilities," to appear in **American Institute of Aeronautics and Astronautics, Journal of Guidance, Control, and Dynamics**.

3. B.H. Wang and G.J. Vachtsevanos, "Fuzzy Dynamic Systems: An Application of Fuzzy Associative Memory with an Intermediate Layer," **Proc. of Automatic Control Conference '91**, pp. 12-13, 1991.

4. B.H. Wang and G. Vachtsevanos, "Fuzzy Logic Control: A Systematic Control Methodology," **Proc. of IEEE Conference on Decision and Control**, Brighton, England, Dec. 1991.

5. H. Kang and G. Vachtsevanos, "Fuzzy Hypercubes: Linguistic Learning/Reasoning Systems for Intelligent Control and Identification," **Proc. of IEEE Conference on Decision and Control**, Brighton, England, Dec. 1991.

6. H. Kang, J. Cheng, I. Kim and G. Vachtsevanos, "An Application of Fuzzy Logic and Dempster-Shafer Theory to Failure Detection and Identification," **Proc. of IEEE Conference on Decision and Control**, Brighton, England, Dec. 1991.

7. B.H. Wang and G. Vachtsevanos, "Learning Fuzzy Logic Control: An Indirect Control Approach," **Proc. of IEEE International Conference on Fuzzy Systems 1992**, pp.297-304, March 1992.

8. H. Kang and G. Vachtsevanos, "Fuzzy Hypercubes: A Possibilistic Inferencing Paradigm," **Proc. of IEEE International Conference on Fuzzy Systems 1992**, pp. 553-560, March 1992.

9. H. Kang and G. Vachtsevanos, "Adaptive Fuzzy Logic Control," **Proc. of IEEE International Conference on Fuzzy Systems 1992**, pp407-414, March 1992.

10. H. Kang and G. Vachtsevanos, "Adaptive Fuzzy Logic Control: Explicit Adaptive Control with Lyapunov Stability and Learning Capability", **Proceedings of 1992 American Control Conference**, pp. 2279-2283, 1992.

11. G. Vachtsevanos and Y.T. Kim, "System Restructuring/Controller Reconfiguration Strategy for Faulted Large Scale Systems," **The First IEEE Conference on Control Applications,** Dayton, Ohio, Sept. 1992.

12. G. Vachtsevanos and Y.T. Kim, "Critical System Units of Large Scale Systems," **The First IEEE Conference on Control Applications,** Dayton, Ohio, Sept. 1992.

13. G. Vachtsevanos, S. Farinwata and H. Kang, " A Systematic Design Method for Fuzzy Logic Control with Application to Automotive Idle Speed Control," to be presented at the **31st IEEE Conference on Decision and Control,** December 1992.

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Final Report for the project :

A HYBRID ANALYTIC/INTELLIGENT APPROACH TO FAULT-TOLERANT CONTROL SYSTEM DESIGN

(Contract No : N00014-89-J-3113)

Sponsor: Office of Naval Research
Applied Research Technology Directorate

Principal Investigator : Dr. George Vachtsevanos
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Tel: (404)894-6252

March 7, 1994

A HYBRID ANALYTICAL/INTELLIGENT
APPROACH TO FAULT-TOLERANT
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Georgia Institute of Technology

Table of Contents

Introduction

Technical Approach

Research Accomplishment

1 The Sixth Order Single Spool Gas Turbine Engine Model

2 The Hybrid Fault Detection and Identification Procedure

3 Sensor Fusion

4 Fault Tolerant Control Reconfiguration

5 Implementation Issues

6 Integrated Shipboard Electric Power System

7 Failure Identification Using Histogram Analysis and A Fuzzy Decision Hypercube

8 Hybrid Time/Frequency Information matrix Approach of Evaluating Fuzzy Membership Function and Fuzzy Decision Hypercube

9 Performance Measures

Test Results

INTRODUCTION:

In September 1989, the Office of Naval Research awarded contract No. N00014-89-J-3113 to the Georgia Institute of Technology to develop fault tolerant control strategies for large scale dynamic systems. Specifically, the technical issues under investigation were: Development of a structure-based modelling methodology of large scale systems which possess feature of structure flexibility, i.e. a component or subsystem may be deleted in a simple way does not substantially disturb the global control strategy; development of robust fault detection and identification for single and multiple failures that are maximally sensitive to the failures and at the same time insensitive to noise and modelling uncertainties thus reducing the possibility of false alarms; a technique to restructure the system dynamics by isolating the faulty components; a method that would lead to the primary performance objective of guaranteed stability while the system is operating in a degraded mode; give the measures of performance assessment of such a system; and finally to demonstrate the systems on actual physical systems.

TECHNICAL APPROACH:

The underlying factor of the fault-tolerant methodology capitalized upon structure features of the large scale system and employed both numerical and symbolic manipulations. This arrangement has allowed us to use the concepts control theory and artificial intelligence. A control law was applied which consisted of a local and a global feedback term, thus resulting in two level hierarchal control strategy. We examined large scale systems that are represented by linear state equations and the analysis was extended to nonlinear systems. A methodology for fault diagnosis was introduced which is based upon the combination of signal redundancy and detection/estimation procedures. An expert system consisting of multi-valued rulebase and an appropriate inferencing mechanism was used to assess the aggregate of fault and determined the sensitivity of failure conditions. An innovative approach to multiple failure detection using hybrid time and frequency analysis was developed using fuzzy decision hypercube. Aggregation of information is used to combine the results from this algorithm to the results obtained from histogram techniques.

In January 1991, an additional task was added to this project with partial funding from NAVSEA/DARPA and David Taylor Research Center. We were given the responsibility of developing a fault tolerant design methodology for a shipboard electric power distribution network. Through appropriate modeling and simulation, we addressed the reliability studies aimed at the "best" from a fault-tolerant standpoint, network topology, the design of protection and control reconfiguration strategies to guarantee maximum power availability to vital sensitive loads and the development of appropriate modeling/simulation tools to assist in the design and validation phases of the project. The methodologies adopted to address these issues were again a hybrid of analytical intelligent techniques the offer optimum performance trade-offs.

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RESEARCH ACCOMPLISHMENTS:

During the period of research, the research team focused their efforts on the following issues.

- Development of a full state model for a single spool turbine jet engine.

- Completion of work to detect axial flow compressor instabilities; conceptualization of a fuzzy active/adaptive control strategy to extend the operating envelop of a jet engine and to dynamic instabilities.

- Development of generic algorithms for fault tolerant control, i.e. system restructuring and controller reconfiguration routines to maintain system stability in the event of a failure.

- Basic research into sensor fusion routines to accommodate maximum availability of sensor data for the engine.

- Conceptualization of a fault tolerant control methodology for an integrated shipboard electric power system and simulation studies to demonstrate proof-of-concept feasibility. The research activity included system modeling techniques, a reliability analysis to identify the optimum network topology, a method to detect and isolate failures and a hybrid analytic/intelligent procedure has been developed to reconfigure the network in the event of a failure and main power continuity to vital loads.

- Extended the fault detection and identification algorithm using the histogram formulation, possibility distribution of failure signatures and Fuzzy Decision Hypercube. The computer programs for these algorithms were developed on a Sun workstation in C-language to achieve high processing speeds.

- Development of a unified fault tolerant control methodology to address complex large scale systems that are subjected to large grain disturbances; a unique feature of the approach is the ability to address the control reconfiguration problem even when one or more of the system's constituent components is nonlinear; verification of the effectiveness and viability of the proposed algorithms has been carried out on a simulation basis.

- An innovative hybrid time/frequency analysis algorithm was developed to accommodate both Fourier Transforms and time series data in the fuzzy paradigm. This technique allowed higher level of detectability and identifiability with less possibility of false alarms.This algorithm was also implemented on a Sun workstation in C-language.

- An algorithm for aggregation of knowledge was developed using Dempster Shafer theory of evidence to combine the results from the two above mentioned techniques.

- The algorithms and techniques thus developed were tested on an actual physical environment. A turbojet engine at the Aerospace department of Georgia Tech was used as a test platform for actual implementation.

- Finally theoretical measures of performance were developed to benchmark the intelligent fault detection and identification techniques. Formulas for the measure of detectability and identifiability were developed and tested in real test environment.

1 THE SIXTH ORDER SINGLE SPOOL GAS TURBINE ENGINE MODEL

We have developed a systematic unified failure detection and identification methodology based on possibility theory [1] and Dempster-Shafer theory [2]. The algorithm has been applied to a motor speed control problem and to the multi-stage axial flow compressor to detect and identify aerodynamic instabilities. The algorithm has been verified through computer simulation studies. An axial flow compressor model was adopted to develop the FDI algorithm from an experimental model constructed at MIT to study aerodynamic instabilities of a gas turbine engine. The experimental facility consists of a slow speed motor driven by a variable plenum volume axial flow compressor. In order to fully investigate the aerodynamic instabilities of the turbo jet engine we need to apply the developed FDI algorithm to a real jet engine so that we may validate the routines. The main focus of our research effort is to solve the real gas turbine engine aerodynamic instabilities problem.

At Georgia Tech's School of Aerospace Engineering, a research team headed by Dr. Carl Nett has recently completed the construction of a single-spool, centrifugal compressor, gas turbine engine. An engine model [3] has been developed based on the experimental stall capable gas turbine engine. The engine can be operated at a speed of 60,000 rpm which is much higher than that of Greitzer's compressor model [4]. The gas turbine engine available at the School of Aerospace Engineering is equipped with a high bandwidth fuel flow, nozzle area, and compressor discharge bleed area servos. The model developed for the experimental engine is based on engine component steady state performance maps and unsteady quasi one-dimensional flow equations. The model has three control inputs, three states, and incorporates the dynamic linkage of the compressor and turbine through the spool. The three states are compressor mass flow rate, plenum pressure and speed of spool.

Our main concern is to apply the FDI routines to an appropriate engine model. For this reason, we borrow C. Nett's basic third order dynamic engine model and expand it to a 6th order engine model. The fuel flow rate, bleed valve, and nozzle dynamics are added to the original third order dynamic model. The objective in increasing the order of the dynamics is primarily to study the system parameter effects on engine instabilities. In Greitzer's compressor model [4], the compressor instabilities are caused by perturbations of the compressor rotor rotating speed and the nozzle area. It is well accepted that engine instabilities are generally caused by the same dynamic variations. The final goal is to apply the hybrid FDI algorithm to the GT experimental engine [3] in order to demonstrate its real-time feasibility on a complex dynamic engineered system - the gas turbine engine. The following section details the development of the 6th order dynamic model for GT's single spool, single stage centrifugal compressor engine.

1.1 The sixth order Single Spool Turbo Jet Engine Model:

Figure 1.1 depicts, in a flow diagrammatic form, this sixth order quasi one-dimensional engine model. The model can be described mathematically in terms of the following ten sections:

1.1.1 Inlet: At the inlet, there is no flow rate, so the inlet pressure P_{t2} and the inlet temperature T_{t2} are the same as ambient pressure P_0 and ambient temperature T_0, respectively,

$$P_0, \quad T_0, \quad given, \quad P_{t2} = P_0, \quad T_{t2} = T_0 \tag{1.1}$$

1.1.2 Compressor : the compressor dynamics are described by the steady state performance map, $C(\dot{m}_c,\omega,P_{t2},T_{t2})$ which is shown in Figure 1.2. $\dot{m}_c$ is the flow rate in the compressor, ω is the rotating speed of the spool, η_c is the efficiency of the compressor, P_3 is the pressure in the compressor duct, γ is the air specific constant, P_{t3} is the plenum total pressure, and T_{t3} is the plenum total temperature, thus

$$P_{t3} = P_{t2}\, C\,(\, \dot{m}_c\,,\, \omega\,,\, P_{t2}\,,\, T_{t2}\,) \tag{1.2}$$

$$T_{t3} = T_{t2}\,\{1 + \frac{1}{\eta_c}\,[\,(\frac{P_3}{P_{t2}})^{\frac{\gamma-1}{\gamma}} - 1\,]\,\} \tag{1.3}$$

1.1.3 Compressor Duct: The flow rate change in the compressor can be described as the additional effects of the pressure difference between the compressor inlet P_3, outlet P_{31} and the axial flow velocity difference between inlet u_3 and outlet u_{31}. A_c is the average area of the compressor duct and L_c is the length of the compressor duct,

$$\frac{d\dot{m}_c}{dt} = \frac{A_c}{L_c}\,(P_3 - P_{31}) + \frac{\dot{m}_c}{L_c}\,(u_3 - u_{31}) \tag{1.4}$$

1.1.4 Plenum : The pressure change in the plenum can be described as mass flow balance and adiabatic compression, where $\dot{m}_b$ is the mass flow of the bleed valve, $\dot{m}_T$ is the flow rate out of

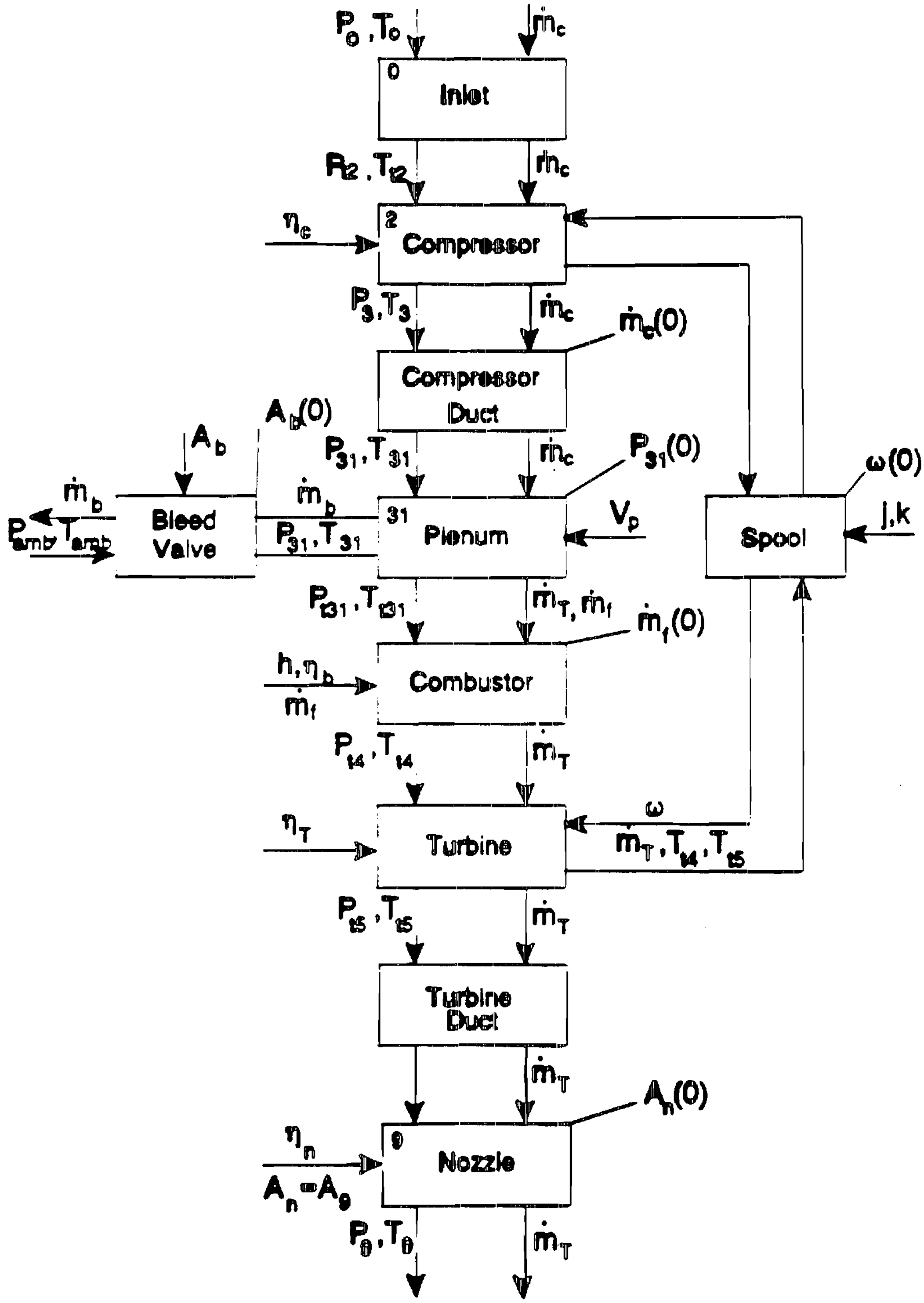


Figure 1.1 Block Diagram of Engine Model

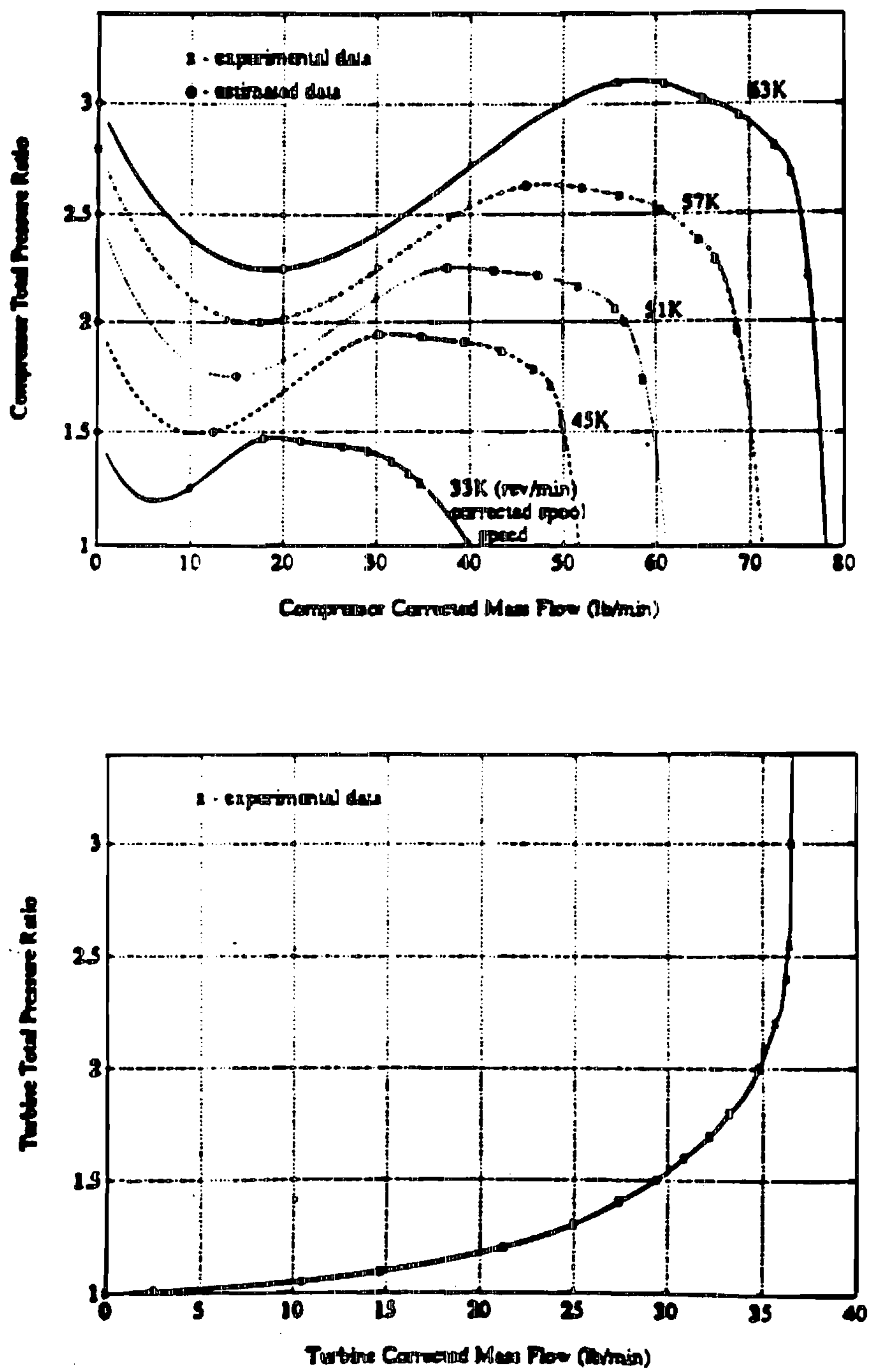


Figure 1.2 Compressor Map

the plenum, $\dot{m}_f$ is the flow rate of the fuel, P_{031} is the total pressure in the plenum, T_{031} is the total temperature of the plenum, R is the air constant, and V_p is the volume of the plenum. The relevant equations are :

$$\frac{dp_{31}}{dt} = \frac{\gamma R T_{31}}{V_p}(\dot{m}_c - \dot{m}_b - \dot{m}_T + \dot{m}_f) \tag{1.5}$$

$$\frac{T_{31}}{T_3} = (\frac{P_{31}}{P_3})^{\frac{\gamma-1}{\gamma}} \tag{1.6}$$

$$P_{t31} = P_{31}, \qquad T_{t31} = T_{31} \tag{1.7}$$

1.1.5 Bleed Valve : This valve is used as a damping device to increase the damping of the system dynamics. The device can be described as a simple orifice. P_{amb} is the ambient pressure, T_{amb} is the ambient temperature, A_b is the opening area of the bleed valve, then

$$\dot{m}_b = A_b \sqrt{\frac{2P_{amb}}{RT_{amb}}(P_{31} - P_{amb})} \tag{1.8}$$

1.1.6 Combustor : The combustor section can be modeled as a constant pressure process with a simple heat balance. The temperature change in the combustor is the heat release of the fuel in the combustion process. C_p is the specific heat of air in the constant pressure process, η_b is the efficiency of the combustor, h_f is the enthalpy of the fuel, and T_{t4} is the total temperature at the combustor, thus

$$T_{t4} = \frac{\dot{m}_T C_p T_{t31} + \eta_b \dot{m}_f h_f}{\dot{m}_T C_p} = T_{t31} + \frac{\eta_b \dot{m}_f h_f}{\dot{m}_T C_p} \tag{1.9}$$

$$P_{t4} = P_{t3} \tag{1.10}$$

1.1.7 Turbine : The turbine section is modelled as an adiabatic expansion process. The turbine is described as a steady state performance map $F(\dot{m}_T, P_{t4}, T_{t4})$. The following two equations depict the characteristics of the turbine. P_{t5} is the total pressure at the turbine outlet, T_{t4} is the total temperature at the turbine outlet, and η_T is the efficiency of the turbine,

$$P_{t5} = \frac{P_{t4}}{F(\dot{m}_T, P_{t4}, T_{t4})} \tag{1.11}$$

$$T_{t5} = T_{t4}\{1 - \eta_T[(\frac{P_{t4}}{P_{t5}})^{\frac{\gamma-1}{\gamma}} - 1]\} \tag{1.12}$$

1.1.8 Turbine Duct or (Throttle Duct) : This section is simply modelled as an insulated conducting duct. The pressure P_{t8} and temperature T_{t8} are kept constant through the duct, i.e.

$$P_{t8} = P_{t5} \tag{1.13}$$

$$T_{t8} = T_{t5} \tag{1.14}$$

1.1.9 Nozzle : The nozzle is separated into an unchoked process and a choked process.

- Unchoked process:

$$\frac{P_9}{P_{t8}} > \frac{P_{cri}}{P_{t8}} \sim P_9 > P_{cri} \tag{1.15}$$

$$\dot{m}_T = \frac{A_9}{R}\frac{P_9}{P_{t8}}\frac{P_{t8}}{\sqrt{T_{t8}}}\sqrt{2C_p\eta_n[1-(\frac{P_9}{P_{t8}})^{\frac{\gamma-1}{\gamma}}][\frac{1}{1-\eta_n[1-(\frac{P_9}{P_{t8}})^{\frac{\gamma-1}{\gamma}}]}]} \tag{1.16}$$

- Choked process:

$$\frac{P_9}{P_{t8}} < \frac{P_{cri}}{P_{t8}} \sim P_9 < P_{cri} \tag{1.17}$$

$$\dot{m}_T = \frac{P_{t8}}{\sqrt{T_{t8}}}\sqrt{\frac{2\gamma R}{\gamma+1}}\,\frac{A_9}{R}\,[1 - \frac{1}{\eta_n}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}\,\frac{\gamma+1}{2} \tag{1.18}$$

where A_9 is the opening area of the nozzle, and η_n is the efficiency of the nozzle.

1.1.10 Spool : The spool is the mechanical linkage between the compressor and the turbine. The compressor is driven by the turbine, and the inertia dynamics are expressed as

$$J\omega\frac{d}{dt}\omega = C_p\dot{m}_T(T_{14} - T_{15}) - C_p\dot{m}_c(T_{13} - T_{12}) - k\omega^2 \tag{1.19}$$

$$\frac{d}{dt}\omega = \frac{1}{J\omega}[C_p\dot{m}_T(T_{14} - T_{15}) - C_p\dot{m}_c(T_{13} - T_{12}) - k\omega^2] \tag{1.20}$$

ω is the rotational speed of the spool, k is the friction coefficient of the mechanical parts, and J is the rotational inertia of the spool.

1.1.11 State Equations :

The 6th order state equations can be described as follows

$$\frac{d\dot{m}_c}{dt} = \frac{A_c}{L_c}(P_3 - P_{31}) + \frac{\dot{m}_c}{L_c}(u_3 - u_{31}) \tag{1.21}$$

$$\frac{dP_{31}}{dt} = \frac{\gamma R T_{31}}{T_{31}}(\dot{m}_c - \dot{m}_b - \dot{m}_T + \dot{m}_f) \tag{1.22}$$

$$\frac{d\omega}{dt} = \frac{1}{J\omega}[C_p\dot{m}_T(T_{14} - T_{15}) - C_p\dot{m}_c(T_{13} - T_{12}) - k\omega^2] \tag{1.23}$$

$$\frac{d\dot{m}_f}{dt} = \frac{1}{\tau_f}(\dot{m}_{f\infty} - \dot{m}_f) \tag{1.24}$$

$$\frac{dA_b}{dt} = \frac{1}{\tau_b}(A_{b\infty} - A_b) \tag{1.25}$$

$$\frac{dA_n}{dt} = \frac{1}{\tau_n}(A_{n\infty} - A_n) \tag{1.26}$$

1.1.12 State Equations : The six state equations can be rearranged in the following form:

$$\dot{x}(1) = \frac{A_c}{L_c}(P_3 - x(2)) + \frac{x(1)}{L_c}(u_3 - u_{31}) \tag{1.27}$$

$$\dot{x}(2) = \frac{\gamma R T_{31}}{V_p}(x(1) - \dot{m}_b - \dot{m}_T + x(4)) \tag{1.28}$$

$$\dot{x}(3) = \frac{1}{Jx(3)}[C_p \dot{m}_T(T_{t4} - T_{t5}) - C_p x(1)(T_{t3} - T_{t2}) - kx^2(2)] \tag{1.29}$$

$$\dot{x}(4) = \frac{1}{\tau_f}(\dot{m}_{f\infty} - \dot{m}_f) \tag{1.30}$$

$$\dot{x}(5) = \frac{1}{\tau_b}(A_{b\infty} - A_b) \tag{1.31}$$

$$\dot{x}(6) = \frac{1}{\tau_n}(A_{n\infty} - A_n) \tag{1.32}$$

$$\frac{P_{t3}}{P_{t2}} = C(\dot{m}_c, \omega, P_{t2}, T_{t2}) \tag{1.33}$$

where τ_f is the time constant of the Fuel Mass Flow, τ_b is the time constant of the Bleed Valve, τ_n is the time constant of the nozzle, $A_{b\infty}$ is the Bleed Valve Opening Command, $\dot{m}_{f\infty}$ is the Fuel Flow Rate Command, and $A_{n\infty}$ is the Nozzle Opening Command.

We have developed an appropriate computer simulation for this sixth order engine model. Simulation studies are currently being conducted to test model validity and its sensitivity to parametric changes. This engine simulation platform will be linked with the fault detection and identification and control routines. The FDI routines are described in the following section of this report.

2 THE HYBRID FAULT DETECTION AND IDENTIFICATION PROCEDURE

FDI techniques are considered for sensor, actuator, and component failures. Both single and multiple faults are treated. The research objective is to develop a systematic and thorough FDI procedure that is maximally sensitive to failures while avoiding false alarms. The approach is systematic because it relies on a modular architecture to (1) efficiently trigger FDI routines, (2) validate sensor data, (3) combine failure evidence from such diverse sources as analytic redundancy, detection/estimation theory, and limit checking, (4) utilize expert system tools and Dempster-Shafer evidential theory to manage uncertainty and assess the symptomatic evidence, i.e. detect and identify faulty components for monitoring and control purposes. Figure 2.1 is a functional block diagram of the proposed fault diagnostics approach.

Fault triggering is based on a deviation of system performance from its planned trajectory. Sensor data validation involves the detection of sensor failures through a combined parity space (redundancy-based) and detection/estimation algorithm. The combined FDI architecture is depicted schematically in Figure 2.2.

The validation procedure compares each reading to all other like readings. The "best" estimate from a set of good measurements is defined as that value which gives the minimum of a particular function of the measurements. Where more than the minimum number of measurements is available, the "best" estimate is taken in the least squares sense, i.e., it is that value which minimizes the square of the length of the measurement error vector.

Fault detection and identification uses validated sensor and analytic data to analyze the status of the system components. Fault diagnosis involves the incremental accumulation of evidence in the form of symptoms to determine the "health" of the system components. Parity space representation and analytic redundancy [5-10]. in addition to limit checking and statistical estimation techniques may be applied to accumulate the required evidence. Parity space representation transforms an array of redundant measurements into a new array, called a parity vector, in such a way that the true value of the underlying variable is suppressed, leaving only components of the measurement errors as elements of the parity vector [6]. In parity space, the parity vector remains near zero when the redundant measurements are consistent, i.e., when no faults are present. When a fault occurs, the parity vector grows in magnitude, and its direction of growth is uniquely associated with the faulty measurement.

The fault detection part of the algorithm capitalizes upon the behavior of the magnitude of the parity vector to detect the presence of a fault. The magnitude of the parity vector P is,

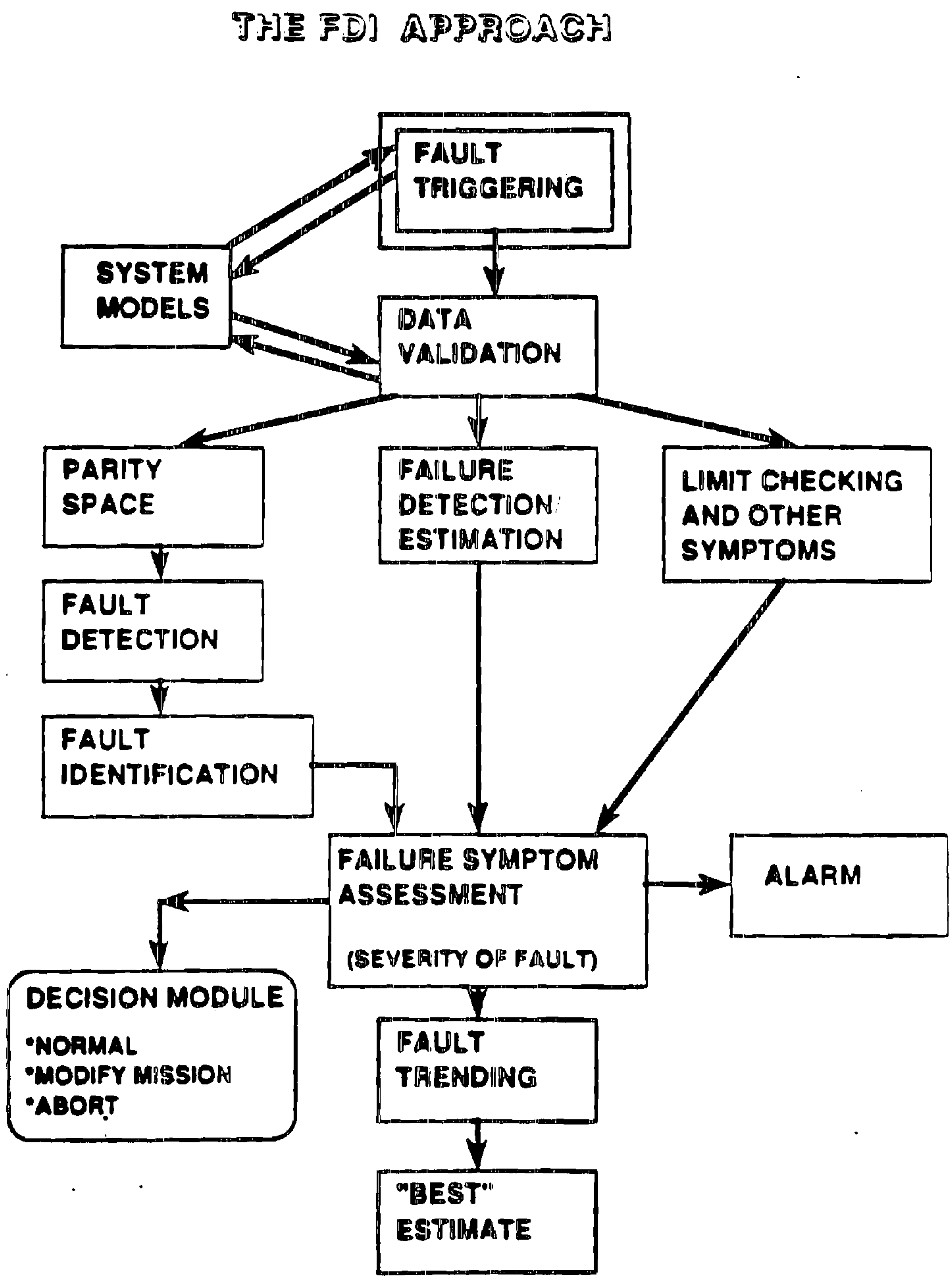


Figure 2.1 The Failure Detection and Identification Approach

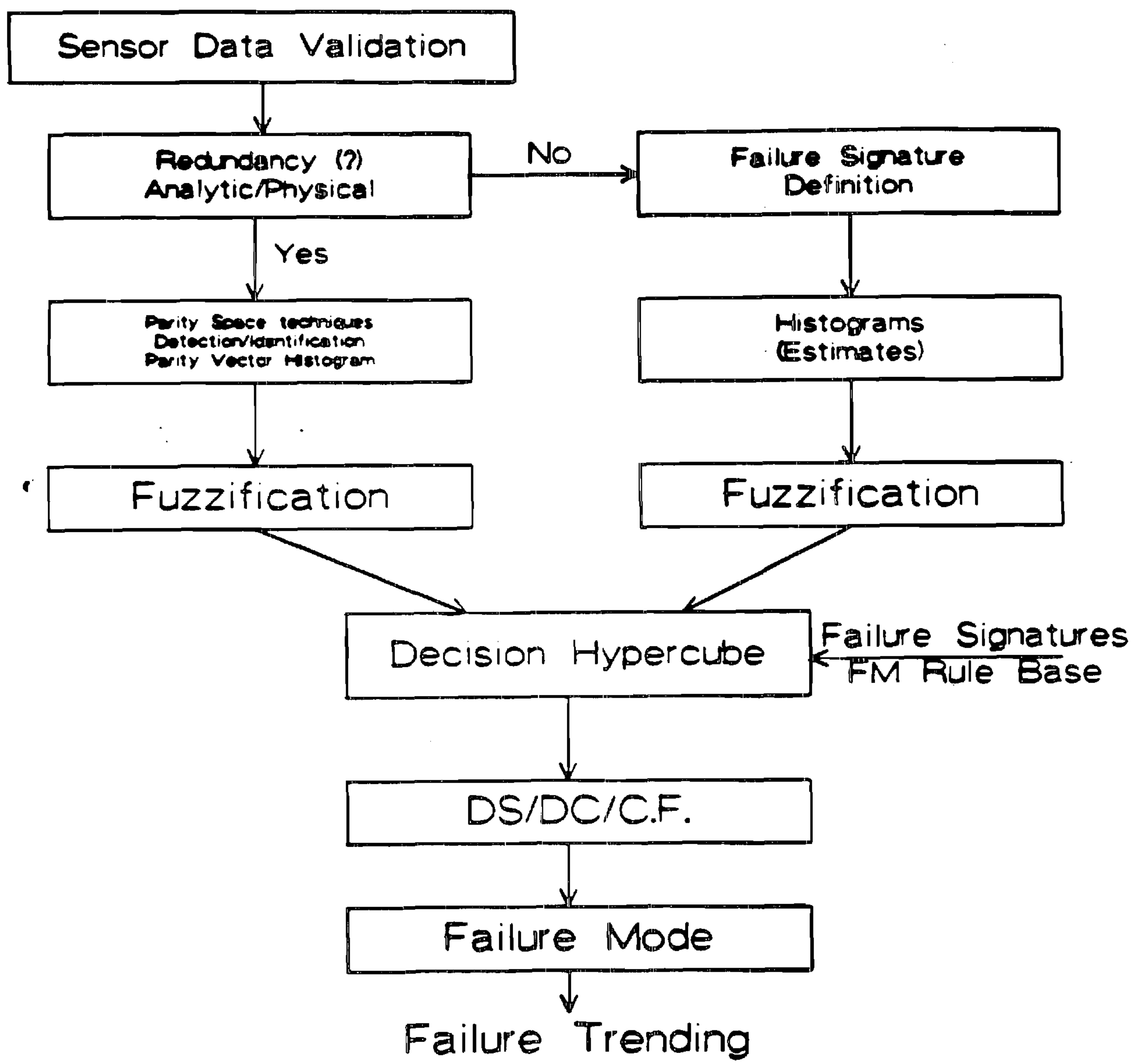


Figure 2.2 The combined FDI architecture

in general

$$P^T P = \sum_{i=1}^{l} \varepsilon_i^2 - \frac{1}{l} (\sum_{i=1}^{l} \varepsilon_i)^2 \tag{2.1}$$

$$\delta_l = \max(P^T P) = \sum_{i=1}^{l} b_i^2 - \frac{1}{l} (\sum_{i=1}^{l} b_i)^2 \tag{2.2}$$

If the sensor noise probability density function is uniform and if the error in each measurement for an unfailed component is assumed to be bounded, i.e., $|\epsilon_i| < b_i$, then A search algorithm is used to locate a minimum of Σb_i and, finally, δ_l is obtained from (2.2).

Fault detection is based upon the relative magnitude od the maximum allowable error bound compared to the length of the parity vector, i.e., δ_l/P^TP. A threshold condition is reached when $P^TP = \delta_l$. In general, both δ_l and P^TP are not known accurately and, therefore, a fuzzy representation of the quantity δ_l/P^TP is most appropriate. The compositional rule of inference [11] is used to infer the degree of severity of the fault condition.

The fault identification part of the program is intended to identify which component is faulted. Following [6], we propose a technique based upon the concurrent checking of the relative consistency of small size subsets of measurements. In the absence of the redundant data, a scheme based upon stochastic estimation is called to perform the fault detection and identification function.

The stochastic estimation scheme consists of two processing units. First, fault triggering plays the role of a failure sensitive filter at the learning state. A recursive parameter estimation technique is used to determine the direction and size of the parameter changes in real-time from the failure signature. Once the FDI system is alert, the histograms of parameter residues are evaluated and transformed into the corresponding fuzzy sets as decision samples at the tracking stage. In the second step, the corresponding fuzzy input vectors are applied to fuzzy relational matrices which undertake the identification of the failure mode utilizing the concept of possibility theory. Thus, multiple failures can be identified using the knowledge base. The combined analytical/intelligent approach relaxes many assumptions raised in other FDI schemes such as the conditions of a linear time-invariant system [12] and left invertibility from the failure mode to the output [11] as well as the requirement of a bank of Kalman filters [13][14]. Figure 2.3 shows the systematic stages of FDI required for potential system failures.

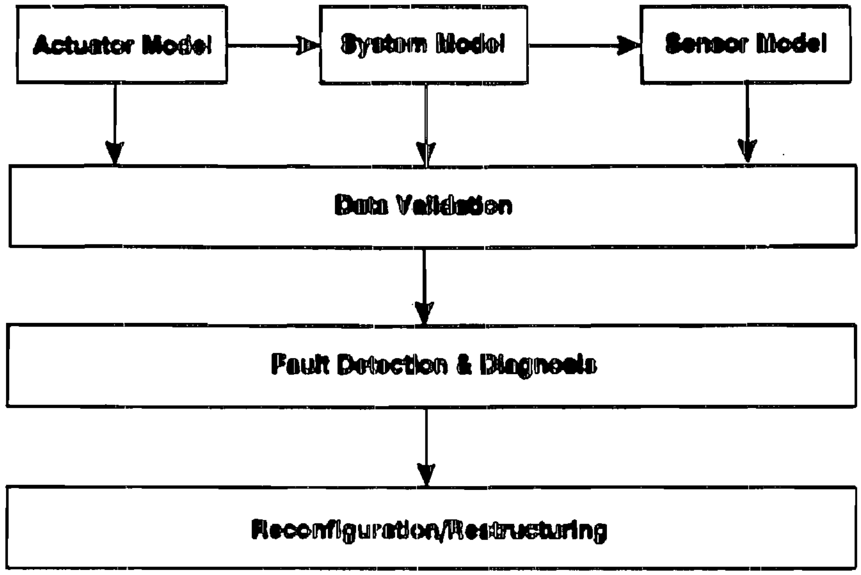


Figure 2.3 Block Diagram of FDI modules

The advantages of knowledge based approaches to FDI are flexibility of failure information, intelligence in the failure representations, and direct description of failure modes. When uncertainty is inevitable, Zadeh's possibility theory [1] can be applied to FDI systems as one of the most efficient methods of coping with uncertainty. Fuzzy semantics can be applied to the intelligent part of FDI where soft bodies of evidence are used for detecting and identifying a potential failure. Let us limit the FDI scheme to the case of completely known failure modes.

We define by "failure" the deviation of some parameters that are physically responsible for the abnormal behavior of the system. The FDI scheme can be expressed as a stochastic parameter estimation problem. A failure mode is defined as a one-to-one mapping relationship with a failure symptom such as "the impulse line is leaking" or "the motor inertia is very large". In fact, the failure mode is directly related to the system parameters and can be additively incorporated in a failure model. Therefore, we can easily interpret a failure mode as a physical representation of a failure signature is defined as a time history of each failure mode.

The actual system may be described by a state space representation in terms of discrete nonlinear vector equations of the form,

$$x(t+1) = f(x(t),\theta(t),u(t)) \tag{2.3}$$

$$z(t) = h(x(t),\theta(t)) \tag{2.4}$$

where $u(t) \in U$ and $z(t) \in Z$ are the scalar input and output, respectively; $\theta(t) \in \Theta$ is the time-varying system parameter vector; $x(T) \in \chi$ denotes the state vector; and f, h represent the nonlinear vector fields for the state and output vectors, respectively. A nonlinear model can be obtained for a class of nonlinear systems where the corresponding equations are linear in the parameter vector $\theta(t) \in \Re^n$ such as

$$z(t) = \phi(t,\bar{\theta})^T \theta(t) \tag{2.5}$$

where $\bar{\theta}$ is the nominal solution for the parameters - $\bar{\theta}$ may be a partially known set of $\theta(t)$ that does not require the estimation process - during the <u>time window</u> $J_T=[t_0,t_0+t]$ and $\phi(t,\bar{\theta}) \in \mathbb{R}^{p\times n}$ is the nonlinear regressor associated with $z(t-i)$ for $i=1\ldots r_1$ and $u(t-d-j)$ for $j=0\ldots r_2-1$ (d is an integer designating the system delay and $n=r_1+r_2$). In the linear case, it is easy to derive the above relation from an auto-regressive moving-average (ARMA) model,

$$A(q^{-1},\theta(t))z(t) = q^{-d}B(q^{-1},\theta(t))u(t) \tag{2.6}$$

where q is the shift operator; $A(q^{-1},\theta)$ is a monic matrix polynomial; and $B(q^{-1},\theta)$ is a matrix polynomial. The transfer function $H(q^{-1},\theta)$ is

$$H(q^{-1},\theta) = q^{-d}A(q^{-1},\theta)^{-1}B(q^{-1},\theta) \tag{2.7}$$

in the linear multivariable case.

The failure model can be represented by

$$\theta(t+1) = \theta(t) + \omega(t) + \xi(t) \tag{2.8}$$

$$z(t) = \phi(t)^T \theta(t) + \nu(t) + \eta(t) \tag{2.9}$$

where $\omega(t) \in \mathbb{R}^n$ and $\nu(t) \in \mathbb{R}^p$ are white gaussian parametric and measurement disturbances, respectively; $\phi(t)=\phi(t,\bar{\Theta}) \in \mathbb{R}^n$ is the regression vector; $\xi(t) \in \mathbb{R}^n$is the system plus actuator

failure signature while $\eta(t) \in \mathbb{R}^p$ represents the sensor failure signature. For initial conditions, it is required that $\theta_0 = E[\theta(0)$ and $P_0 = \mathrm{Cov}[\theta(0) = \theta_0)$.
We assume that failure modes are unique and the associated failure signatures do not occur simultaneously in one parameter. It is required that the failure should occur inside the time-window $J_T = [t_0, t_0 + T]$ in order to get the nominal parameter values

$$\bar{\theta} = \lim_{t \to t_f^-} \theta(t) \tag{2.10}$$

where t_f^- denotes the time just before the failure. Let the parameter residue be $\theta(t) \in \tilde{\theta}$. $\theta(t+1) = \theta(t) + \omega(t)$ when there is no failure in $(t < t_f)$,

$$\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta} \tag{2.11}$$

where $\hat{\theta}(t)$ is the estimate of the parameter $\theta(t)$. The parameter residues are quantized in a finite number of bins, q_i, with respect to the i-th element of $\tilde{\theta}(t)$ in order to generate a histogram of the parameter residues by counting the frequency of each bin $\tilde{\theta}_k$, $f^i = \{f^i\}Q_i$ for $k = 1 \ldots q_i$ and for $i = 1 \ldots n$ and θ_k is the bin symbol with a total number of q_i bins. Here, we call the vector $f^i \in \mathbb{R}^{q_i}$ the failure signature histogram (FSH) of the i-th system parameter. The time-window function W_T is introduced such that

$$W_T : \tilde{\Theta} \to Q \tag{2.12}$$

In the normal case, these failures are

$$\xi(t) = 0, \quad \eta(t) = 0. \tag{2.13}$$

<u>Failure model operation</u>: Let us assume that either one of $\xi(t)$ or one of $\eta(t)$ is non-zero and let $\xi(t) \neq 0$ for simplicity. The steady state assumption assures that

$$\hat{\theta}(t) \to \bar{\theta} \quad as \quad t \to \infty \tag{2.14}$$

but if the failure occurs at $t = t_f < T$,
Therefore, the parameter residue $\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta}$ plays an important role in representing the unique failure signature by the certainty equivalence principle. This failure signature can be obtained

$$\hat{\theta}(t) \rightarrow \bar{\theta} + \sum_{r=t_f}^{T} \xi(r) \quad as \quad t \rightarrow \infty \tag{2.15}$$

by the windowing function W_T that stores the residue $\tilde{\theta}(t)$ in J_T into the histogram f^{θ}. Note that $\xi(t)$ of the impulse function shows a biased failure signature in the parameter while $\xi(t)$ of the step function corresponds to a parameter drift. The block diagrams of the FDI scheme are shown in Figures 2.4 and 2.5.

The failure identification method is based on a rulebase implemented by a multi-dimensional decision making system called "Fuzzy Decision Hyper-Cube" (FDHC). Given a FSH f^{θ}, there exists a transformation from the probabilities to the corresponding fuzzy sets μ^{θ} which is based on possibility theory. Thus, each failure mode is represented uniquely by a failure signature fuzzy set (FSF). We also define a priori failure mode fuzzy sets (FMF) which correspond to each crisp failure mode.

The fuzzy hyper-cube is constructed along each rule on the basis of a priori knowledge of FSF and FMF. A matching or inferencing process is used to derive the resultant and FMF output which is excited by FSF vector μ^{θ}. There may be several rules fired by the matching process, but only one failure mode is obtained. It is suggested that the number of rules for each parameter FSF may exceed the number of failure modes for almost perfect recall.

If there are more than one FSF available, then we combine the fuzzy output vectors (FMF's) together in order to make certain that we arrive at the true failure mode using Dempster's rule of combination of evidence.

We demonstrate the proposed approach with a simple example. A motor speed control system is considered with a two-dimensional parameter vector $\theta(t)=[\alpha,\kappa]$ describing possible failure modes such as "the motor has stalled". The fuzzy relational matrix is constructed from statements of the form "if κ is high and α is low, the motor has stalled". "κ is high" is an FSF derived from the failure signature histogram. "The motor stalls" is a failure mode fuzzy set (FMF). If inferencing declares the failure motor "motor stalls", then Dempster-Shafer theory assigns a certainty factor to this failure mode. A decision on the occurrence of a specific failure mode is made only when the certainty factor exceeds a predetermined threshold limit. Thus, the scheme accounts for both ignorance (limited data base) and uncertainty (noise, unmodeled dynamics, parameter variations) resulting in a reliable and robust fault detection method.

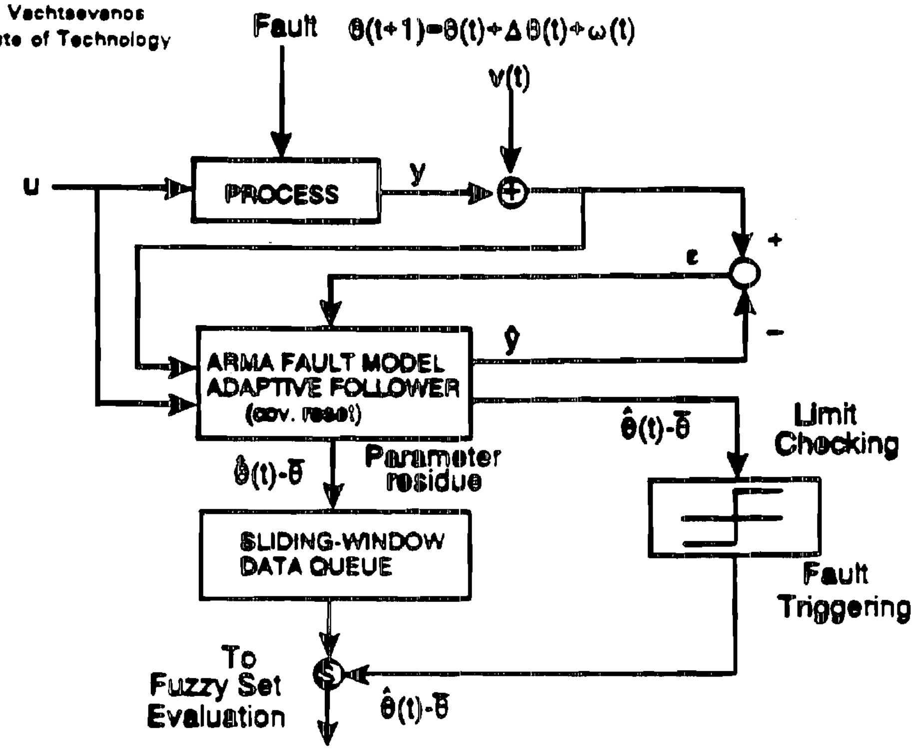


Figure 2.4 Analytic part of FDI

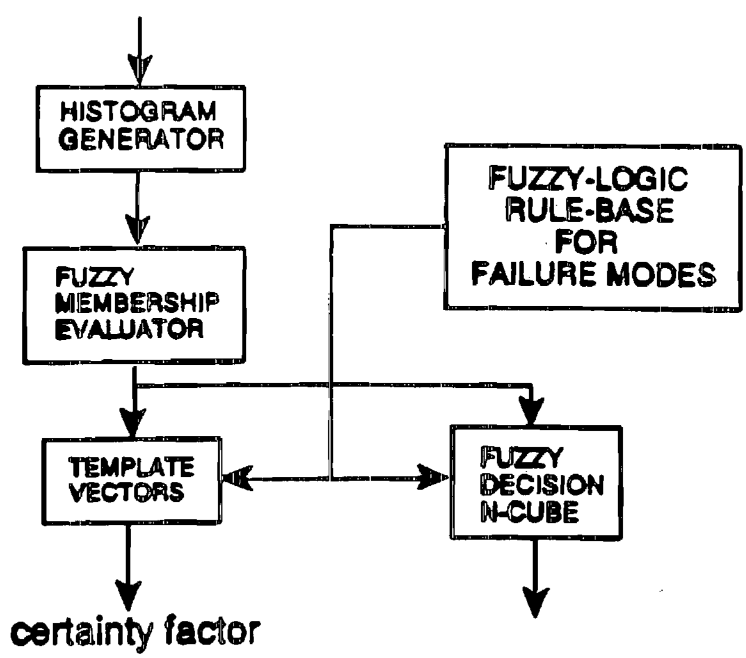


Figure 2.5 Intelligent part of FDI

Figures 2.6 and 2.7 depict a block diagram of the FDI modules and computer results that illustrate the fuzzy relational matrix for this example, fuzzy membership functions, the failure signature detection (single-fault, multiple-faults, and false alarm cases) and the failure mode identification algorithm. Figures 2.8, 2.9 and 2.10 show typical computer simulation results for an impending surge, a rotating stall, and an abrupt surge instability condition in an axial flow compressor. Since the fuzzy decision hyper-cube and the template vector use the same fuzzy failure signature distributions, the results are both consistent and complementary.

Our research team has developed a unified methodology to the fault-tolerant control problem that addresses not only FDI issues, as discussed above, but also such basic concerns as fault propagation, system restructuring and controller reconfiguration. These concerns are essential in the design of control logic for autonomous systems that demand a high degree of survivability.

For large scale complex dynamical systems, such as the jet engine or the integrated shipboard electric power system, efficient application of FDI and control strategies requires the accurate and timely processing of data from multiple sensors. It is essential, therefore, for us to consider the basic problem of integrating or fusing and processing reliably real-time monitored information for intelligent decision-making processes. We examine this topic briefly in the next section. Sensor fusion has been an active area of our research endeavor and we are currently developing efficient algorithms to accomplish this objective.

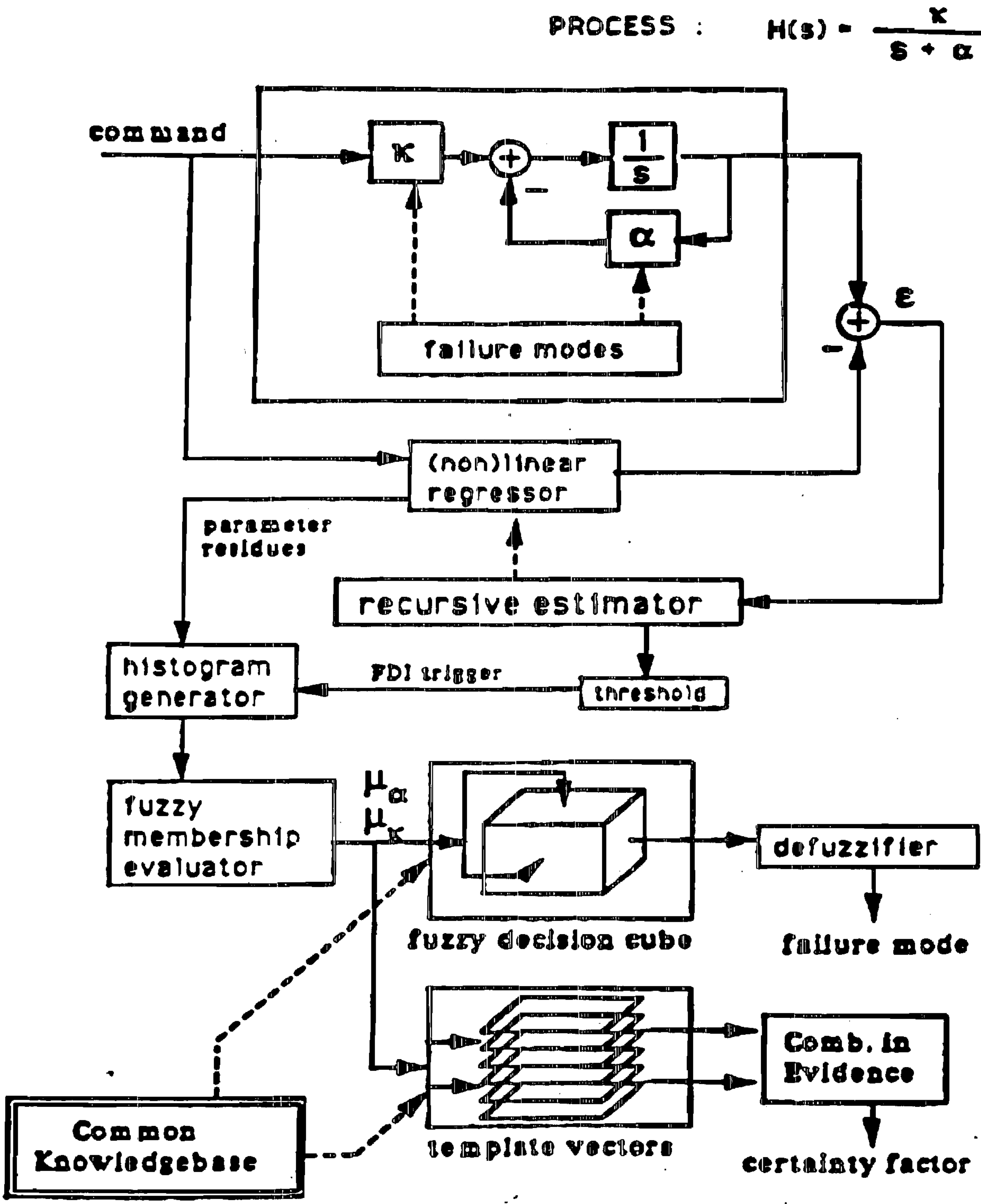


Figure 2.6 FDI structure for the Demonstration Example

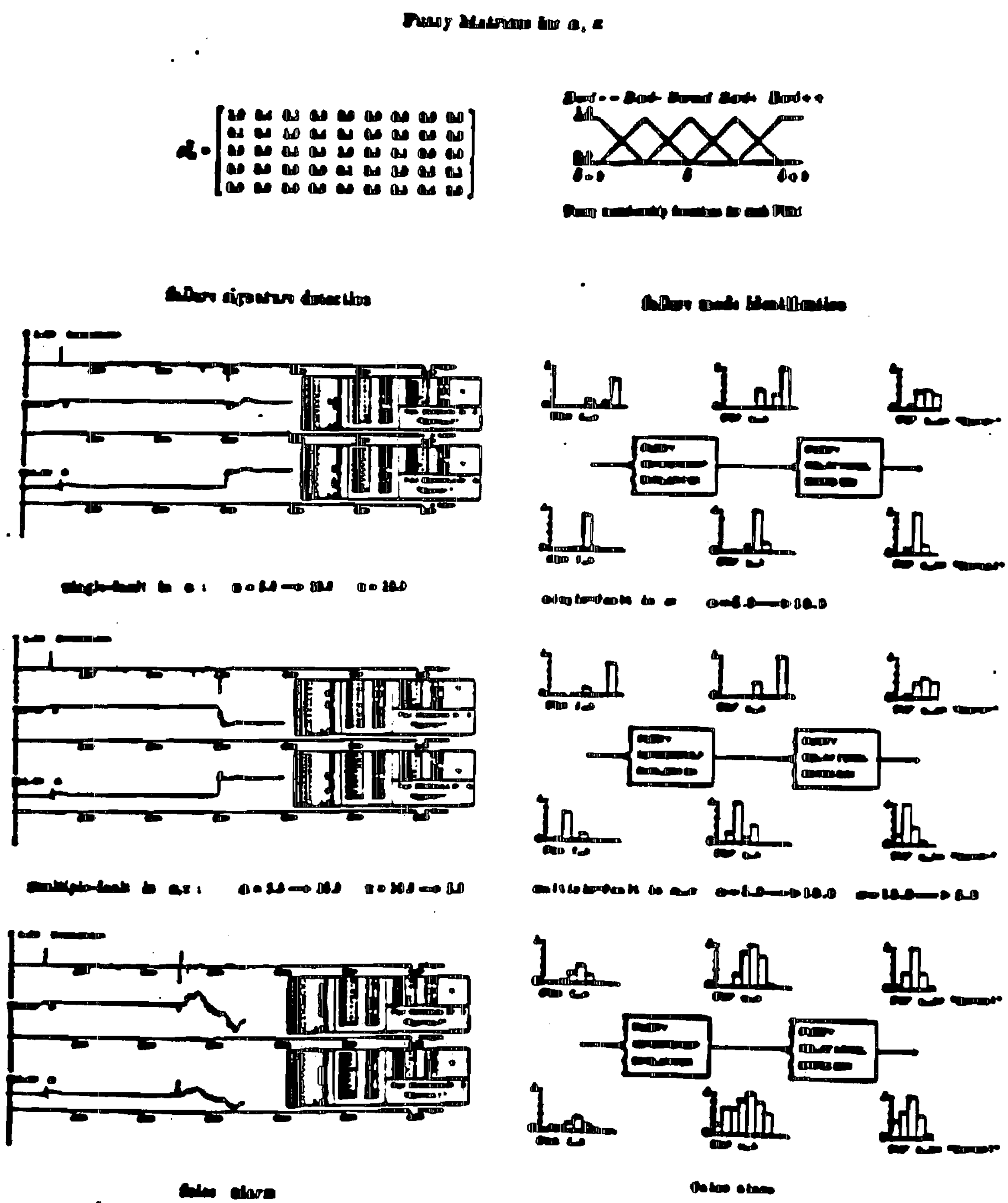


Figure 2.7 Simulation Results for Demonstration Example

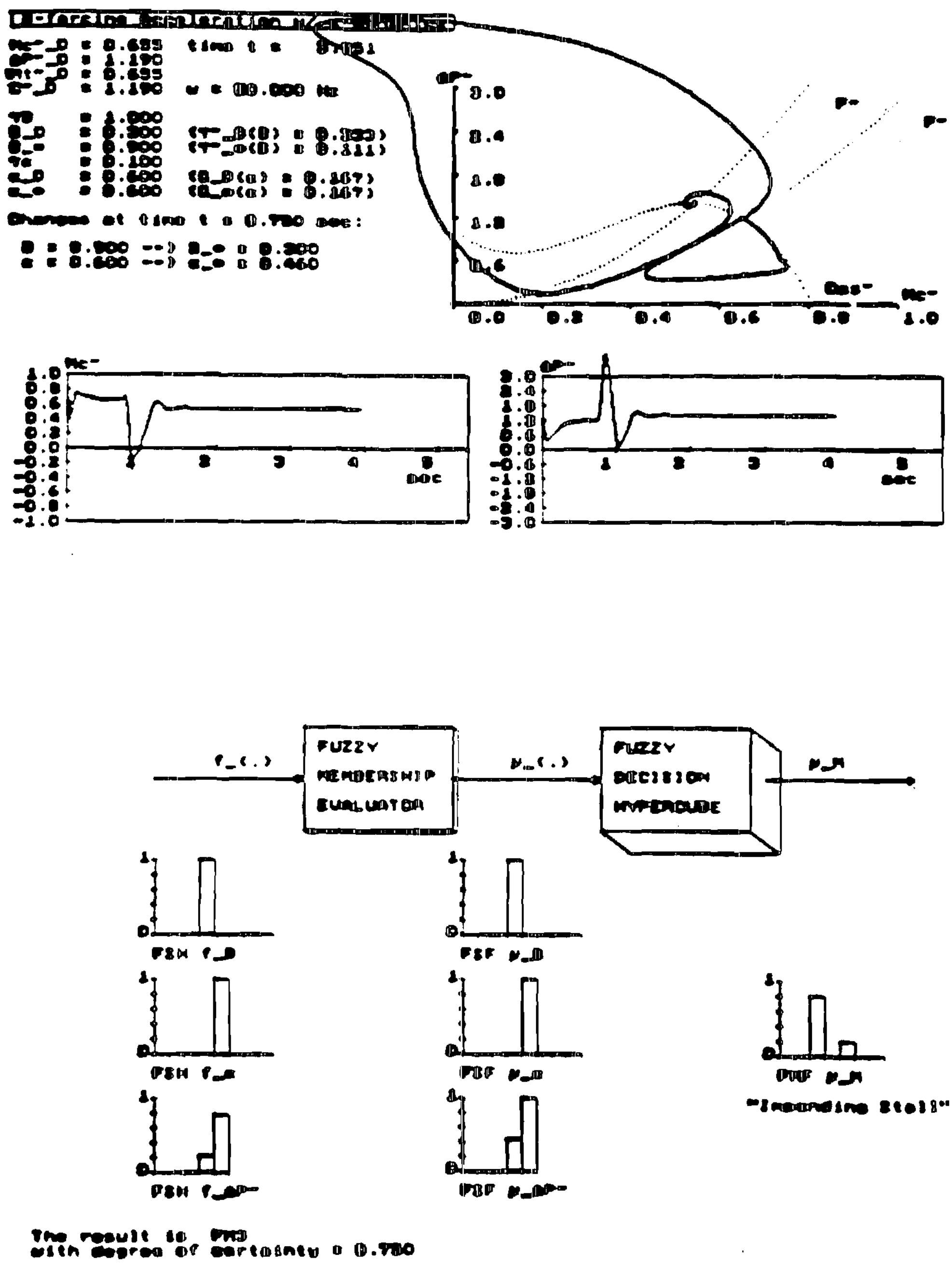


Figure 2.8 Simulation Result for an Impending Surge Instability

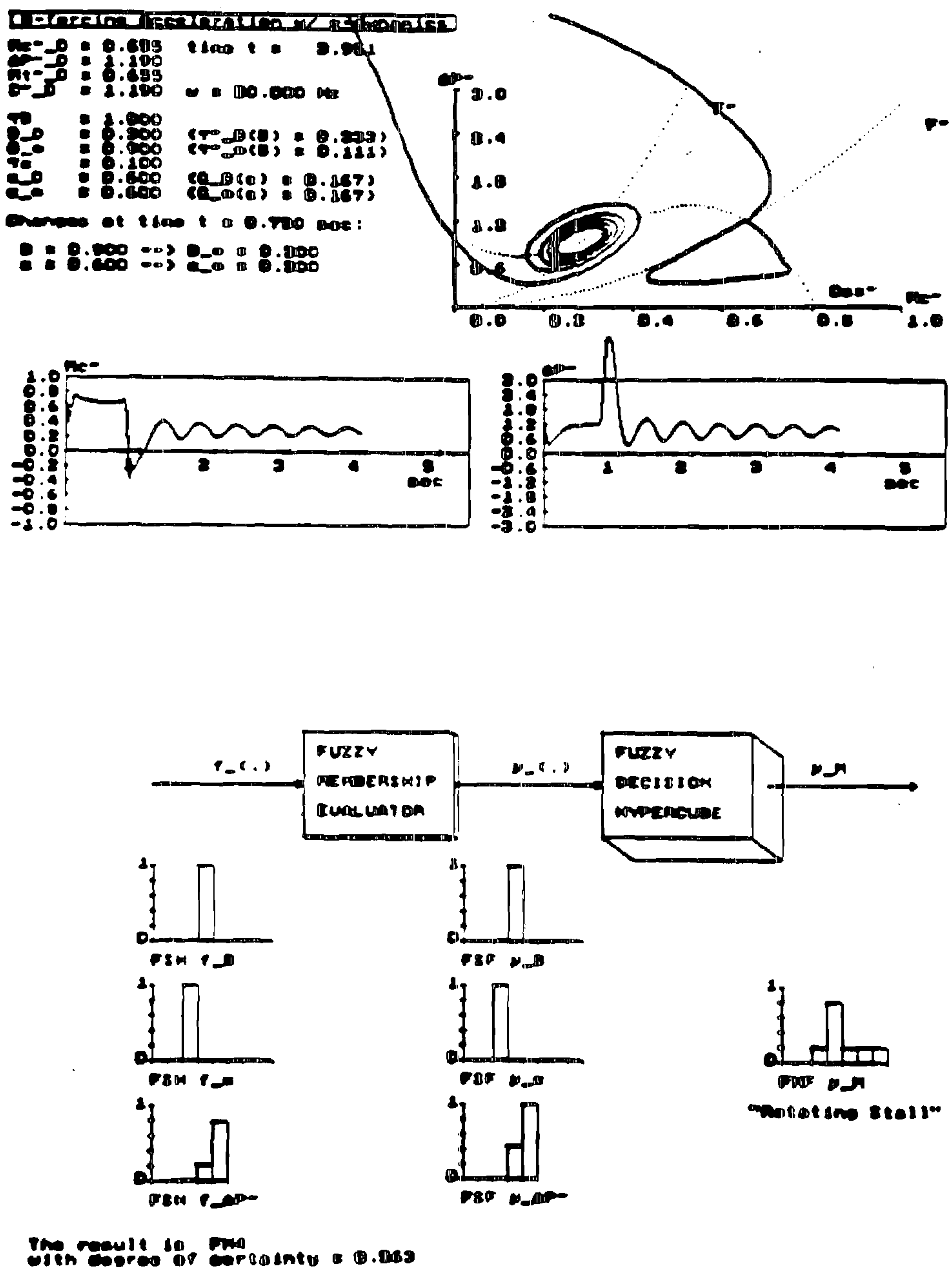


Figure 2.9 Simulation Result for a Rotating Stall Instability

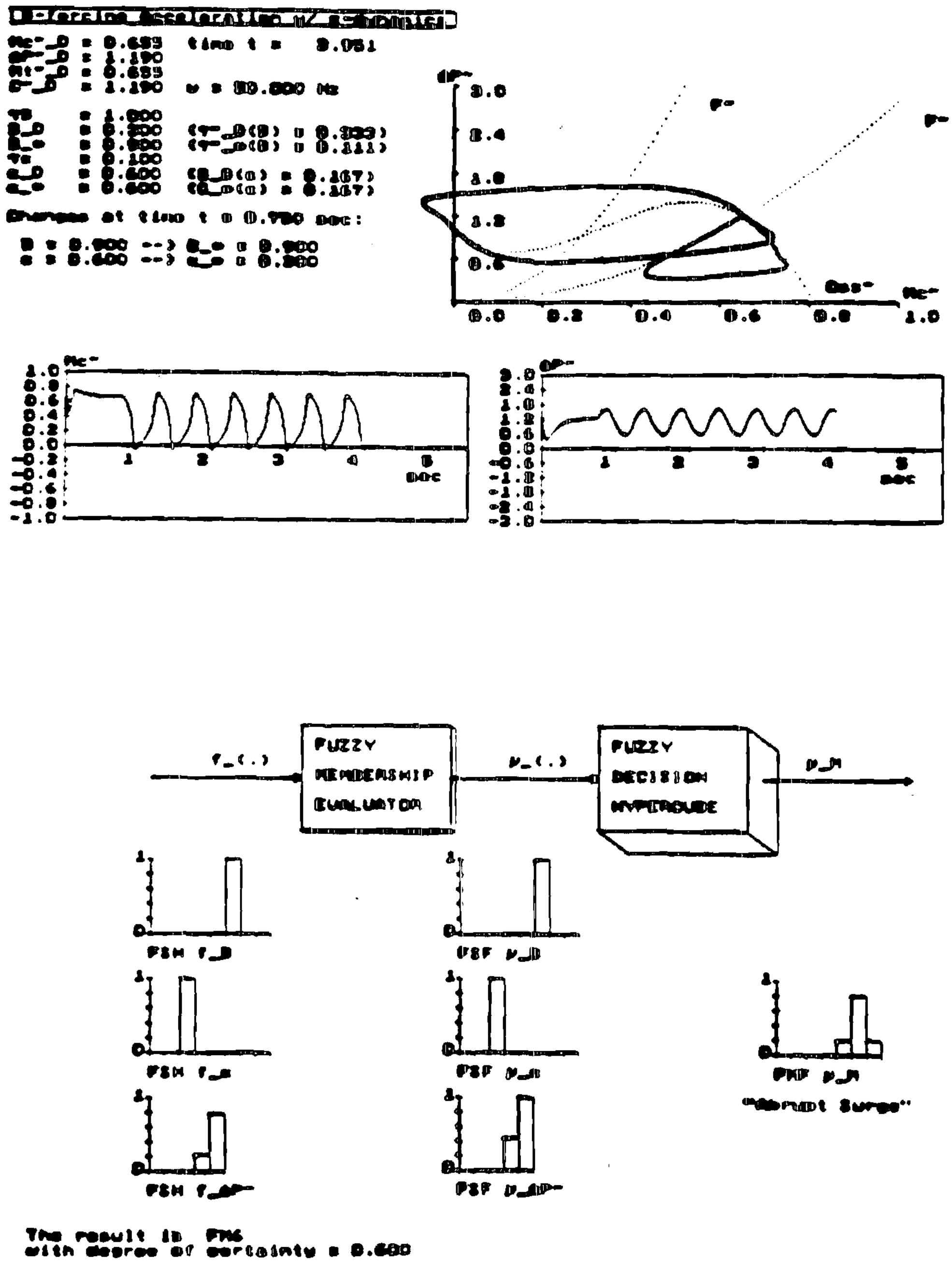


Figure 2.10 Simulation Result for an Abrupt Surge Instability

3 SENSOR FUSION

The concept of sensor fusion can be applied to sensor data from two viewpoints: sensor data validation and noise reduction. It is of course assumed that the sensors are somehow dependent on each other such that a measurement of a specific sensor can be obtained from those of other sensors through analytic relations. This technique is known as analytic redundancy which, in contrast to physical redundancy, is an alternative method to generate redundant data.

The fusion of data from several sensors, whether they constitute analytic redundancy, or physical redundancy, may bring about a number of advantages. Sensor data validation and noise reduction tend to improve the accuracy and the efficiency of the FDI and control routines.

3.1 Sensor data validation

The FDI routine assumes that sensors are operating in a normal state and the mechanism of inferencing is based on the same assumption. It is very important to ascertain whether sensors function properly, because any sensor failure may result in a catastrophic event.

The FDI routine makes a decision when the degree of certainty(DOC) is above a preset threshold. Failure on a sensor produces a low DOC value which prolongs the decision making process. Accordingly, it is not possible to make a timely decision and the system under observation will be at the risk of deteriorating to a catastrophic state. In order to exclude data from a failed sensor, it is necessary to check the validity of the sensor data.

We may rely on it to derive the available FDI rule base to assess the validity of sensor data in terms of conflicting factors between active sensors. The conflicting factor, κ, is obtained by

$$\kappa = \sum_{A \cap B = \emptyset} m_1(A)\, m_2(B) \qquad (3.1)$$

where A and B are any focal elements of sensors 1 and 2, respectively.

[example]
Assume that there are three sensors employed for a measurement. The result in terms of basic assignments is the following:

$m_1(A)=0.5$	$m_1(A,B)=0.3$	$m_1(A,B,C)=0.2$
$m_2(C)=0.6$	$m_2(C,A)=0.3$	$m_2(C,A,B)=0.1$
$m_3(A)=0.7$	$m_3(A,B,C)=0.3$	

where A, B, and C indicate the failure modes. Then the conflicting factors κ's are given by (3.1). Since the common number of subscripts for κ is 2, it is suspected that sensor 2 has failed.

One of the most important reasons for using Dempster-Shafer theory is to resolve conflicting information as shown in the above example. It is difficult to distinguish whether the conflict occurs due to a sensor failure or noise in the corrupted signal, but using a rule of thumb, a larger value of the conflicting factor typically implies a sensor failure. For the efficient use of sensors, it is recommended that specific sensors, whose conflicting factors are large, are excluded from consideration, otherwise the faulty data may procrastinate the decision making process.

3.2 Noise reduction

The most primitive form of sensor fusion can be found in averaging measurements from sensors to derive an accurate signal representation without noise,

$$x(t) = s(t) + n(t)$$

where $x(t)$, $s(t)$ and $n(t)$ are the measurement, signal and noise, respectively. It is generally assumed that the statistics satisfy a Gaussian distribution with a zero mean. Therefore, it can be claimed that for large n, the average of n measurements is close to the actual signal level. Thus, for the purpose of obtaining more accurate data, data fusion or simply averaging data from several analytic redundant sources, may be considered.

4. Fault Tolerant Control-Reconfiguration

Suppose that a failure occurs in a subsystem or component of a complex large scale dynamic system. The FDI routine, described in the previous sections of this report, identifies the failure (or impending failure) and reports this event to the control logic. The failure(s) may impair the system's operational capability or may even lead to a catastrophic event, if left unattended. It is desirable, therefore, to assess first the impact of the fault on other healthy system units and to restructure and/or reconfigure the system and its control authority to assure survivability. This section of the report describes a systematic methodology that we have developed to address the fault-tolerant control problem.

With the fault information provided by the fault detection and identification scheme, the faulted large scale system can remain in a survivable state via system restructuring and/or controller reconfiguration. Typical possible transitions, from a fault tolerance point of view, of a large scale system are shown in Figure 4.1, where

S: Normal large scale system
S^f: Faulty large scale system
S^{rs}: Restructured large scale system
S^{rc}: Reconfigured large scale system

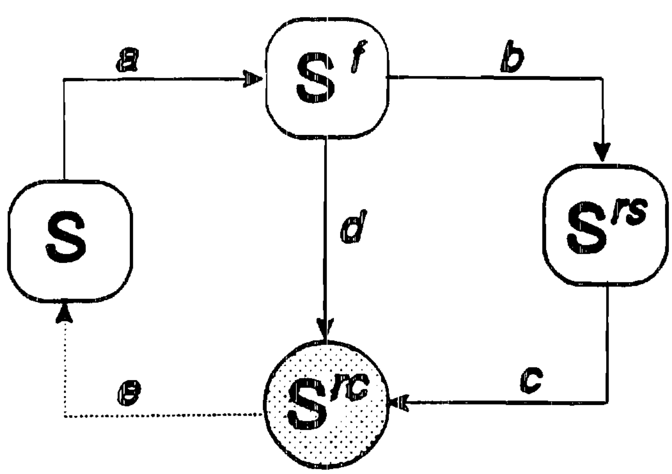


Figure 4.1 Transition of Large Scale System under failure

and the fault-tolerant issues are classified as

Path (a) $S \Rightarrow S^f$ Fault Diagnosis
Path (b) $S^f \Rightarrow S^{rs}$ Restructuring
Path (c) $S^{rs} \Rightarrow S^{rc}$ Reconfiguration
Path (d) $S^f \Rightarrow S^{rc}$ Reconfiguration without restructuring
Path (e) $S^{rc} \Rightarrow S$ Repairing

Our focus is on path (b), path (c), and path (d), which are described below.

Restructuring Problem: Determine which subsystem(s) should be isolated in order for the overall system to be controllable with the available control authority and to block the effect of any failure on other healthy subsystems.

Reconfiguration Problem: Reconfigure the controllers so that the overall (restructured) system is stable (survivable) with possibly maximum performance.

Control of a complex large scale system typically involves three distinct but interrelated modules:

- Mission Planner (MP)
- Decision Making (DM)
- Control Action (CA)

The mission planner sets a mission plan based on the functional capability of the system and the conditions of the environment in which the system operates. The functional capability of the LSS is described in terms of various operational constraints. The mission plan is modified on the basis of the environmental changes and the limits of various operational variables. The task of estimating the limits of the operational variables is undertaken by DM. Obviously, these limits are quite dependent on the system status. Based on the modified mission plan, the next main task of DM is to determine the proper controller-reconfiguration and to command the CA to perform the specfied action.

In practice, a failure event may cause failure(s) of healthy system units sequentially, and eventually the LSS may enter a catastrophic state. In this respect, analysis of the impact of the failure on healthy system units is required in order for the reconfiguration action to be determined and completed accurately and efficiently. With fault information from the Fault Propagation module, DM re-estimates the limits of various operational variables and sends such information to MP. Next, the mission planning module sets a modified possible mission plan under the occurred failure. Figure 4.2 shows the functional block diagram of the proposed fault tolerant control philosophy. Before detailing the reconfiguration algorithm, some useful definitions need to be stated.

Definition 1 (Survivability, Fault-Tolerance) The large scale system is said to be survivable with respect to the *i*th subsystem if the system has the capability to perform the certain critical functions without the *i*th subsystem; it is then said to be fault tolerant with respect to the *i*th subsystem.

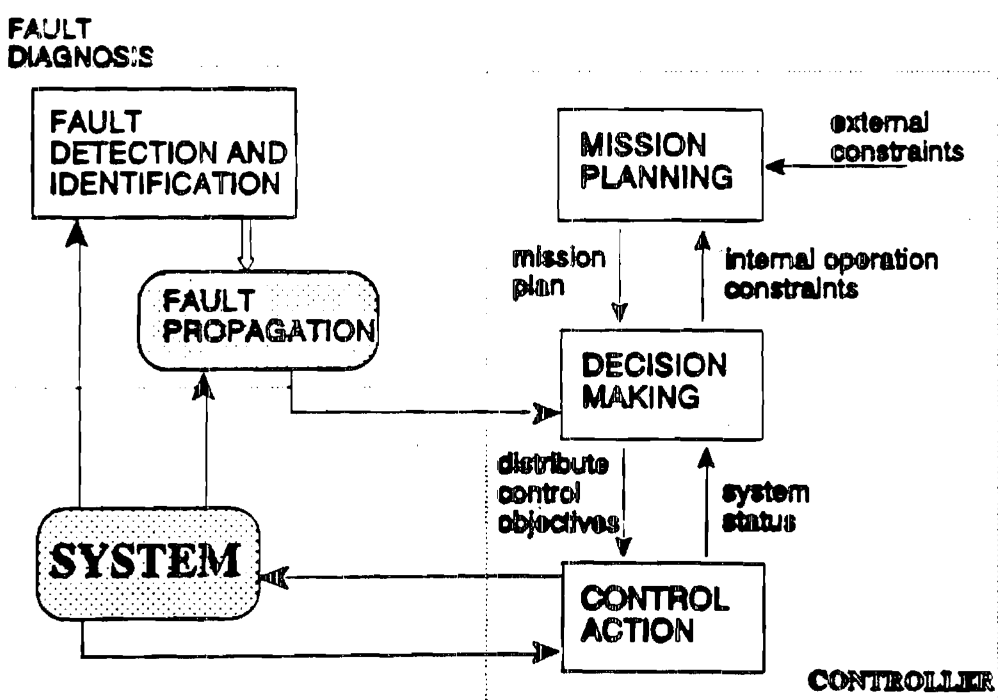


Figure 4.2 Block Diagram of Fault Tolerant Control System

<u>Definition 2</u> **(Restructurability, Reconfigurability, Restructurable System)**

- The large scale system is said to have the property of restructurability with respect to the *i*th subsystem if the *i*th subsystem can be isolated.
- The large scale system is said to have the property of reconfigurability with respect to the *i*th subsystem if the restructured system with respect to the *i*th subsystem is structurally controllable, and stabilizable.
- The large scale system is said to be restructurable with respect to the *i*th subsystem, if the large scale system has both properties of restructurability and reconfigurability with respect to the *i*th subsystem.

<u>Definition 3</u> (Critical Units)

The set of subsystems and appropriate interconnections which are required to absolutely perform those functions that allow the whole system to be in a survivable state are called critical units of the system. The removal of a critical unit results in a catastrophic event.

As for the faulted large scale system, the specific tasks required to perform the proper fault accommodating action are as follows:

Based on the failure information provided by the FDI module,

1. Analyze the impact of the failure on other healthy subsystems.
2. If necessary, restructure the system by isolating system units.
3. Determine (modify) the control objectives.
4. Perform a properly reconfigured control action to meet the (modified) control objectives.

Although the proposed fault accommodating philosophy promises to accomplish an effective control action, it is necessary that the large scale system possess certain structural properties for restructurability and reconfigurability. We have investigated a structured system which possesses the required structural properties. Block triangular structure large scale systems (BTS LSS) are considered in detail in the following subsection.

4.1 Block Triangular Structure Large Scale System

Consider the unforced large scale system with l subsystems described as

$$\begin{aligned} \dot{X} &= F(X,t) \\ \dot{x}_i &= f_i(x_i,t) + h_i(X,t) = f_i(x_i,t) + \sum_{j=1}^{l} h_{ij}(x_j,t) \end{aligned} \tag{4.1}$$

It may be rewritten in matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_l \end{bmatrix} = \begin{bmatrix} f_1(x_1,t) & h_{12}(x_2,t) & \cdots & \\ h_{21}(x_1,t) & f_2(x_2,t) & & \\ \vdots & & \ddots & \\ h_{l1}(x_1,t) & \cdots & & f_l(x_l,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{4.2}$$

With respect to (4.2), the *interconnection matrix (E)* is determined as

$$E = [e_{ij}] \quad where \quad e_{ij} = \begin{cases} 1, & if\ h_{ij} \neq 0 \\ 0, & otherwise \end{cases}$$

The structure of the system is dictated by the interconnection matrix E. A class of systems is said to have block triangular structure (BTS), if $e_{ij}=0$ $(\forall\ i<j)$. The BTS LSS is described in matrix form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_l \end{bmatrix} = \begin{bmatrix} f_1(x_1,t) & 0 & \cdots & 0 \\ h_{21}(x_1,t) & f_2(x_2,t) & & \\ \vdots & & \ddots & \\ h_{l1}(x_1,t) & \cdots & & f_l(x_l,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{4.3}$$

As shown in Figure 4.3, the subsystems in a BTS LSS are interconnected in a uni-directional form. In general, sparsity is one of the structural properties of a large scale system. Many engineered systems can be realized as block triangular structure large scale systems (BTS LSS) by rearranging their subsystems via a square permutation matrix, P, with only one non-zero element exactly equal to 1 in each row and in each column,

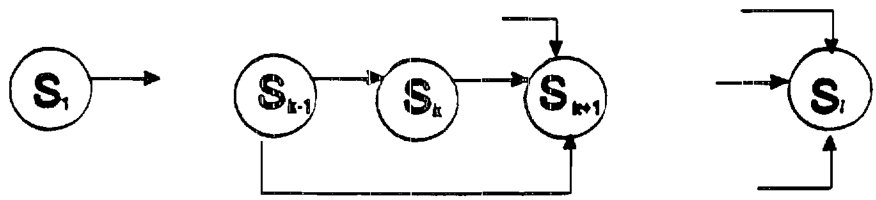


Figure 4.3 Block Triangular Structure (One-way interconnected) Large Scale System.

$$\begin{gathered} PEP^T = \hat{E} \\ \hat{E} = [\hat{e}_{ij}] \quad where \quad \hat{e}_{ij} = 0 \quad \forall\ i<j \end{gathered} \tag{4.4}$$

Similarly, large scale systems can be realized in BTS form by acyclic decomposition employing graph theoretic techniques. The following example illustrates how to realize a given large scale system as BTS LSS.

(Example 1) [BTS realization]

Consider the large scale system with 5 subsystems described by

$$\dot{X} = F(X,t), \quad or$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} f_1(x_1,t) & h_{12}(x_2,t) & h_{13}(x_3,t) & 0 & h_{15}(x_5,t) \\ 0 & f_2(x_2,t) & 0 & 0 & h_{25}(x_5,t) \\ 0 & h_{32}(x_3,t) & f_3(x_3,t) & 0 & h_{35}(x_5,t) \\ h_{41}(x_1,t) & 0 & 0 & f_4(x_4,t) & h_{45}(x_5,t) \\ 0 & 0 & 0 & 0 & f_5(x_5,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.5}$$

The direct graph (digraph) of the given system is shown in Figure 4.4(a). Even though it may appear complicated, there are no cycles or loops. Thus, it can be realized in BTS form by rearranging the subsystems, i.e.,

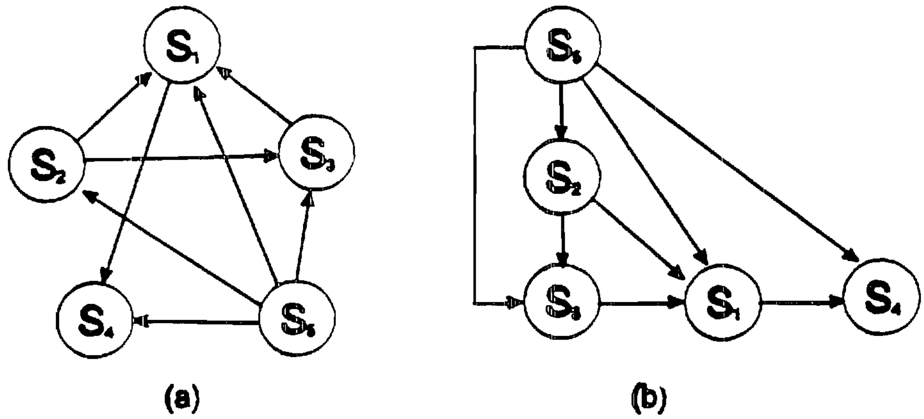


Figure 4.4 Digraphs of Example 1

$$\bar{X} = P X, \quad \bar{E} = PEP^T$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \bar{X} = \begin{bmatrix} x_5 \\ x_2 \\ x_3 \\ x_1 \\ x_4 \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Then (4.5) becomes

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_1 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} f_5(x_5,t) & 0 & 0 & 0 & 0 \\ h_{25}(x_5,t) & f_2(x_2,t) & 0 & 0 & 0 \\ h_{35}(x_5,t) & h_{32}(x_2,t) & f_3(x_3,t) & 0 & 0 \\ h_{15}(x_5,t) & h_{12}(x_2,t) & h_{13}(x_3,t) & f_1(x_1,t) & 0 \\ h_{45}(x_5,t) & 0 & 0 & h_{41}(x_1,t) & f_4(x_4,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{4.6}$$

and its digraph is shown in Figure 4.4(b). ††

As illustrated, a large scale system is BTS realizable if there is no cyclic path among its subsystems. A large scale system with cyclic connections between some of its subsystems may also be BTS realizable by aggregating such subsystems into one subsystem. The following example illustrates this case.

(Example 2) [BTS realization]

Consider the system (4.5) in the previous example. Suppose that there is one more path from S_4 to S_1, as shown in Figure 4.5(a). There is only one cycle in the digraph, consisting of S_1 and S_4. By aggregating S_1 and S_4 into one subsystem, the given system can be realized in BTS form as shown in Figure 4.5(b).

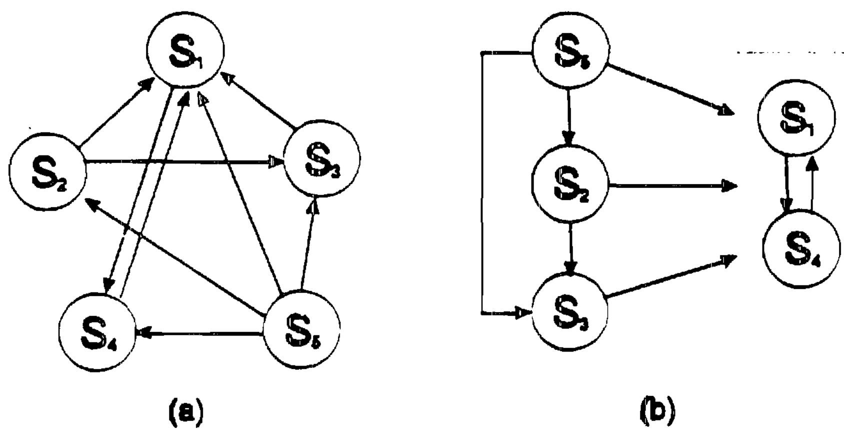


Figure 4.5 Digraphs of Example 2

The resulting system equations in BTS form are

$$\begin{bmatrix} \dot{x}_5 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_1 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} f_5(x_5,t) & 0 & 0 & 0 & 0 \\ h_{25}(x_5,t) & f_2(x_2,t) & 0 & 0 & 0 \\ h_{35}(x_5,t) & h_{32}(x_2,t) & f_3(x_3,t) & 0 & 0 \\ h_{15}(x_5,t) & h_{12}(x_2,t) & h_{13}(x_3,t) & f_1(x_1,t) & h_{14}(x_4,t) \\ h_{45}(x_5,t) & 0 & 0 & h_{41}(x_1,t) & f_4(x_4,t) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{4.7}$$

For convenience, let us consider the linear LSS described by

$$\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX \end{aligned} \tag{4.8}$$

where

$$X \in \Re^n, \quad U \in \Re^m, \quad Y \in \Re^p$$
$$A \in \Re^{n \times n}, \ B \in \Re^{n \times m}, \ C \in \Re^{p \times n}$$

Then the ith subsystem is

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i \mu_i + \sum_{j=1}^{l} A_{ij} x_j \\ y_i &= C_i x_i \end{aligned} \tag{4.9}$$

where

$$X_i \in \Re^{n_i}, \quad U_i \in \Re^{m_i}, \quad Y_i \in \Re^{p_i}$$
$$A_i \in \Re^{n_i \times n_i}, \ B_i \in \Re^{n_i \times m_i}, \ C_i \in \Re^{p_i \times n_i}$$

$$\sum_{i=1}^{N} n_i = n, \quad \sum_{i=1}^{N} m_i = m, \quad \sum_{i=1}^{N} p_i = p$$

The interconnection matrix (E) is

$$E = [e_{ij}] \quad where \quad e_{ij} = \begin{cases} 1, & if\ A_{ij} \neq 0 \\ 0, & otherwise \end{cases}$$

if there exists a permutation matrix (P) described by (4.9), then the following transformed system has block triangular structure

$$\frac{d\tilde{x}}{dt} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}$$
$$where \ x = P^T\tilde{x}, \ u = P^T\tilde{u}, \ \tilde{A} = PAP^T, \ \tilde{B} = PBP^T$$

Suppose that the controller consists of a local and a global controller, i.e.

$$u_i = u_i^l + u_i^g$$

then (4.9) is rewritten as

$$\dot{x}_i = A_i x_i + B_i u_i^l + \sum_{j=1}^{l} A_{ij} x_j + B_i u_i^g \tag{4.10}$$

the *i*th isolated subsystem is described by

$$\dot{x}_i = A_i x_i + B_i u_i^l$$

With the state feedback controller as

$$u_i^l = -k_i x_i, \quad u_i^g = -\sum_{i=1, i \neq j}^{l} k_{ij} x_{ij}$$

The following theorem gives a sufficient condition for transforming an LSS into BTS form, if physically feasible.

<u>Theorem 1</u> (Transformation to BTS) A linear LSS (4.8)-(4.10) is restructurable (reconfigurable) to a BTS LSS, by applying a global controller, if the following condition is satisfied,

$$\text{Range } A_{ij} \in \text{Range } B_i \quad \forall \; i<j \quad (i,j = 1, \ldots, l)$$

(proof) We can prove this by finding appropriate global feedback gains such that

$$A_{ij} - B_i k_{ij} = 0 \qquad \forall \; i<j$$
$$E = [e_{ij}] \quad \text{where } e_{ij} = 0 \quad \forall \; i<j$$

Since Range $A_{ij} \in$ Range B_i,

$$A_{ij} = B_i L_{ij}$$

Thus

$$B_i L_{ij} - B_i k_{ij} = 0 \quad \forall \; i<j$$

$$k_{ij} = L_{ij}$$

Then, the resulting system has block triangular structure

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_l \end{bmatrix} = \begin{bmatrix} A_1 - B_1 k_1 & 0 & \cdots & 0 \\ A_{21} - B_2 k_{21} & A_2 - B_2 k_2 & & \vdots \\ \vdots & & \ddots & 0 \\ A_{l1} - B_l k_{l1} & & \cdots & A_l - B_l k_l \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_l \end{bmatrix} \tag{4.11}$$

4.1.1 Stability of BTS LSS

In general, it is difficult to check the stability of an LSS because of its high dimensionality and complex interconnections. However, for the BTS LSS, this is not difficult when local stability (the stability of ISS) can be verified with ease. The following theorem reveals the relation between the stability of LSS and that of ISS.

Theorem 2 (Stability of BTS LSS) If all of the subsystems are stable, then the linear BTS LSS is stable. In other words, if the linear LSS with l subsystems has the block triangular structure, the overall system is stable if all of its subsystems are stable, i.e.

$$\text{stable } S_i \;\; \forall\, i \rightarrow \text{stable } S$$

(proof) Consider the linear LSS described in (4.11). The ith isolated subsystem is

$$\dot{x}_i = (A_i - B_i k_i) x_i$$

From the stability of S_i, let us assume that

$$A_i - B_i k_i = \Lambda_i = diag\{ \begin{bmatrix} -\sigma_1^i & \omega_1^i \\ -\omega_1^i & -\sigma_1^i \end{bmatrix}, \cdots, \begin{bmatrix} -\sigma_p^i & \omega_p^i \\ -\omega_p^i & -\sigma_p^i \end{bmatrix}, -\sigma_{p+1}^i, \cdots, -\sigma_{n_i-p}^i \}$$

Then, an appropriate Lyapunov function for the ith isolated subsystem is chosen as

$$V_i = (x_i^T H_i x_i)^{\frac{1}{2}}, \qquad V_i: R^n \to R_+$$

where

$$A_i^T H_i + H_i A_i = -G_i$$

$$G_i = 2\alpha_i diag\{\sigma_1^i, \sigma_1^i, \cdots, \sigma_p^i, \sigma_p^i, \sigma_{p+1}^i, \cdots, \sigma_{n_i-p}^i\}$$
$$H_i = \alpha_i I_i$$

Since the α_i's are arbitrary positive constants, choose $\alpha_i = 1$ for simplicity. Then the relations

$$V_i = \|x_i\|$$
$$\dot{V}_i \leq -\rho_i \|x_i\|$$

are satisfied, where

$$\rho_i = \frac{1}{2}\lambda_m(G_i) = \min_j \sigma_j^i$$

With regard to the interconnected system

$$\dot{x}_i = (A_i - B_i k_i)x_i + \sum_{j=1}^{l} (A_{ij} - B_i k_{ij})x_j \tag{4.12}$$

the derivative of the Lyapunov function is described by the following inequality

$$\begin{aligned} \dot{V}_i &\leq -\rho_i \|x_i\| + \sum_{j=1}^{l} \|(A_{ij} + B_i k_{ij})x_j\| \\ &\leq -\rho_i \|x_i\| + \sum_{j=1}^{l} \|A_{ij} - B_i k_{ij}\| \, \|x_j\| \\ &\leq -\rho_i V_i + \sum_{j=1}^{l} \xi_{ij} V_j \end{aligned} \tag{4.13}$$

For the overall system, choose a vector Lyapunov function as

$$V = (V_1, V_2, \cdots, V_l)^T, \qquad V: R^n \to R_+^l \tag{4.14}$$

From (4.13) and (4.14), we have the following aggregation form

$$\dot{V} \leq WV$$

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_l \end{bmatrix} \leq \begin{bmatrix} -\rho_1 & 0 & \cdots & 0 \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & 0 \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_l \end{bmatrix} \tag{4.15}$$

where W is a Metzler matrix. By the comparison principle, if W is stable, then the LSS is stable. For stability of a Metzler matrix, necessary and sufficient conditions are expressed by the Sevastyanov-Kotelyanskii theorem as follows

The $n \times n$ Metzler matrix $A = (a_{ij})$ is stable if and only if

$$(-1)^n \det \begin{bmatrix} a_{11} & a_{12} & & a_{1n} \\ a_{21} & a_{22} & & \\ \vdots & & \ddots & \\ a_{n1} & & & a_{nn} \end{bmatrix} > 0$$

The aggregation matrix for the BTS LSS, W in (4.15), satisfies the Sevastyanov and Kotelyanskii condition, since

$$(-1)^l \det \begin{bmatrix} -\rho_1 & 0 & \cdots & 0 \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & 0 \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} = (-1)^{2l} \rho_1 \rho_2 \cdots \rho_l > 0$$

Thus, the BTS LSS consisting of l stable interconnected subsystems is stable. ††

<u>**Theorem 3**</u> **(Stabilizability of BTS LSS)** For the LSS with BTS, the overall system is stabilizable simultaneously when the local system is stabilizable, i.e.,

$$S_i \text{ is stabilizable } \forall\, i \rightarrow S \text{ is stabilizable}$$

(proof) It follows from Theorem 2. ††

<u>**Theorem 4**</u> **(Stabilization of linear LSS)** The linear LSS is globally stabilized by stabilizing the local systems using a decentralized multilevel state feedback controller if the following conditions are satisfied,

(i) (A_i, B_i) are completely controllable
(ii) Range $A_{ij} \in$ Range B_i $\forall\, i<j$ $(i,j = 1, \ldots, l)$

(proof) Condition (i) implies stabilizability (pole assignability) in the local subsystem and condition (ii) implies that the given system is restructurable to BTS LSS. For BTS LSS, the whole system is stabilizable simultaneously when the local system is stabilizable (Theorem 3) ††

4.2. System Restructuring and Controller Reconfiguration

As mentioned above, in order for the faulted system to survive a failure event, a new control law, U_P, must be found to stabilize faulted large scale system and to retain its performance as much as possible. Based on the failure information and the available control authority, the failure event is accomodated via one of the following procedures

- Controller reconfiguration
- System restructuring and controller reconfiguration

With the remaining control authority, if the new control law can be achieved, the failure event can be accommodated via controller reconfiguration only (i.e., re-distribute the available local & global control authority to maintain the system within its stable operational envelope). Otherwise, it is required to restructure the system in order for the remaining control authority to save the system. The system restructuring strategy is described as follows,

- If the failed subsystem is unstable and cannot be stabilized, then restructure the faulted system by isolating the unstabilizable subsystem.
- Otherwise, isolate certain subsystem(s) based on their degree of criticality.

The main issue of the proposed fault-tloerant control strategy is how to reconfigure the remaining control authority. Suppose that the remaining control authority is reconfigurable and capable assuring that the system survives the failure event. Then the following proposed procedure can guarantee the stability of the large scale system

<u>Controller Reconfiguration</u>: With the available failure information,

I. <u>Stabilize the overall system</u>: use both the global and local controller *in parallel*,

- <u>local controller reconfiguration</u>: stabilize the local subsystem.
- <u>global controller reconfiguration</u>: restructure the faulted system into Block Triangular Structure form.

II. <u>Improve performance</u> - Recover the interconnection strength as much as possible by reconfiguring the global controller.

For a given LSS, which is stable under normal operating conditions, in case of a failure event, stabilization of the isolated local subsystems can not, in general, guarantee the stability of the overall system. However, if the system has block triangular structure, local stability of all isolated subsystems can, indeed, guarantee the overall stability, as explained in the previous subsection. That is the reason why reconfiguration of the global controller is proposed as a means to restructure the given system into block triangular structure. Thus, the system with both local and global controllers can be reconfigured to guarantee stability.

The performance of the reconfigured system may be degraded because of loss of the interconnection strength among its subsystems. In order to improve the system performance, for the stabilized system, we recover the interconnection strength as much as possible via reconfiguration of the global controller.

For purposes of illustration, let us consider the following linear large scale system with state feedback control laws determined at the design stage,

$$U = -KX, \quad u_i = -k_i x_i - \sum_{j=1, i\neq j}^{l} k_{ij} x_j$$

$$\begin{array}{ll} (LSS:\ S\ ) & \dot{X} = (A - BK)X \\ (CSS:\ S_i\ ) & \dot{x}_i = (A_i - B_i k_i)x_i + \sum_{i=1, i\neq j}^{l} (A_{ij} - B_i k_{ij})x_j \\ (ISS:\ S_i\ ) & \dot{x}_i = (A_i - B_i k_i)x_i \end{array}$$

The block diagram of Figure 4.6 shows the ith subsystem (composite subsystem). Without loss of generality, suppose that the given system is stable under normal operating conditions, i.e., the following inequality is satisfied,

$$(-1)^l \det \begin{bmatrix} -\rho_1 & \xi_{12} & \cdots & \xi_{1l} \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & \xi_{(l-1)l} \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} > 0, \quad \begin{cases} -\rho_i = \min_j Re[\lambda_j(A_i - B_i k_i)] \quad j=1,\ldots,n_i \\ \xi_{ij} = \lambda_M^{\frac{1}{2}}[(A_{ij} - B_i k_{ij})^T (A_{ij} - B_i k_{ij})] \end{cases}$$

Let us further suppose that an actuator failure in S_m,(i.e., (A_m,B_{mp})) occurs. For the failed subsystem, one of the available stabilization schemes can be applied since each isolated

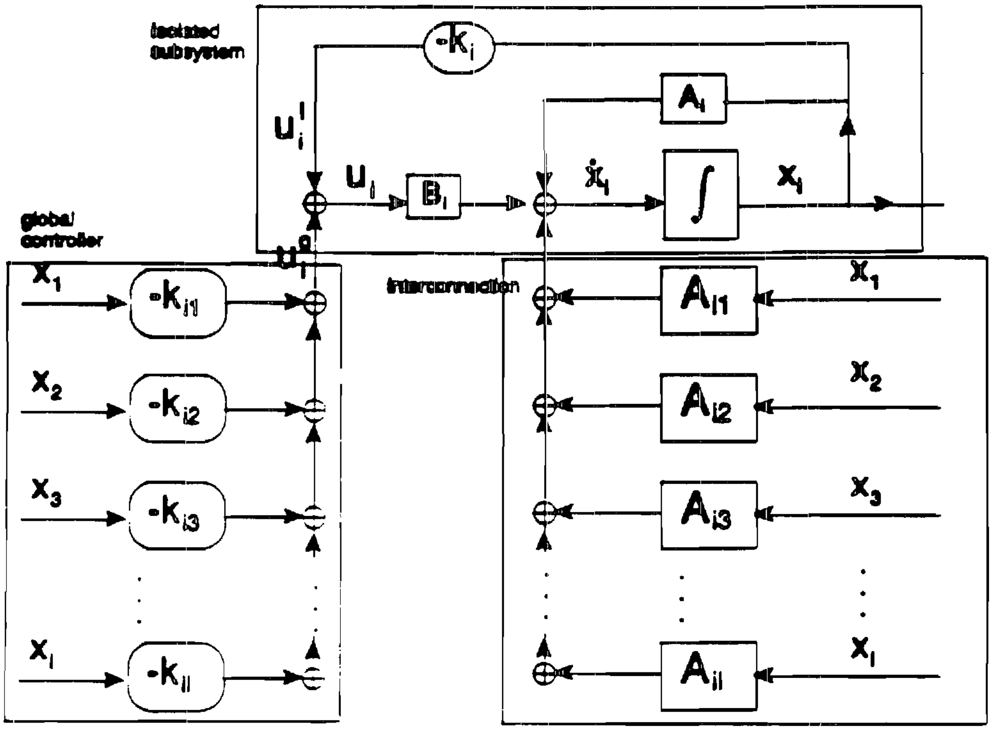


Figure 4.6 Block Diagram of *i*th composite subsystem

subsystem has a relatively small dimension. Let us consider the reconfiguration scheme based on the Pseudo Inverse method [15,16]

<u>Stabilize the overall system</u>

Find new feedback gains K_F such that $Re[\lambda_j\ (A-B_F K_F)] < 0\ (j=1,\dots, n)$

1. Local controller reconfiguration - local stabilization:

   Find new feedback gains k_{mF} such that

   $$Re[\ \lambda_j\ (A_m - B_{mF}\ k_{mF})]\ <\ 0\ ,\ (j=1,\dots,n_j)$$

$$\dot{x}_m = (A_m - B_m k_m) x_m$$
$$\dot{x}_{mF} = (A_m - B_{mF} k_{mF}) x_{mF}$$

In robust control,

$$\dot{x} = (A + \Delta A)x$$

$$A \text{ is stable}, \ \|\Delta A\| < \sigma \Rightarrow (A+\Delta A) \text{ is stable}$$

So, the faulted system is stable if

$$\|B_{mF}k_{mF}\| < \sigma_m \Rightarrow \|k_{mF}\| < \hat{\sigma}_{mF}$$

The feedback gain, k_{mF}, is determined from

$$\min \|(A_m - B_m k_m) - (A_m - B_{mF}k_{mF})\|$$

$$k_{mF} = (B_{mF})^{+} B_m k_m \qquad \|k_{mF}\| < \hat{\sigma}_{mF}$$

2. Global controller reconfiguration - Transform to BTS

Apply new global feedback gains k_{ijF} $(i<j)$ such that

$$A_{ij} - B_i k_{ijF} = 0 \qquad \forall \ i<j, \ i \neq m$$
$$A_{mj} - B_{mF}k_{mjF} = 0 \qquad \forall \ m<j$$

$$k_{ijF} = (B_i)^{+} A_{ij} \quad \forall \ i<j, \ i \neq m$$
$$k_{mjF} = (B_{mF})^{+} A_{mj} \ \forall \ m<j$$

$$(-1)^l \ \det \begin{bmatrix} -\rho_1 & 0 & \cdots & 0 \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & 0 \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} = (-1)^{2l}\rho_1\rho_2 \cdots \rho_l > 0$$

<u>Improve performance</u>: Recover the interconnection dynamics as much as possible.

1. Find a new degree of stability for the local subsystem.

Find the degree of stability of the stabilized subsystem, S_m

$$-\hat{\rho}_m = \min_i Re[\lambda_i(A_m - B_{mF} k_{mF})] < 0$$

2. Reconfigure the global controller to meet the stability condition (i.e., the interconnection strength is less than the degree of local stability).

$$A_{ij} - B_i \hat{k}_{ijF} = A_{ij} - B_i k_{ij} \quad \forall\ i<j,\ i \neq m$$
$$A_{mj} - B_{mF} \hat{k}_{mjF} = A_{mj} - B_m k_{mj} \quad \forall\ m<j$$

$$\hat{k}_{ijF} = k_{ij} \quad \forall\ i<j, i \neq m$$
$$\hat{k}_{mjF} = (B_{mF})^{+} B_{mj} k_{mj} \ \forall \ \ m<j$$

check the stability condition

$$(-1)^l \ \det \begin{bmatrix} -\rho_1 & \xi_{12} & \cdots & \xi_{1l} \\ \xi_{21} & -\rho_2 & & \vdots \\ \vdots & & \ddots & \xi_{(l-1)l} \\ \xi_{l1} & \cdots & \xi_{l(l-1)} & -\rho_l \end{bmatrix} > 0$$

As illustrated, the stabilization problem of faulted large scale system is viewed as that of a relatively small scale isolated subsystem. Thus with the proposed reconfiguration strategy, faulted large scale system can maintain its overall stability in a limited time, since it is possible to make use of parallel implementation for each isolated subsystem. Furthermore, this proposed methodology can be applied to a large scale system in which each free subsystem is controlled by the different type of control laws. For many of practical large scale systems, the fault-tolerance can be achieved by applying additional global controller.

5 IMPLEMENTATION ISSUES

Fuzzy reasoning or inferencing in a rule base system is one possible application of fuzzy logic and fuzzy set theory, as introduced first by Zadeh [17,18]. Main concerns in this section are to introduce a hardware structure for implementing a fuzzy rule base system, to demonstrate the compositional rule of inference and to assess the performance of this fuzzy hardware.

An approach to fuzzy inferencing has been reported by Togai [19], but it does not provide flexibility from a hardware viewpoint. Since the same fuzzy rule can be realized in several ways, it is very important to consider a number of aspects which govern the overall system performance. As the number of premises or rules increases, our approach to fuzzy implementation can be realized efficiently and effectively. Important factors may include the processing time, hardware complexity, and flexibility in changing rules or adding a premise part.

The basic mathematics of a fuzzy rule base system employ maximum and minimum operations, it is generally desirable to reduce the number of these operations in order to minimize processing time. Another factor is related to hardware complexity, which determines the storage capacity of the rule base system. Thus, it is important to store data compactly without loss of information. Flexibility is crucial in incrementing or modifying the rules thus avoiding complicated learning processes. Flexibility is gained by making the rule base modular, that is, insertion of a new rule is accomplished by simply augmenting the matrix which stores information not by re-calculating its elements each time.

5.1 Fuzzy Hypercubes

A fuzzy hypercube, H^F, is a fuzzy system characterized by a fuzzy Cartesian product that can store a set of possibilistic if-then propositional rules and which approximately matches the applied stimulus (input) fuzzy sets so that response (output) fuzzy sets may be obtained via fuzzy inferencing. The degree of possibility is modelled in terms of membership functions for each linguistic rule that represents the relative degree of extent. Each rule is a fuzzy implication of an 'IF–THEN' proposition. In a fuzzy hypercube, the k-th rule is of the general form,

rule k: IF (x^k_1) and ... and (x^k_N) THEN (y^k_1) and ... and (y^k_M).

x^k_i, y^k_i are fuzzy I/O vectors for the k-th rule. x^k_i, y^k_i are assumed to satisfy certain convexity and normality conditions [20]. The premises and consequences in this fuzzy relation are, therefore, the inputs (stimuli) and outputs (responses) of H^P.

5.2 Realization of Fuzzy Hypercubes: Split-and-Merge Method

We can decompose a fuzzy hypercube so that only one premise is associated with each rule by using premise matrices and consequence matrices that enable only one consequence from each rule. This method is based on Mamdani's fuzzy inferencing technique [21] as extended from continuous fuzzy sets to discretized fuzzy systems. The premise vector from the i-th premise and the consequence vector from the j-th consequence, in each rule, can be expressed as follows:

$$P_i = \begin{bmatrix} x_i^{1T} \\ \vdots \\ x_i^{KT} \end{bmatrix} , \qquad C_i = \begin{bmatrix} y_j^{1T} \\ \vdots \\ y_j^{KT} \end{bmatrix}$$

where K is the number of rules; x^{pT}_i is the transpose of the i-th premise vector and y^{pT}_i is that of the j-th consequence vector for the p-th rule.

Now we consider the fuzzy inferencing mechanism. The degree of fulfillment $d^i \in [0,1]^K$ for the i-th premise vector is

$$d^i = P_i \circ \tilde{x}_i$$

and this intermediate degree of fulfillment d^i indicates how close the input is to the i-th rule. Combining d^i with each premise yields the resultant degree of fulfillment $d \in [0,1]^K$:

$$d[i] = d^1[i] \otimes .... \otimes d^N[i]$$

where $\otimes$ is the minimum operation. Finally, the j-th fuzzy output vector $\tilde{y}_j \in [0,1]^{n_j}$ is obtained using the max-min operation with consequence matrix C_j,

$$\tilde{y}_j = C_j \circ d \tag{5.1}$$

<u>Defuzzification Strategy</u>

The fuzzy output vector $\tilde{y}$ is converted to the corresponding crisp data j^* using the maximum finder:

Defuzzification Strategy : (the peak detector)

$s^* = s[j^*] = s[\arg\max_j y[j]]$

where j^* stands for the index of y in the quantized bin, and s[j] is the representative value of index j. This is suitable for the above decomposition case in (5.1).

5.3 Implementation: Storage and Timing Requirements

The implementation of fuzzy hypercubes using the SM method is shown in Figure 5.1. Each input is max-mined with its corresponding premise matrix and the intermediate degree of fulfillment(DOF) is calculated. Then the resultant DOF is again max-mined with the consequence matrices and the final result is produced. There are several advantages to employing the SM method in implementing a fuzzy rule base system. For the purpose of comparison, let the direct implementation of a fuzzy hypercube be FDHC and assume that there are K rules, N premises, m_i bins for the i-th premise, M consequences, and n_j bins for the j-th consequence. The most important advantage gained by using the SM method is the saving in memory space required for constructing the rule base. In realizing the SM method, there are two types of matrices: premise and consequence matrices. Since the bins of the i-th premise matrix and that of the j-th consequence matrix are n_i and m_j, respectively, the total space allocation required is $K(\Sigma_{i=1}^{N} m_i + \Sigma_{j=1}^{M} n_j)$. The hardware implementation of the FDHC requires $\Pi^N_{i=1} m_i$ memory allocation for each rule for a crisp consequence. Therefore, the total number of memory units is $\Sigma^M_{j=1} m_j\, \Pi^N_{i=1} n_i$

It should be emphasized that there exists a trade-off between the amount of storage and the processing time in realizing a specific hardware configuration. Thus, using a larger capacity or storage we may reduce the processing time and the reverse is also true. With the storage space as shown above, the processing time for the SM implementation is $\max_j\{[(m_i\,\Delta_{min}) + (m_i - 1)\,\Delta_{max}]K + [K\,\Delta_{min} + (K-1)\,\Delta_{max}]n_j\}$, whereas the FDHC needs $\Delta_{max} + \Delta_{min}$, where Δ_{max} and Δ_{min} indicate the processing time for the maximum and minimum operations, respectively. Table 5.1 summarizes the comparison between the FDHC and the SM methods, where m_i and n_j are simplified to L and $\Delta_{max} = \Delta_{min} = T$ for convenience. Figure 5.2(a) shows the required memory ratio between the FDHC and the SM, based on Table 5.1, with respect to the number of bins, where it is assumed that $m_i = n_j$ for any i,j and N=4, M=3 and K=10. Figure 5.2(b) shows the same ratio with respect to the number of premises with a fixed number of bins(L=5), consequences($M=3$) and rules($K=10$). Another advantage of the SM over the FDHC technique is its structural flexibility. In contrast to the FDHC, there is no coupling between premises in the SM structure and the content of a rule is preserved until the last stages, d^i; modification or increment of a rule is achieved by replacing a component of the matrix d^i with a new one, or by augmenting a vector to the matrix. Figure 5.3 is a practical circuit diagram to realize fuzzy hypercubes based on the SM method.

	FDHC	SM
storage	ML^{N+1}	$KL(N + M)$
time	$2T$	$T(4LK - K - L)$

Table 5.1 Comparison of FDHC and SM

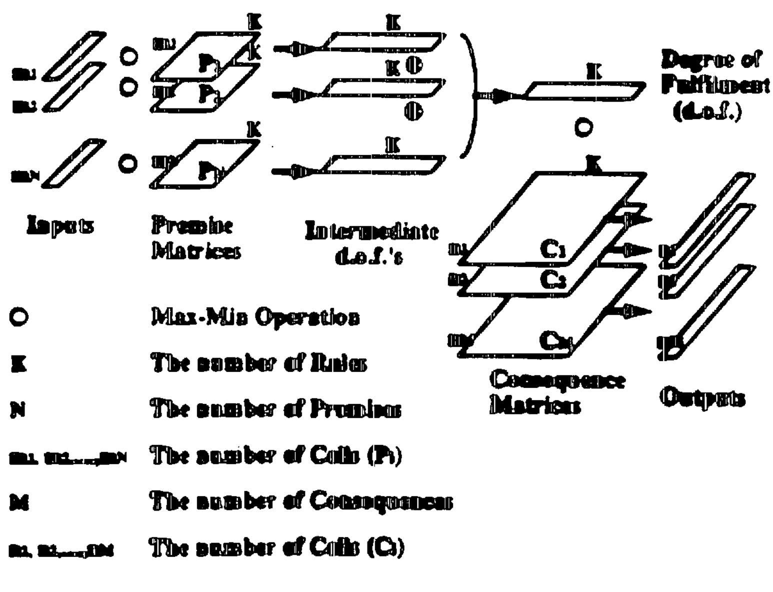


Figure 5.1 Fuzzy Hypercube Inferencing Architecture

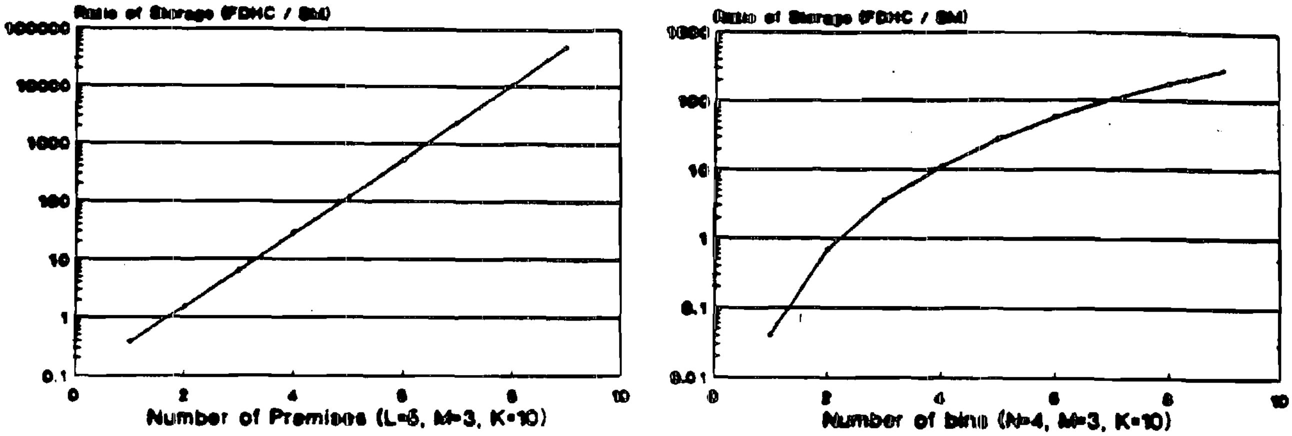


Figure 5.2 The required memory ratio (FDHC/SM) with respect to the number of (a) Premises and (b) Bins

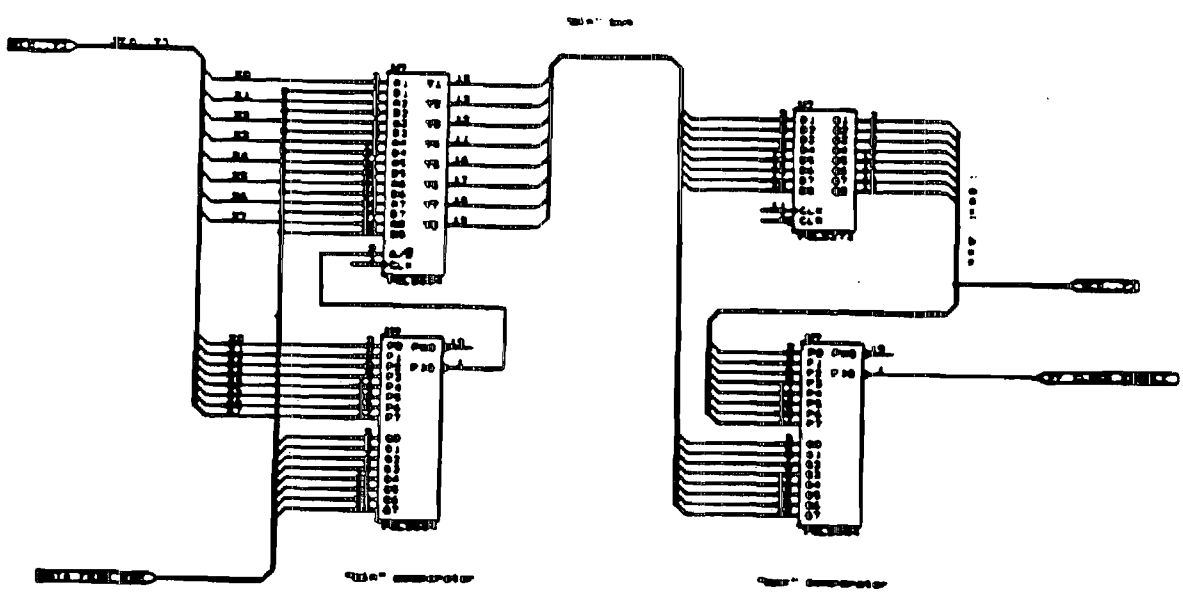


Figure 5.3 The circuit diagram for realization of fuzzy hypercubes

6 THE INTEGRATED SHIPBOARD ELECTRIC POWER SYSTEM

A subproject to contract No. N00014-89-J-3113 was initiated in June 1991 to support work conducted by the David Taylor Research Center and other Navy contractors towards the design of a new electrical distribution system of the integrated shipboard electric power system. Georgia Tech's participation involves the development of fault-tolerant control strategies for the shipboard distribution network. Specifically, in the event of a failure, such as a line short circuit or voltage perturbation, the system is called upon to provide certain critical loads whose uninterrupted operational integrity is essential to the Navy. The reconfiguration strategy addresses those and control actions that will assure system survivability. The detailed work scope of this activity is as follows :

1. Develop a fault-tolerant control strategy. For a particular distribution network topology, we intend to develop a theoretical basis for fault detection and identification, systemrestructuring, and controller reconfiguration. The primary objective of this task is to assist system designers in incorporating fault-tolerant concepts to the shipboard electric power system design.

2. Develop appropriate Fault Detection and Identification routines to be applied to the reconfiguration task and to be used for status monitoring of critical mechanical subsystems. These routines will be demonstrated via simulation studies.

3. Develop an analytical framework for the analysis and control design of the AC/DC interface. Attention may to focused here on the development of an active control strategy to address such problems as power quality and dynamic stability. The power quality problem might be an issue of immediate concern to the Navy. The objective is to damp out potentially harmful oscillations arising from fault conditions or to cancel out harmonic currents.

Our research effort will focus essentially on the reconfiguration task, since it constitutes a high priority item for the DTRC.

6.1 Accomplishments During this Reporting Period

We have been pursuing the definition of a possible topology for the integrated shipboard electric power system that embodies two basic requirements : simplicity and reliability. Both simplicity and cost-effectiveness in terms of minimum parts and interconnections, and an extended life span for critical system units. A preliminary topology that incorporates major system units and aggregates similar functions has been defined and is shown in Figure 6.1. DTRC is currently providing us a new distribution system architecture that is based on a two-bus system. This new architecture will be used as a benchmark to test and validate the reconfiguration and FDI routines.

Our efforts have been focused on two detailed activities : The first one involves the development of a robust FDI algorithm that can be applied to a major subsystem of the shipboard electric power system, and the second is concerned with the selection of an appropriate simulation platform to assist in the development process as well as in testing and validating the FDI and control routines. Here, we have been exploring the possibility of a hybrid simulation environment using C or TURBO-PASCAL to be developed by our research team. Figures 6.2 and 6.3 depict some generic SPICE simulation results for a three phrase power generation results for a three phase power generator and a double tuned transformer, respectively.

The basic FDI architecture is complete and we anticipate to apply this tool to a subsystem of the electrical power system (the propulsion mechanism) for verification purposed. Details of the hybrid FDI algorithm are presented in another section of this report.
Finally, some preliminary work have been conducted on the problem of defining the AC/DC interface requirements and the dynamic stability/control problem.

We anticipate that, over the next reporting period, we will be collaborating closely with DTRC personnel to define fully and to develop the fault-tolerant control laws. Thus, both design and control development activities will take place to optimize the system configuration from a reliability and a cost-effectiveness point of view.

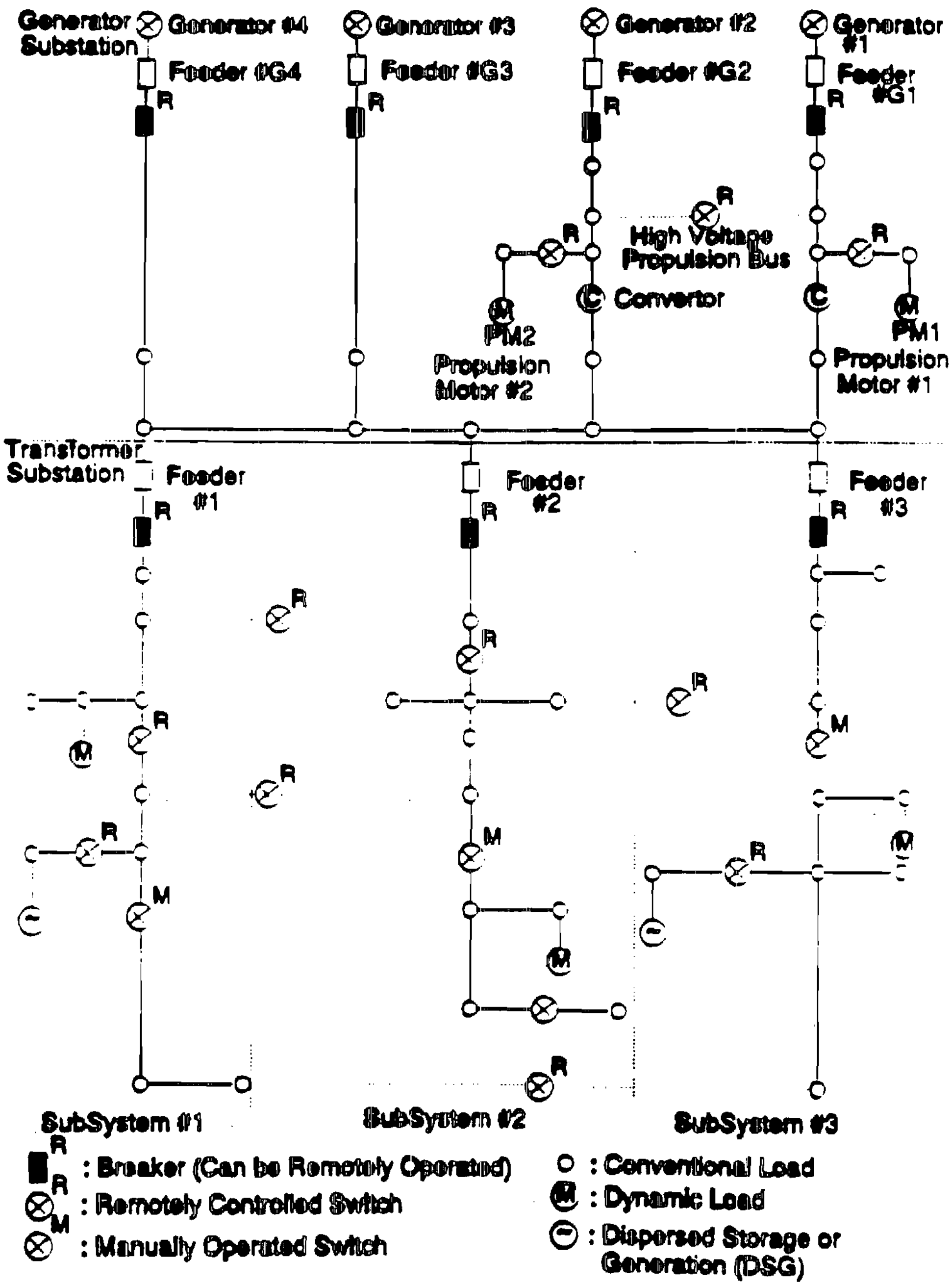
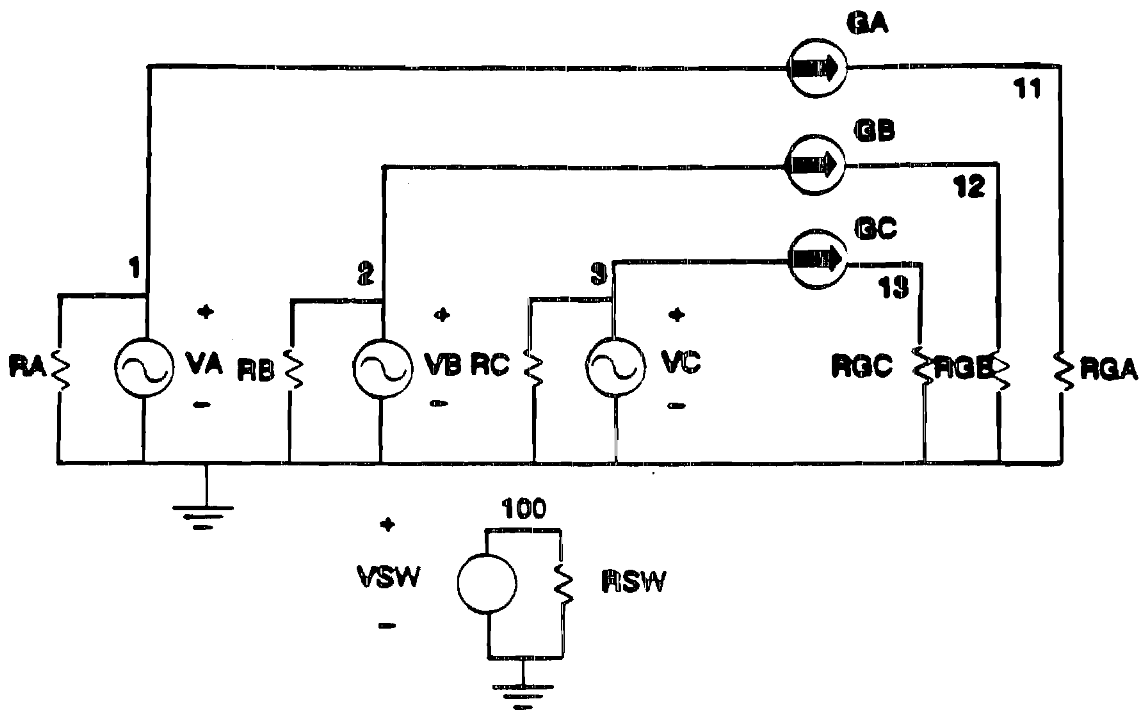


Figure 6.1 Distributed Power System Network



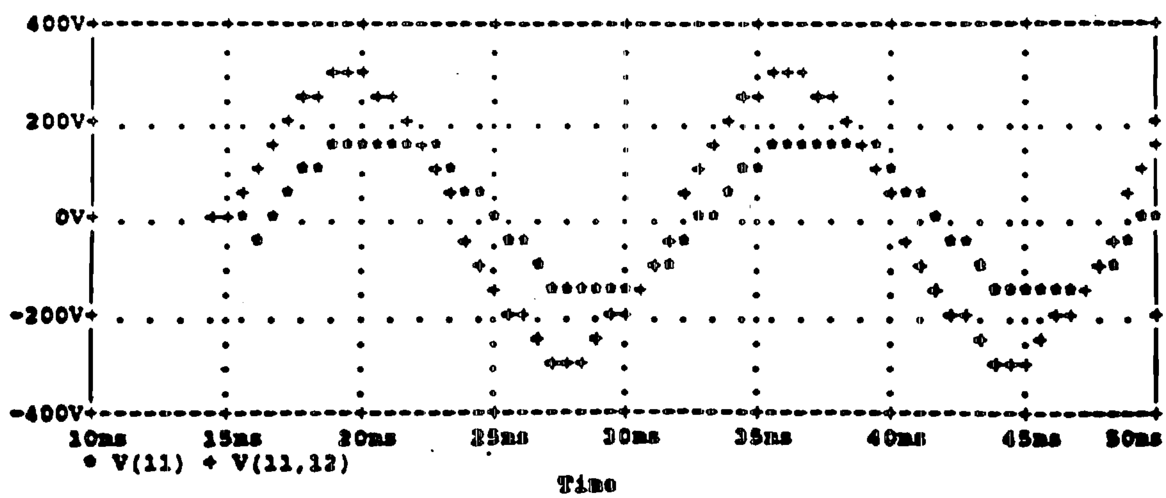


Figure 6.2 Switched Three-phase generator: Example for SPICE simulation

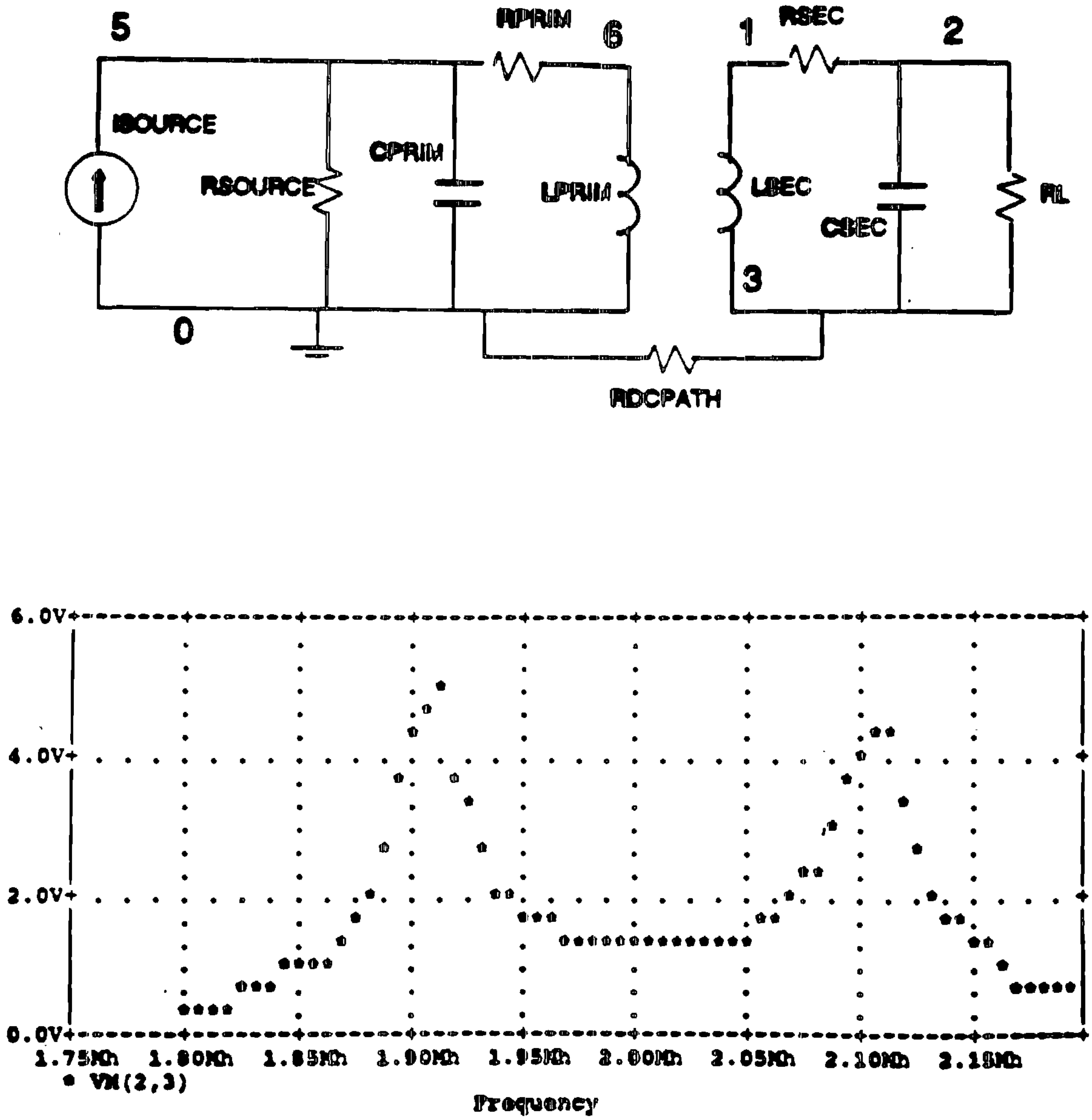


Figure 6.3 Doubly tuned transformer: Example for SPICE simulation

7 FAILURE IDENTIFICATION USING HISTOGRAM ANALYSIS AND A FUZZY DECISION HYPERCUBE.

This method is used for failure identification by comparing the fuzzified histogram of real time sampled data with a rule base implemented by a multi-dimensional decision making system called "Fuzzy Decision Hyper-Cube" (FDHC). The sequence of operations is described below.

7.1 Window Operation:

A windowing operation (as shown in Fig. 1(b)) is performed to allow the computing system to operate on finite data. The size of the window is a compromise between computational effort and reliability/accuracy. This windowed data is $\theta(n)$. Although the shape of the window can be of any form, like Gaussian, Hamming, Hanning, raised cosine, etc., usually rectangular or triangular windows are used for simplicity.

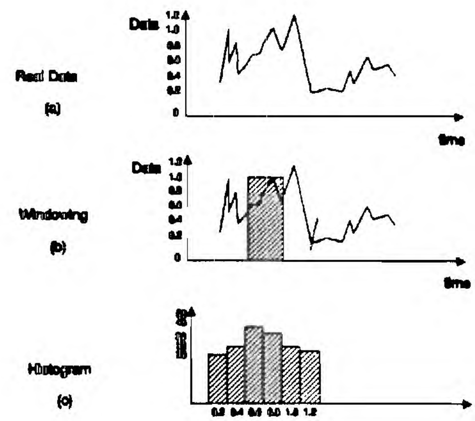


Figure (1)

7.2 Histogram Formation:

The windowed data $\theta(n)$ is divided into bins representing discrete values over the range of data. For example, if a bin size of 0.2 is chosen then $\theta(n) = 0.27$ is placed in the second bin and if $\theta(n) = 0.01$ is placed in the first bin. (as shown in Fig. 1(c)) The bin size is a compromise between resolution and computational complexity. Failure Signature Histogram(FSH) formation is basically a discretization process and the resultant histogram is denoted by f_k^θ where $k = 1,2,\ldots\ldots,N$ where N is the total number of bins.

7.3 Normalization:

The histogram f_k is normalized so as to satisfy the following condition

$$\sum_{k=1}^{N} f_k^\theta = 1$$

this is achieved by dividing the histogram values by the size of the window. Thus, the probability distribution is obtained.

7.4 Fuzzification:

The probability distribution, as formed above, is mapped into a possibility distribution (fuzzy data) by the following transformation.

The possibility transformation π provides the maximum probability of the parameter by mapping

$$\pi(\{\theta_k\}) = \sum_{j=1}^{N} \min\{f_k^{\theta}, f_j^{\theta}\}$$

$$\pi : Q \rightarrow I = [0,1]$$

where $\mu_k^{\theta} = \pi(\{\theta_k\})$ is stored as a fuzzy input vector into a Fuzzy Decision Hyper-Cube. This is called Failure Signature Fuzzy(FSF) set.

Example:

Let the FSH be as shown in Fig. 1 (c)

$$\tilde{f}_k^{\theta} = \{20, 30, 50, 45, 30, 25\}$$

By normalizing we obtain

$$f_k\theta = \{0.1, 0.15, 0.25, 0.225, 0.15, 0.125\}$$

Applying the transformation π we get

$$\mu_k^{\theta} = \{0.6, 0.825, 1.0, 0.975, 0.825, 0.725\}$$

7.6 Fuzzy Decision Hyper-Cube (FDHC):

A rule base is defined a priori called Failure Mode Fuzzy (FMF) which relates the failure symptoms to crisp failure modes. The FDHC is constructed along each rule on the basis of a priori information about FSF and FMF. In general, the j-th rule is described by a fuzzy implication whose premise consists of conjunctive fuzzy sets.

The input-output relationship of the FDHC can be written as

$$B = \Sigma \circ A$$

where '$\circ$' denotes the max-min operation and $A \subset I$ is FSF acquired from FSH f^{θ} and Σ represents the FMF.

8 HYBRID TIME/FREQUENCY INFORMATION MATRIX APPROACH OF EVALUATING FUZZY MEMBERSHIP FUNCTION AND FUZZY DECISION HYPERCUBE.

This is a new development in the field of Fault Identification and Detection at Georgia Tech and most of the tome during past six months has been devoted to the development of this technique for FDI approach. The input data is compared with the data base in both time and frequency domains using the Short Time Fourier Transform (STFT) or the Wavelet Transform (WT). This greatly helps in increasing the identifiability of the system as some fault features are more prominent in the time domain and other in the frequency domain.

8.1 Windowing:

The input data stream *'x'* is collected in groups (windows) of n (typical 30-40) samples. The size of the window is chosen by considering the average duration of the fault. A very long window would mask the fault by averaging out it with normal operation, while a very short window will have inadequate data and will not truly represent the fault.

8.2 Short Time Fourier Transform:

Fourier Transform of these samples is evaluated using FFT algorithms. Since the Fourier Transform is symmetric for a real signal, half of it is discarded to avoid redundancy. This vector is labeled as X_i having $n=n/2$ components.

8.3 Hybrid Time/Frequency Domain Matrix:

A stream of data $x(t)$ from a particular sensor is extracted by window function W_N to get N samples of data (typical values of N range from 30 to 40). The Discrete Fourier Transform of this vector is stored as X_1. The window is moved T_{inc} samples in time and another vector X_2 is obtained. Likewise m such X_i's $(i=1..m)$ are obtained. Since the input data is real, the Fourier transform is symmetric, so X_i's are truncated to half to get an $(n \times m)$ matrix, where $n=N/2$.

The matrix A thus obtained is referred to as information matrix. Its elements a_{ij} have the following characteristics:

- For a fixed $i=u$, a_{uj} gives the fourier transform at a particular time.
- For a fixed $j=v$, a_{iv} gives the relative level of a particular frequency over a period of time.
- The first column of this matrix, i.e. a_{i1} is the D.C. Component of the signal over time m x T_{inc}, where T_{inc} is the time increment between two subsequent windows.
- a_{i1} is used to detect failure modes that are more prominent in the time domain.
- $a_{i,(2...n)}$ is used to detect failure modes that are more prominent in the frequency domain.
- $a_{i,(2...n)}$ are compressed (by averaging) into a vector A_f and a_{i1} is referred to as A_t.

A_f is used for the frequency domain analysis and A_t is used for the time domain analysis.

The matrix **A** can be represented as follows.

$$\begin{array}{c} A \\ \\ Time \\ \downarrow \end{array} = \left[\begin{array}{c|ccccc} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{12} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{13} & a_{32} & a_{33} & \cdots & a_{3n} \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{array}\right]$$

Frequency→

8.4 Identification Process:

The identification of a failure mode is accomplished by comparison with a rulebase which has all the anticipated failure modes r_k $(k=1...M)$ defined, where m is the number of failure modes.

The fuzzy membership for the k^{th} mode is evaluated by the following rule:

$$\mu(K)=\frac{1}{m}\sum_{i=1}^{m}\bigwedge(A_T(i),R_k(i))$$

a more robust formula for calculating the fuzzy membership is as follows

$$\mu(k)=\frac{1}{m}\sum_{i=1}^{m}T(R_k(i),A_T(i))$$

where T is a norm defined as

$$T(\mu_A(x),\mu_B(x))=\frac{1-|\mu_A(x)-\mu_B(x)|}{1+\alpha|\mu_A(x)-\mu_B(x)|}$$

where α is a constant.

8.5 Fuzzy Decision Hypercube:

Fuzzy Hypercube is constructed using the membership functions from the stochastic estimation process and parity space techniques.

The parity space is mainly for detection while stochastic estimation process is used for identification.

The failure modes are labeled as follows:

DR	-	DRIFT
HO	-	HARD OVER
CY	-	CYCLIC
ER	-	ERRATIC
NL	-	NON-LINEAR
ST	-	STUCK

THE FUZZY RULES USED ARE :

if DR is HIGH and CY is LOW then	DRIFTING FAULT
if DR is MEDIUM and CY is HIGH then	OSCILLATORY FAULT
if HO is LOW and ER is HIGH then	LOOSE CONNECTION
if DR is HIGH and CY is MEDIUM then	LOW FREQUENCY OSCILLATION

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.
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.

These results are then tested upon Dempster Shafer rule to obtain a degree of certainty for the fault classification.

9 PERFORMANCE MEASURE

9.1 Detectability:

The detectability coefficient *d(x)* is given by the following heuristic formula

$$D(x) = \frac{mF(x)}{N(x)X_{DC}T_{inc}}(1-\beta|W_i-L(x)|)$$

where

$f(x)$ = *magnitude of the fault feature*
$n(x)$ = *mean squared value of the random noise*
x_{dc} = *dc value of the input signal*
m = *number of windows stacked together*
w_i = *length of the window*
t_{inc} = *time increment for the next window*
$l(x)$ = *length of the fault feature*
β = *constant less than 1*

it is clear from the above equation that the optimum length of the window is equal to the length of the fault. Since this is unknown, an a priori assumption has to be made.

9.3 Identifiability:

Identifiability is a subset of detectability and aims at recognizing the type of failure that has occurred.

A measure of identifying the k^{th} fault is given by

$$I_k(x) = \frac{D(x)\sigma_{R_k,\,X}}{\sum_{p=1,\,p\neq k}^{M} \sigma_{R_k,\,R_p}} \qquad (14)$$

where

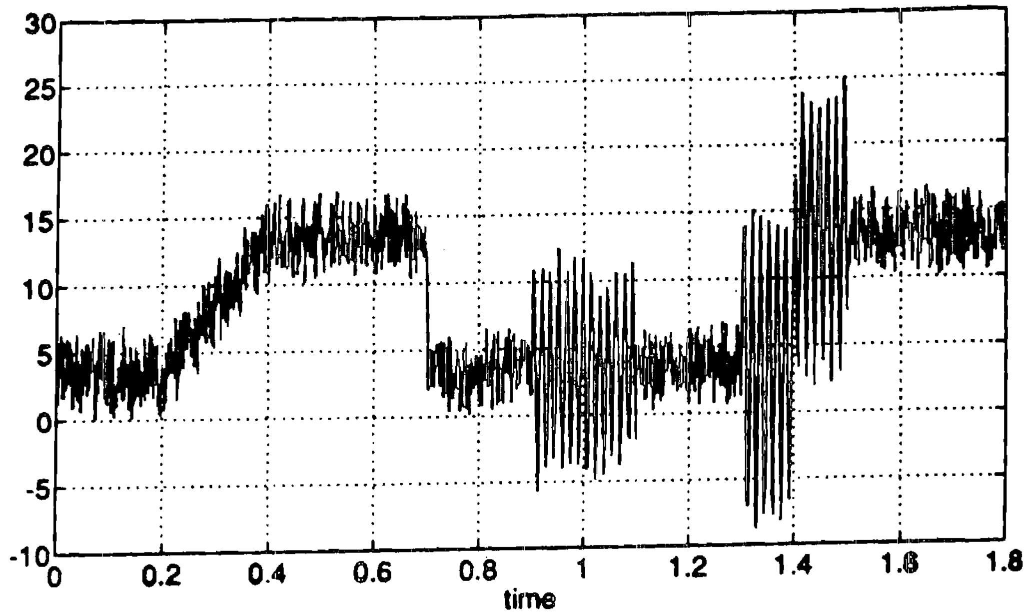
$d(x)$ is the detectability measure

$\sigma_{rk,x}$= *covariance of measured signal with template of same fault stored in rulebase*

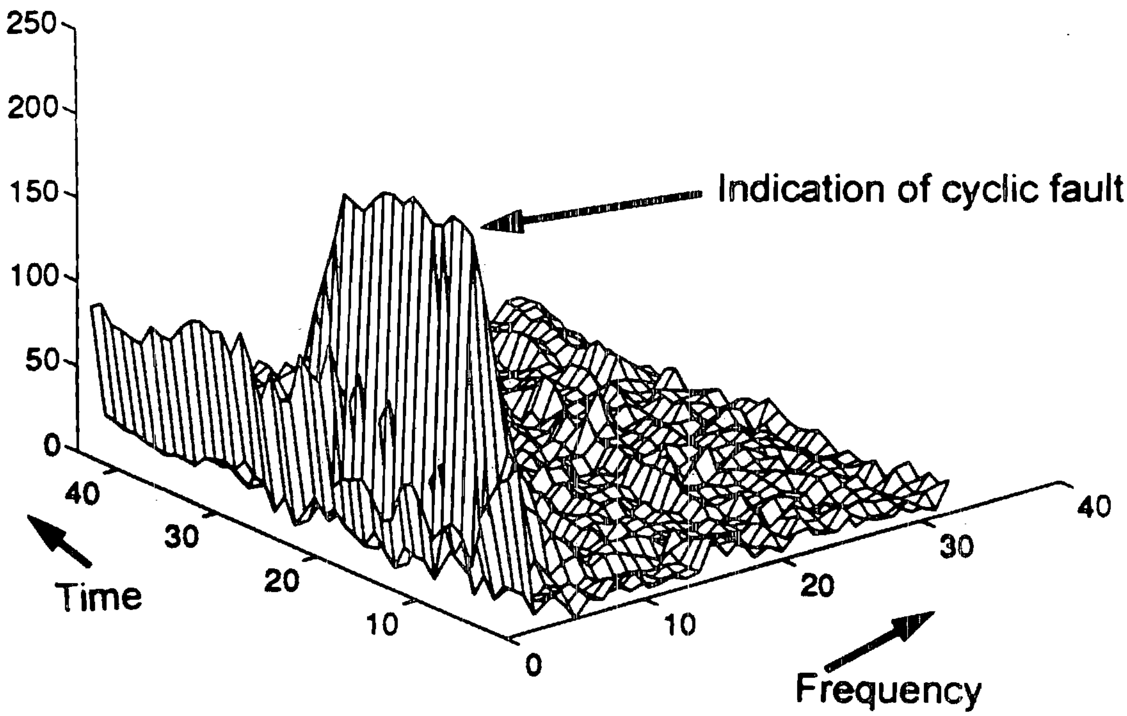
$\sigma_{rk,rp}$= *covariance of measured signal with templates of other signals*

Test Results:

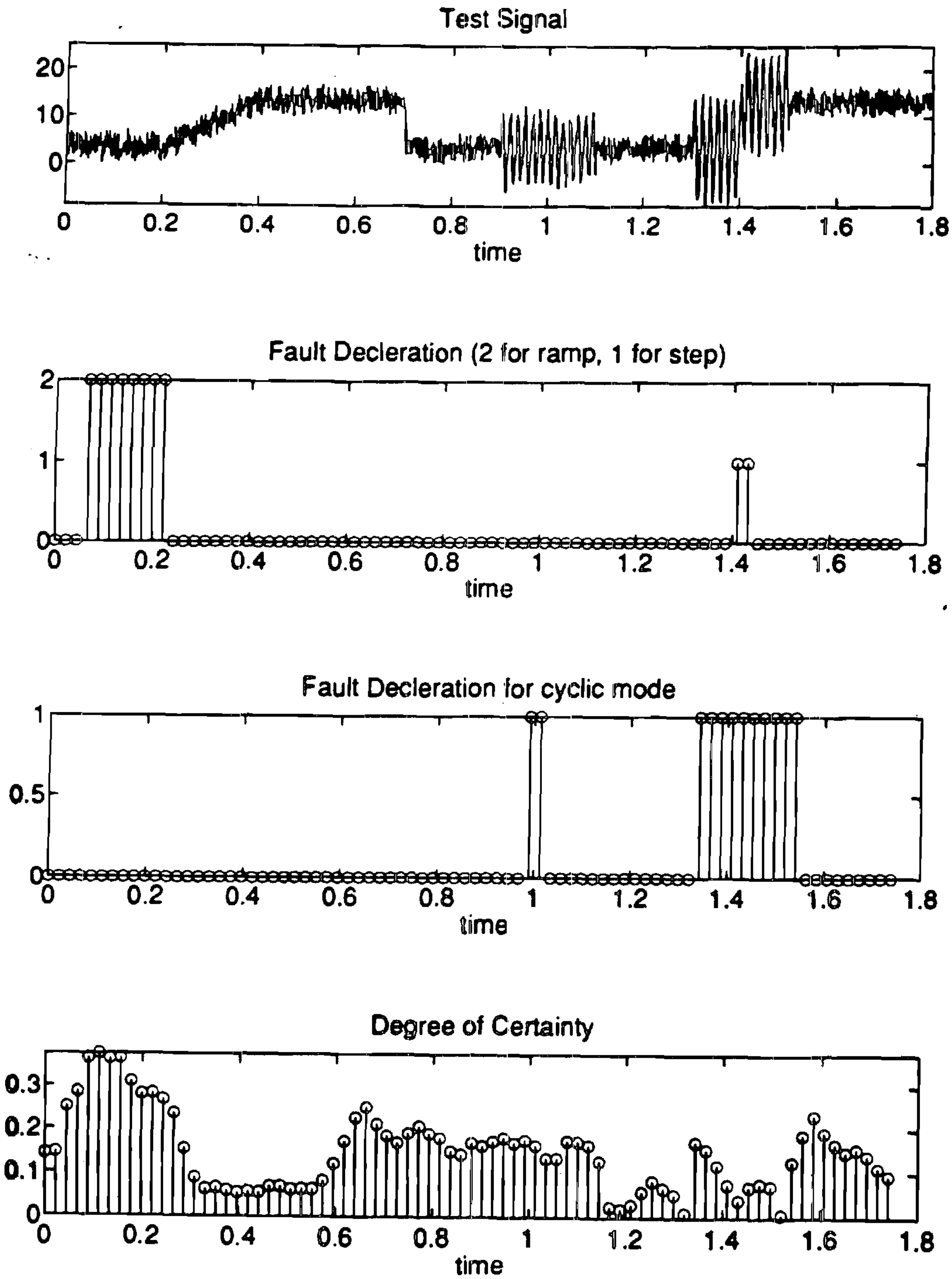
The algorithms developed were first tested on a test signal which had a blend of major anticipated faults including multiple failures. The test signal is shown below:



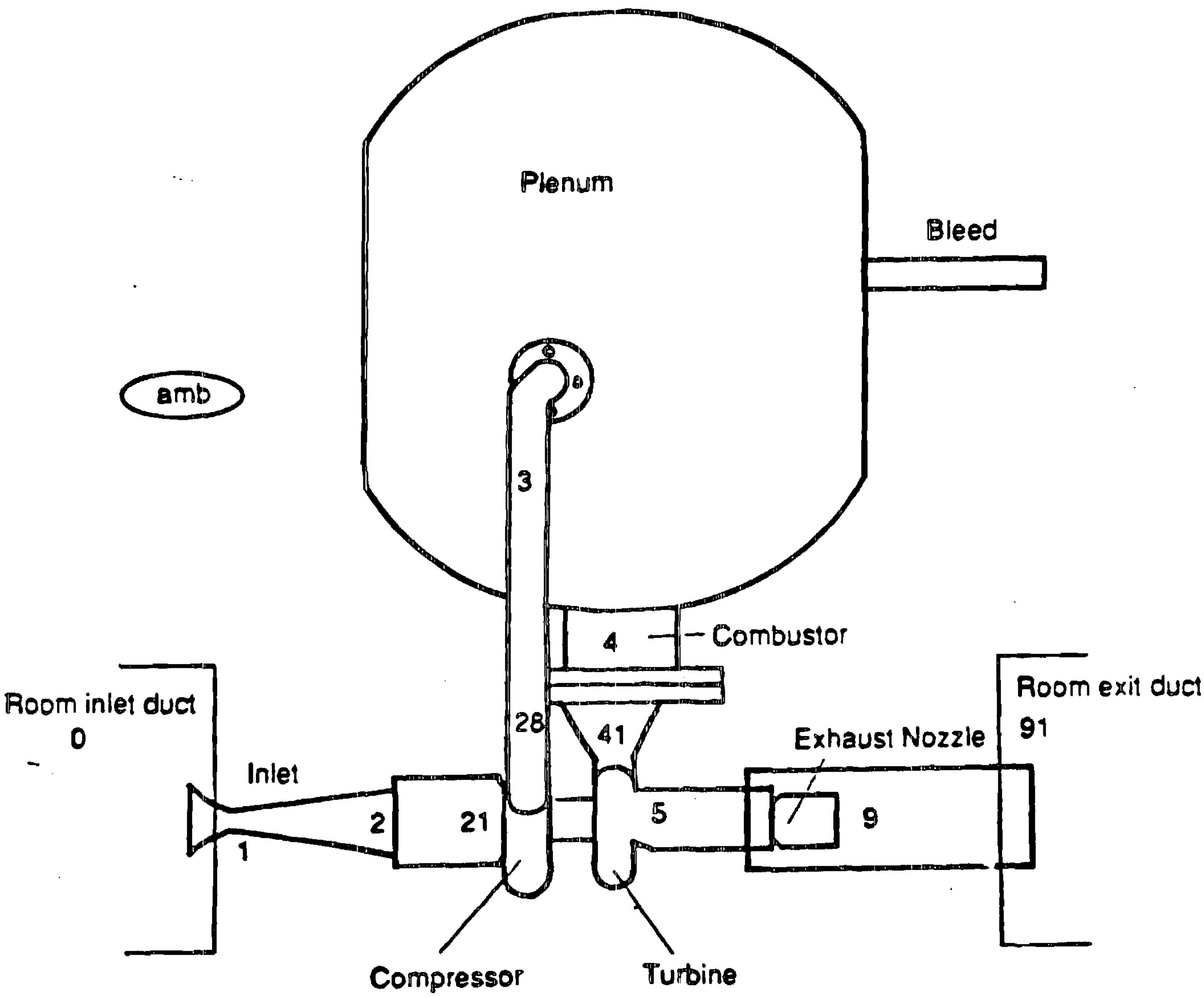
The Short Time Fourier Transform (STFT) for this signal from time 1.2 seconds to 1.6 seconds is as follows:



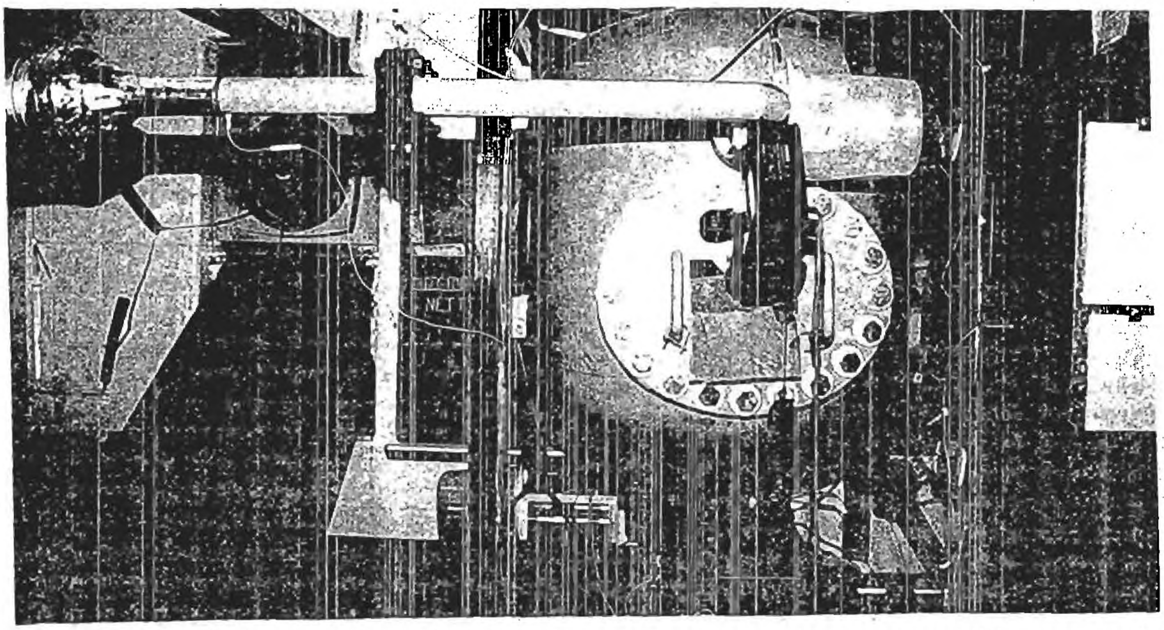
The test signal was tested for step, cyclic and drifting (ramp) faults and a degree of certainty was associated with the hypotheses H_0:No fault, H_1:Step fault, H_2:Drifting failure, H_3:Cyclic fault. This is given below



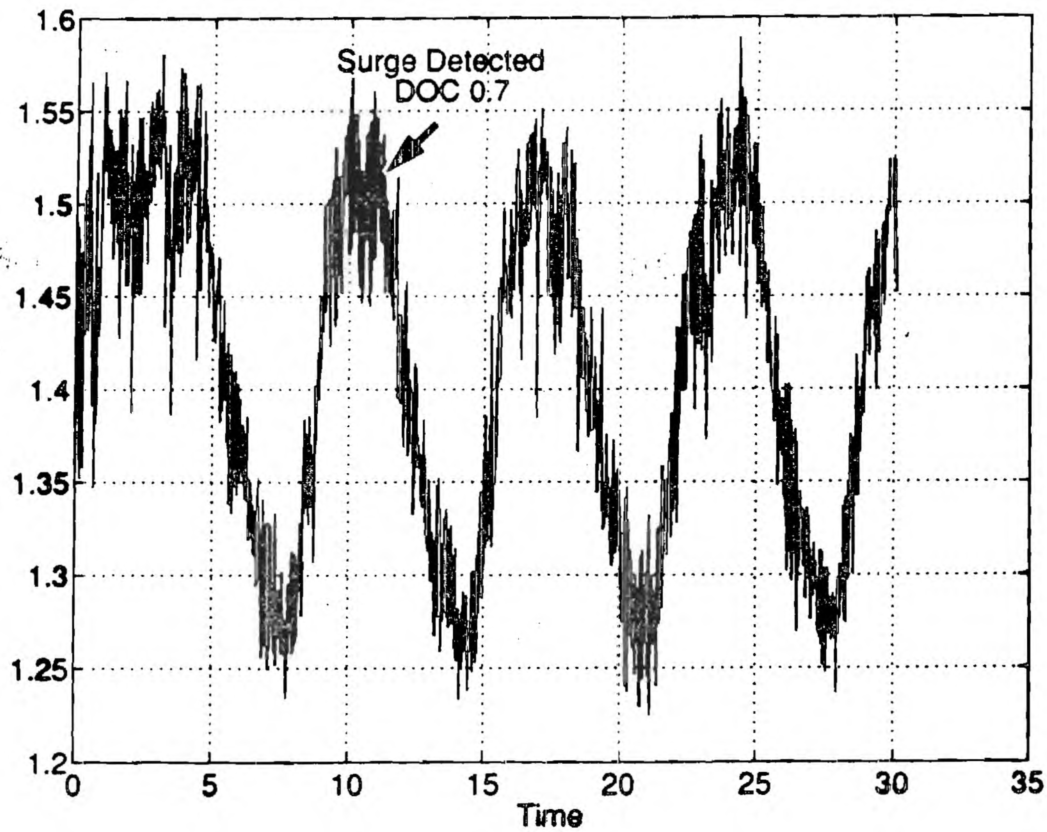
The algorithm was then tested on the actual jet engine. A diagram of test engine facility is as follows:



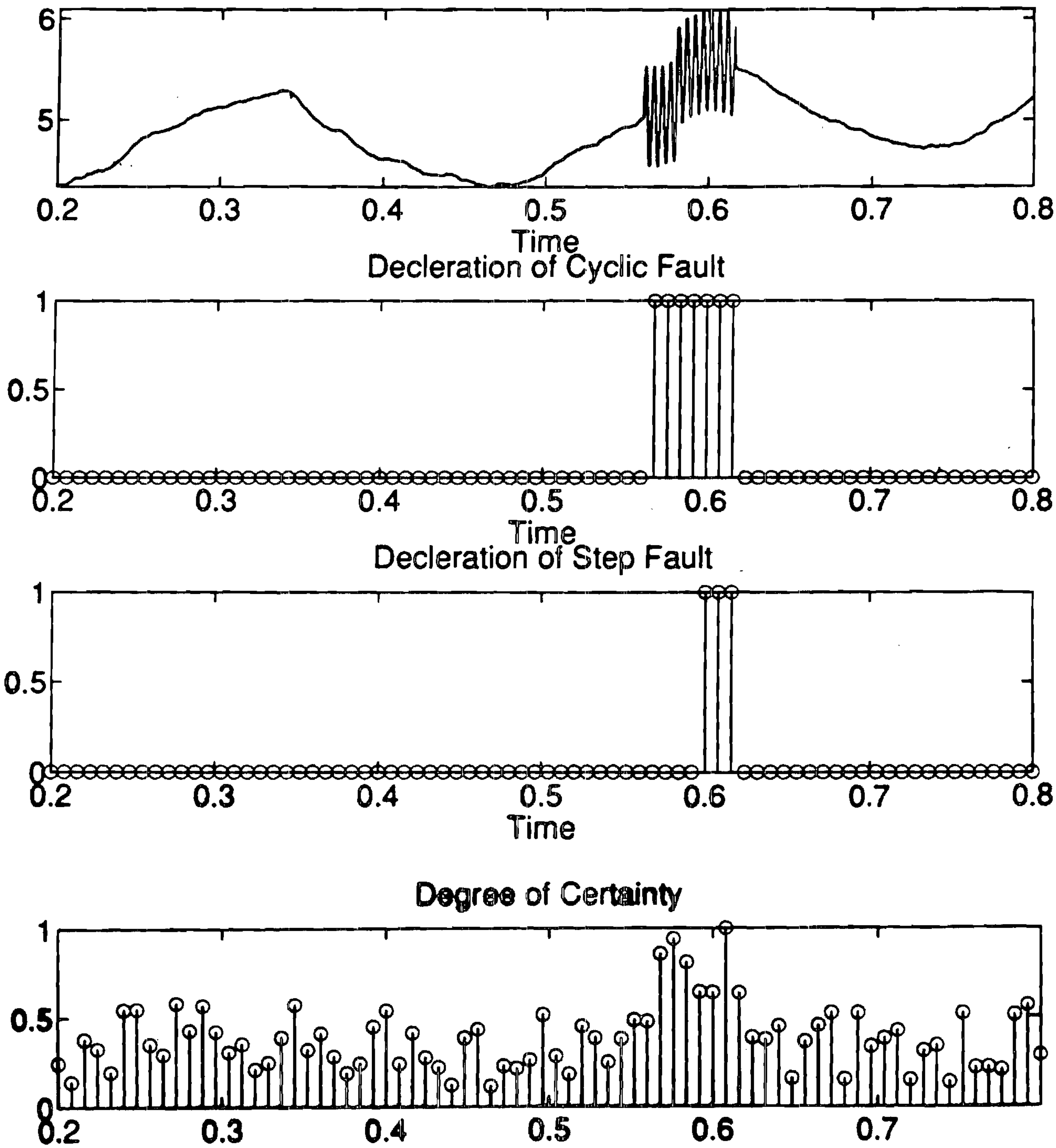
A view of the test facility is shown below



Using the pressure in the combustor for fault signature analysis the surge condition (operational failure) in the engine was detected with a degree of certainty of 0.7.

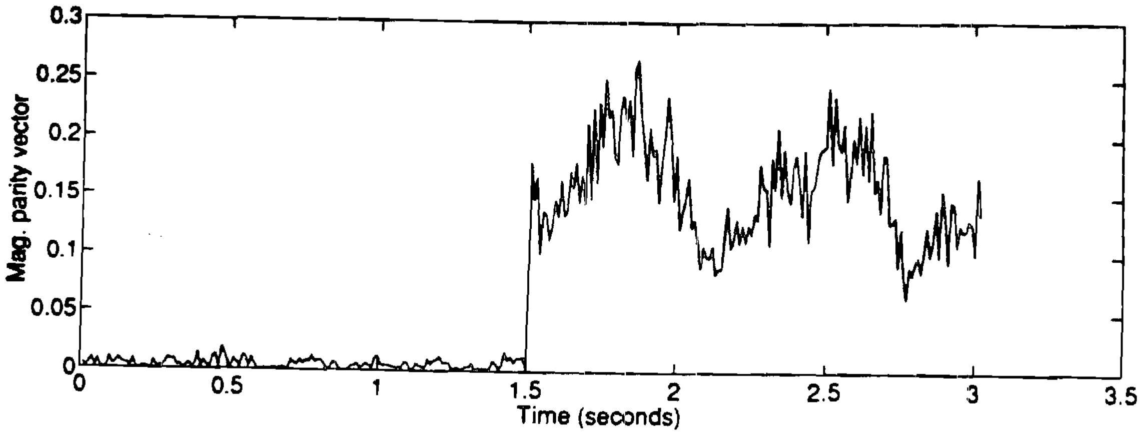


Following is an example of detection of multiple failures (Step bias and oscillatory) in the turbine temperature signal.

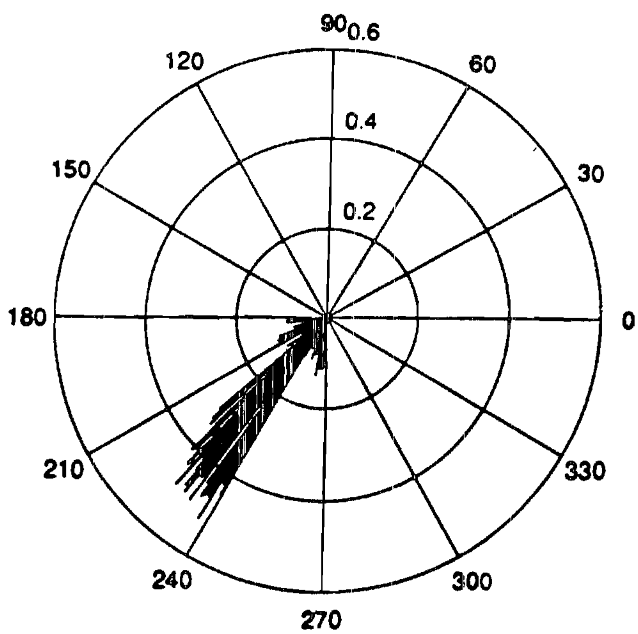


Using Parity Space approach (hardware redundancy) for fault triggering. As an example consider the dynamic pressure p3 in the engine.

Magnitude of Parity Vector due to a Bias Fault



Parity Space and Different Values of Parity Vector



Simulation results for the Test Signal

Step H :	Level of classifaction of step fault using histogram technique
Step A :	Level of classifaction of step fault using Information matrix technique
Ramp H:	Level of classifaction of ramp fault using histogram technique
Ramp A:	Level of classifaction of ramp fault using Information matrix technique
Cyclic H:	Level of classifaction of cyclic fault using histogram technique
Cyclic A:	Level of classifaction of cyclic fault using Information Matrix technique

Time	Step H	Step A	Ramp H	Ramp A	Cyclic H	Cyclic A
0.00	0.57	0.49	0.90	0.64	0.84	0.05
0.02	0.57	0.50	0.90	0.65	0.84	0.04
0.05	0.57	0.51	0.90	0.66	0.84	0.04
0.07	0.55	0.53	0.88	0.67	0.79	0.07
0.09	0.51	0.56	0.87	0.70	0.78	0.03
0.11	0.53	0.59	0.90	0.74	0.81	0.04
0.13	0.55	0.63	0.92	0.78	0.84	0.06
0.16	0.53	0.64	0.90	0.82	0.82	0.05
0.18	0.53	0.66	0.84	0.87	0.77	0.03
0.20	0.55	0.67	0.83	0.91	0.76	0.07
0.22	0.54	0.67	0.82	0.92	0.77	0.09
0.24	0.53	0.66	0.80	0.88	0.76	0.04
0.27	0.53	0.65	0.85	0.83	0.80	0.02
0.29	0.56	0.63	0.89	0.78	0.84	0.09
0.31	0.59	0.60	0.90	0.74	0.85	0.09
0.33	0.62	0.57	0.89	0.71	0.87	0.06
0.35	0.55	0.54	0.86	0.68	0.80	0.05
0.38	0.57	0.52	0.89	0.66	0.84	0.09
0.40	0.57	0.51	0.86	0.64	0.85	0.07
0.42	0.58	0.50	0.92	0.64	0.86	0.07
0.44	0.54	0.50	0.99	0.64	0.85	0.05
0.46	0.55	0.51	0.92	0.64	0.86	0.06
0.49	0.53	0.51	0.90	0.64	0.85	0.04
0.51	0.54	0.50	0.89	0.63	0.85	0.07
0.53	0.60	0.50	0.90	0.63	0.91	0.06
0.55	0.59	0.48	0.88	0.61	0.90	0.04
0.57	0.55	0.42	0.85	0.55	0.86	0.12
0.60	0.60	0.35	0.88	0.50	0.86	0.07
0.62	0.57	0.28	0.87	0.47	0.85	0.04
0.64	0.54	0.21	0.84	0.45	0.83	0.04
0.66	0.52	0.18	0.82	0.43	0.80	0.11
0.68	0.51	0.24	0.82	0.45	0.82	0.05
0.71	0.49	0.31	0.78	0.48	0.77	0.04
0.73	0.51	0.38	0.83	0.54	0.80	0.04
0.75	0.48	0.46	0.78	0.60	0.76	0.12
0.77	0.49	0.51	0.89	0.64	0.80	0.05
0.79	0.49	0.52	0.89	0.65	0.80	0.11
0.82	0.49	0.52	0.89	0.66	0.80	0.23
0.84	0.52	0.51	0.91	0.66	0.80	0.25
0.86	0.52	0.50	0.90	0.66	0.81	0.24

Time	Step H	Step A	Ramp H	Ramp A	Cyclic H	Cyclic A
0.88	0.52	0.49	0.89	0.66	0.80	0.31
0.90	0.52	0.49	0.89	0.66	0.81	0.44
0.93	0.52	0.48	0.90	0.64	0.80	0.44
0.95	0.53	0.48	0.89	0.63	0.81	0.42
0.97	0.53	0.50	0.91	0.65	0.82	0.48
0.99	0.53	0.50	0.91	0.65	0.84	0.62
1.01	0.54	0.50	0.91	0.64	0.84	0.50
1.04	0.57	0.51	0.93	0.65	0.86	0.42
1.06	0.60	0.51	0.90	0.65	0.86	0.41
1.08	0.57	0.50	0.92	0.65	0.86	0.43
1.10	0.57	0.50	0.91	0.65	0.84	0.31
1.12	0.67	0.50	0.50	0.66	0.57	0.21
1.15	0.70	0.51	0.53	0.65	0.60	0.22
1.17	0.75	0.51	0.55	0.65	0.62	0.22
1.19	0.80	0.51	0.55	0.66	0.63	0.11
1.21	0.86	0.51	0.57	0.67	0.64	0.02
1.23	0.89	0.50	0.57	0.66	0.66	0.16
1.26	0.92	0.50	0.57	0.66	0.68	0.34
1.28	0.95	0.50	0.56	0.66	0.68	0.36
1.30	0.98	0.50	0.54	0.66	0.68	0.34
1.32	1.00	0.52	0.54	0.68	0.68	0.48
1.34	0.96	0.59	0.55	0.74	0.67	0.57
1.37	0.93	0.65	0.57	0.81	0.67	0.65
1.39	0.90	0.71	0.59	0.85	0.67	0.67
1.41	0.86	0.78	0.60	0.89	0.64	0.78
1.43	0.83	0.80	0.58	0.89	0.61	0.88
1.45	0.76	0.73	0.55	0.85	0.62	0.84
1.48	0.72	0.67	0.55	0.81	0.59	0.68
1.50	0.68	0.61	0.53	0.74	0.57	0.66
1.52	0.63	0.53	0.54	0.67	0.57	0.56
1.54	0.45	0.50	0.65	0.64	0.61	0.50
1.56	0.40	0.50	0.70	0.64	0.61	0.38
1.59	0.35	0.50	0.62	0.64	0.53	0.36
1.61	0.38	0.50	0.66	0.64	0.57	0.34
1.63	0.40	0.51	0.68	0.64	0.59	0.22
1.65	0.42	0.51	0.72	0.63	0.63	0.08
1.67	0.41	0.50	0.73	0.64	0.64	0.07
1.70	0.43	0.50	0.77	0.64	0.67	0.03
1.72	0.45	0.50	0.79	0.63	0.70	0.04
1.74	0.47	0.49	0.83	0.63	0.74	0.08

Final Results After Aggregation

(Aggregation combines the outputs of the histogram method and the information matrix method)

Step :	Level of classifaction of step fault after aggregation
Ramp :	Level of classifaction of ramp fault after aggregation
Cyclic :	Level of classifaction of cyclic fault after aggregation
Fault :	fault decleration ; (1) Step, (2) Ramp, (3) Cyclic
DOC :	Degree of Certainty as calculated using Demster Shafer theory

	1	2	3			
Time	**Step**	**Ramp**	**Cyclic**	**Fault**	**Fault**	**DOC**
0.00	0.46	0.60	0.05			0.14
0.02	0.49	0.63	0.04			0.14
0.05	0.53	0.78	0.04			0.25
0.07	0.55	0.83	0.07	2		0.28
0.09	0.51	0.87	0.03	2		0.37
0.11	0.53	0.90	0.04	2		0.38
0.13	0.55	0.92	0.06	2		0.36
0.16	0.53	0.90	0.05	2		0.37
0.18	0.53	0.84	0.03	2		0.31
0.20	0.55	0.83	0.07	2		0.28
0.22	0.54	0.82	0.09	2		0.28
0.24	0.53	0.80	0.04			0.27
0.27	0.53	0.77	0.02			0.24
0.29	0.56	0.72	0.09			0.15
0.31	0.59	0.68	0.09			0.09
0.33	0.57	0.64	0.06			0.06
0.35	0.54	0.61	0.05			0.06
0.38	0.52	0.58	0.09			0.06
0.40	0.51	0.57	0.07			0.05
0.42	0.51	0.56	0.07			0.06
0.44	0.50	0.56	0.05			0.05
0.46	0.51	0.57	0.06			0.07
0.49	0.51	0.57	0.04			0.07
0.51	0.51	0.57	0.07			0.06
0.53	0.50	0.57	0.06			0.06
0.55	0.48	0.54	0.04			0.06
0.57	0.41	0.49	0.12			0.08
0.60	0.33	0.45	0.07			0.12
0.62	0.26	0.43	0.04			0.17
0.64	0.18	0.41	0.04			0.23
0.66	0.16	0.41	0.11			0.25
0.68	0.22	0.43	0.05			0.21
0.71	0.29	0.47	0.04			0.18
0.73	0.37	0.54	0.04			0.17
0.75	0.44	0.63	0.12			0.19
0.77	0.49	0.70	0.05			0.21
0.79	0.49	0.68	0.11			0.19
0.82	0.49	0.67	0.23			0.18
0.84	0.52	0.67	0.25			0.15

Time	Step	Ramp	Cyclic	Fault	Fault	DOC
0.86	0.50	0.64	0.24			0.14
0.88	0.47	0.63	0.31			0.17
0.90	0.48	0.64	0.44			0.16
0.93	0.44	0.61	0.44			0.17
0.95	0.44	0.63	0.42			0.18
0.97	0.49	0.66	0.48			0.17
0.99	0.49	0.67	0.62		1	0.17
1.01	0.49	0.65	0.50		1	0.16
1.04	0.52	0.65	0.42			0.13
1.06	0.53	0.66	0.41			0.13
1.08	0.49	0.67	0.43			0.17
1.10	0.51	0.68	0.31			0.17
1.12	0.52	0.50	0.21			0.16
1.15	0.53	0.53	0.22			0.13
1.17	0.53	0.55	0.22			0.02
1.19	0.54	0.55	0.11			0.02
1.21	0.54	0.57	0.02			0.03
1.23	0.52	0.57	0.16			0.06
1.26	0.49	0.57	0.34			0.08
1.28	0.49	0.56	0.36			0.07
1.30	0.49	0.54	0.34			0.05
1.32	0.53	0.54	0.48			0.01
1.34	0.60	0.55	0.57		1	0.17
1.37	0.67	0.57	0.65		1	0.15
1.39	0.74	0.59	0.67		1	0.12
1.41	0.81	0.60	0.78	1	1	0.07
1.43	0.82	0.58	0.88	1	1	0.04
1.45	0.75	0.55	0.84		1	0.07
1.48	0.69	0.55	0.68		1	0.08
1.50	0.62	0.53	0.66		1	0.07
1.52	0.54	0.54	0.56		1	0.01
1.54	0.45	0.58	0.50		1	0.13
1.56	0.40	0.58	0.38			0.19
1.59	0.35	0.58	0.36			0.23
1.61	0.38	0.58	0.34			0.19
1.63	0.40	0.57	0.22			0.17
1.65	0.42	0.57	0.08			0.15
1.67	0.41	0.57	0.07			0.16
1.70	0.43	0.57	0.03			0.14
1.72	0.45	0.56	0.04			0.11
1.74	0.47	0.56	0.08			0.09

Presentations and Technical Contributions

1. In October 1990, Dr. Vachtsevanos presented a paper on fuzzy logic control at the Instrument Society of American 1990 Conference in New Orleans, Louisiana.

2. In December 1990, Dr. Vachtsevanos presented two papers at the 29th IEEE Conference on Decision and Control in Honolulu, Hawaii and participated as a panelist in a panel discussion on intelligent control.

3. In January 1991, Dr. Vachtsevanos participated in an Advanced Process Control Workshop at the invitation of Honeywell, Inc. in Phoenix, Arizona.

4. Dr. Vachtsevanos attended the kickoff meeting for the NAVSEA/DARPA project in Washington, DC on 1/30/91 and made a presentation on fault-tolerant control work at Georgia Tech.

5. In April 1991, Dr. Vachtsevanos attended the annual ONR review meeting and presented the results of our research activity.

6. In May 1991, Dr. Vachtsevanos chaired a session on Process Control and presented a paper on fuzzy logic control at the 1991 IEEE Conference on Instrumentation and Measurements Technology.

7. In June 1991, Dr. Vachtsevanos presented a paper at the 1991 Automatic Control Conference.

8. On June 26, 1991, Dr. Vachtsevanos attended the DARPA review meeting at MIT.

9. In June 1991, Dr. Vachtsevanos chaired a technical session on Intelligent Manufacturing and presented a paper at the EURISCON '91 Conference.

The following technical papers authored or co-authored by the research team were published during the reporting period:

1. H. Kang and G. Vachtsevanos, "Nonlinear Fuzzy Control Based on the Vector Fields of the Phase Portrait Assignment Algorithm," **Proceedings of the 1990 American Control Conference, San Diego, California, pp. 1479-1484, 1990.**

2. R. Youngblood, J. Curtis, and G. Vachtsevanos, "Fuzzy Logic Control: New Approaches and Applications," **Proceedings of ISA 90 Conference**, New Orleans,

Louisiana, pp. 1129-1136, 1990.

3. H. Kang and G. Vachtsevanos, "Fuzzy Hypercubes: Linguistic Learning/Reasoning System for Intelligent Control and Identification," to appear in International Journal of Intelligent and Robotic Systems.

4. G. Vachtsevanos, H. Kang, J. Cheng and I. Kim, "On the Detection and Identification of Axial Flow Compressor Instabilities," to appear in American Institute of Aeronautics and Astronautics, Journal of Guidance, Control, and Dynamics.

5. G. Vachtsevanos, "Fuzzy Logic Control Applications in Textile Manufacturing," IEEE Instrumentation and Measurements Technology Conference, IMTC/91, Atlanta, GA, May 1991.

6. H. Kang and G. Vachtsevanos, "An Intelligent Strategy to Robot Coordination and Control," **Proc. of 29th IEEE Conference on Decision and Control**, pp. 2208-2213, 1990.

7. R.C. Arkin and G. Vachtsevanos, "Qualitative Fault Propagation in Complex Systems," **Proc. of 29th IEEE Conference on Decision and Control**, pp. 1509-1510, 1990.

8. B.H. Wang and G. Vachtsevanos, "Fuzzy Logic Control: A Systematic Control Methodology," Proc. of IEEE Conference on Decision and Control, Brighton, England, Dec. 1991.

9. H. Kang and G. Vachtsevanos, "Fuzzy Hypercubes: Linguistic Learning/Reasoning Systems for Intelligent Control and Identification," **Proc. of IEEE Conference on Decision and Control**, Brighton, England, Dec. 1991.

10. B.H. Wang and G.J. Vachtsevanos, "Fuzzy Dynamic Systems: An Application of Fuzzy Associative Memory with an Intermediate Layer," **Proc. of Automatic Control Conference '91**, pp. 12-13, 1991.

11. G. Vachtsevanos and P. Groumpos, "Fault-Tolerant Design Issues for A Manufacturing Cell," European Robotics and Intelligent Systems Conference, June 27-28, 1991.

12. H. Kang, J. Cheng, I. Kim and G. Vachtsevanos, "An Application of Fuzzy Logic and Dempster-Shafer Theory to Failure Detection and Identification," Proc. of IEEE Conference on Decision and Control, Brighton, England, Dec. 1991.

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